Kentucky Academic Standards for Mathematics: Conceptual Category Functions

Functions Overview

Interpreting Functions	Building Functions	Linear, Quadratic and Exponential Models	Trigonometric Functions
• Understand the concept of a function and use function notation. • Interpret functions that arise in applications in terms of the context. • Analyze functions using different representations.	• Build a function that models a relationship between two quantities. • Build new functions from existing functions.	• Construct and compare linear, quadratic and exponential models and solve problems. • Interpret expressions for functions in terms of the situation they model.	• Extend the domain of trigonometric functions using the unit circle. • Model periodic phenomena with trigonometric functions. • Prove and apply trigonometric identities.

Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice and specific modeling standards appear throughout the high school standards indicated by a star symbol (★). The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to all standards in that group.

Plus (+) Standards: Additional mathematics concepts students should learn in order to take advanced courses such as calculus, advanced statistics or discrete mathematics are indicated by (+) symbol.

Functions-Interpreting Functions	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Understand the concept of a function and use function notation.	
Standards	**Clarifications**
KY.HS.F.1 Understand properties and key features of functions and the different ways functions can be represented. a. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. b. Using appropriate function notation, evaluate functions for inputs in their domains and interpret statements that use function notation in terms of a context. c. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities and sketch graphs showing key features given a verbal description of the relationship. d. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. e. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). **MP.2**, **MP.4**, **MP.7**	a. When describing relationships between quantities, the defining characteristic of a function is the input value determines the output value or, equivalently, the output value depends upon the input value. In some situations where two quantities are related, each can be viewed as a function of the other. c. A function is often described and understood in terms of the output behavior, or over what input values is it increasing, decreasing, or constant. Important questions include, “For what input values is the output value positive, negative, or 0? What happens to the output when the input value gets very large in magnitude?” Graphs become useful representations for understanding and comparing functions because these behaviors are often easy to see in the graphs of functions. Key features include, but are not limited to: intercepts; intervals where the function is increasing, decreasing, or remaining constant; relative maxima and minima; symmetries; end behavior; periodicity. e. Students compare characteristics from various representations for one type of family of function at a time. For quadratics, students might determine which function has the larger maximum when given two different representations of quadratic functions.

Standards	Clarifications
KY.HS.F.2 Recognize that arithmetic and geometric sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. **MP.7**, **MP.8**	Sequences are functions with a domain consisting of whole numbers.

Attending to the Standards for Mathematical Practice
Students reason quantitatively about the relationship between domain and range of functions across abstract and concrete representations (**MP.2**). Students look closely to discern arithmetic and geometric relationships as patterns with additive and multiplicative changes, respectively **(MP.7)**. Students notice the regularity in the pattern to write a general formula for arithmetic or geometric sequence (**MP.8**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Functions-Interpreting Functions	
Standards for Mathematical Practice	
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Cluster: Interpret functions that arise in applications in terms of the context.

Standards	Clarifications
KY.HS.F.3 Understand average rate of change of a function over an interval. a. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. b. Estimate the rate of change from a graph. ★ **MP.2**, **MP.4**	The rate of change over an interval is equivalent to the slope between the endpoints of the interval. For linear functions, the rate of change is constant, over all intervals. However, for nonlinear functions, the average rate of change may vary depending on the interval.

Attending to the Standards for Mathematical Practice

Students make sense of the rate of change, recognizing it captures how the input and the output of a function vary simultaneously (**MP. 2**). For example, students explain that the rate of change for nonlinear functions is not constant. Students use equations, tables and graphs to analyze rate of change in applied and mathematical contexts (**MP.4**).

Functions-Interpreting Functions	
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Cluster: Analyze functions using different representations.	
Standards	**Clarifications**
KY.HS.F.4 Graph functions expressed symbolically and show key features of the graph, with and without using technology (computer, graphing calculator). ★ a. Graph linear and quadratic functions and show intercepts, maxima and minima. b. Graph square root, cube root and absolute value functions. c. Graph polynomial functions, identifying zeros when suitable factorizations are available and showing end behavior. d. Graph exponential and logarithmic functions, showing intercepts and end behavior. e. (+) Graph trigonometric functions, showing period, midline and amplitude. f. (+) Graph piecewise functions, including step functions. g. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available and showing end behavior. **MP.4**, **MP.5**	Within a family, the functions often have commonalities in the shapes of their graphs and in the kinds of features important for identifying and describing functions. This standard indicates the function families in students' repertoires, detailing which features are required for several key families. Students demonstrate fluency with linear, quadratic and exponential functions, including the ability to graph without using technology. In other function families, students graph simple cases without technology and more complex ones with technology.
KY.HS.F.5 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Identify zeros, extreme values and symmetry of the graph within the context of a quadratic function.	a. Quadratic functions provide a rich playground for developing this ability, since the three principal forms for a quadratic expression (expanded, factored and completed square) each give insight into different aspects of the function. b. Students examine real-world situations with constant multiplicative change, represented as expressions, such as growth or decay.

Standards	Clarifications
b. Use the properties of exponents to interpret expressions for exponential functions and classify the exponential function as representing growth or decay. **MP.3**, **MP.6**	

Attending to the Standards for Mathematical Practice
Students use graphs to answer questions and/or make predictions for a given context (**MP. 4**). Students use technology to explore concepts of function families and show key features of the graph (**MP. 5**). Students compare and contrast different characteristics of functions to connect features of the graph with different real-world contexts (**MP.6**). Students manipulate expressions, being careful to preserve equivalence and describe why a particular expression provides insights into the function (**MP.3, MP.6**).

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Functions-Building Functions	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Build a function that models a relationship between two quantities.	
Standards	**Clarifications**
KY.HS.F.6 Write a function that describes a relationship between two quantities. ★ a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. c. (+) Compose functions. **MP.4**, **MP.7**	b. Use real-world examples when appropriate. c. Consider contextual examples for composition functions, such as, if $T(y)$ is the temperature in the atmosphere as a function of height and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.
KY.HS.F.7 Use arithmetic and geometric sequences to model situations and scenarios. a. Use formulas (explicit and recursive) to generate terms for arithmetic and geometric sequences. b. Write formulas to model arithmetic and geometric sequences and apply those formulas in realistic situations. ★ c. (+) Translate between recursive and explicit formulas. **MP.4**, **MP.8**	Examples include, but are not limited to: • calculating mortgages • drug dosages • simple interest
Attending to the Standards for Mathematical Practice	
For real-world problems, students formulate the problem, make assumptions, define variables and create functions to model the situation (**MP.4**). Students notice the regularity in real-world growing patterns and use these insights to write a general formula to describe arithmetic or geometric sequences (**MP.8**).	

Functions- Building Functions

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Build new functions from existing functions.

Standards	Clarifications
KY.HS.F.8 Understand the effects of transformations on the graph of a function. a. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$ and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. b. Experiment with cases and illustrate an explanation of the effects on the graph using technology. **MP.3**, **MP.5**	a. Mastery of this standard includes recognizing even and odd functions from their graphs and algebraic expressions.
KY.HS.F.9 Find inverse functions. a. Given the equation of an invertible function, find the inverse. b. (+) Verify by composition that one function is the inverse of another. c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. d. (+) Produce an invertible function from a non-invertible function by restricting the domain. **MP.2**, **MP.6**	a. Students can complete the process of finding the inverse when given an equation of a function that is invertible. b-d. Students need a formal sense of inverse functions. Students understand a function and its inverse describe the exact same relationship but in different ways.
KY.HS.F.10 Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents with the use of technology. **MP.1**, **MP.7**	Students can use inverses of simple logarithmic and exponential equations in order to solve those equations. The inverse relationship between logarithmic and exponential functions is special in that each function's inverse is also a function

Attending to the Standards for Mathematical Practice

Students use technology to explore how changing the value of k impacts the graph of the function (**MP.5**). Students use the graphical representation to create plausible arguments about the effects of transformations, instead of relying on computational rules (**MP.3**).

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Functions-Linear, Quadratic and Exponential Functions	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Construct and compare linear, quadratic and exponential models and solve problems.	
Standards	**Clarifications**
KY.HS.F.11 Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Recognize and justify that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. **MP.3**, **MP.8**	Linear functions have the same average rate of change over same-sized intervals; the same value is added to the output over each interval. In contrast, the outputs of exponential functions grow or decay by the same percent over same-sized intervals; the same value is multiplied by the output over each interval.
KY.HS.F.12 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). **MP.7**, **MP.8**	Students construct functions with and without technology.
KY.HS.F.13 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. **MP.7**, **MP.8**	Students compare functions by focusing on how the output values change over intervals of equal length. Even though a linear function may initially be increasing faster than an exponential function, an increasing exponential function always eventually exceeds an increasing linear function.
Attending to the Standards for Mathematical Practice	
Students reason about particular characteristics of linear, quadratic and exponential functions, for example comparing how rates of change across different types of functions (**MP.3**). Students recognize families of functions in a more general sense to discern that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically (**MP.8**).	

Functions- Linear, Quadratic and Exponential Functions	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Interpret expressions for functions in terms of the situation they model.	
Standards	**Clarifications**
KY.HS.F.14 Interpret the parameters in a linear or exponential function in terms of a context. **MP.1**, **MP.2**	More than just substituting values into a given formula, this requires students to understand how changing specific parameters will change the function output. An example of this with an exponential function ($f(x) = a \bullet b^x$) might be changing the "b" from a number greater than 1 to a number between 0 and 1. Students should recognize this creates a decay problem instead of a growth problem. Similarly, changing the "a" parameter creates corresponding changes to the graph and has different implications within the realistic context.
Attending to the Standards for Mathematical Practice	
Students quantitatively reason to consider the units, limitations and parameters in linear and exponential functions in terms of a context (**MP.2**). When solving problems, students ask themselves, "Does this make sense?" (**MP.1**).	

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Functions-Trigonometric Functions	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Extend the domain of trigonometric functions using the unit circle.

Standards	Clarifications
KY.HS.F.15 (+) Understand the relationship of radian measure of an angle to its arc length. **MP.1**, **MP.6**	Understanding radian measure of an angle as arc length on the unit circle enables students to build on their understanding of trigonometric ratios associated with acute angles and to explain how these ratios extend to trigonometric functions whose domains are included in the real numbers.
KY.HS.F.16 (+) Understand and use the unit circle. a. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. b. Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π /4 and π /6 and use the unit circle to express the values of sine, cosine and tangent for π –*x*, π +*x* and 2 π –*x* in terms of their values for *x*, where *x* is any real number. c. Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. **MP.7**, **MP.8**	This standard is sometimes called "unwrapping the unit circle." For each function, the angle θ is represented by values on the horizontal axis and the resulting outputs are graphed on the vertical axis. c. Students understand symmetry exists within the unit circle for paired reference angles: sin (-θ) = -sin(θ), so sine is an odd function; and cos (-θ) = cos(θ), so cosine is an even function.

Functions-Trigonometric Functions	
Standards for Mathematical Practice	
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Cluster: Model periodic phenomena with trigonometric functions.	
Standards	**Clarifications**
KY.HS.F.17 (+) Choose trigonometric functions to model periodic phenomena with specified period, midline and amplitude. ★ **MP.4**, **MP.5, MP.6**	A function is described as sinusoidal or is called a sinusoid if it has the same shape as the sine graph, for example, has the form $f(t) = A + B(\sin Ct + D)$. Many real-world phenomena can be approximated by sinusoids, including sound waves, oscillation on a spring, the motion of a pendulum, tides and phases of the moon. Because $\sin(t)$oscillates between 1 and 1, $A + B(\sin Ct + D)$will oscillate between $A - B$ and $A + B$. Thus, $y = A$ is the midline and B is the amplitude of the sinusoid. Students can obtain the frequency of f: the period of $\sin(t)$ is 2π, so (knowing the effect of multiplying t by C) the period of $\sin(Ct)$ is $2\pi/C$ and the frequency is its reciprocal. When modeling, students have the sense that C affects the frequency and that C and D together produce a phase shift, but finding a correct solution might involve technological support, except in simple cases.
KY.HS.F.18 (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. **MP.2**, **MP.3**	Students experience restricting the domain of a function so it has an inverse. For trigonometric functions, a common approach to restricting the domain is to choose an interval on which the function is always increasing or always decreasing.

Standards	Clarifications
KY.HS.F.19 (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology and interpret them in terms of the context. ★ **MP.4**, **MP.5**	Include $\sin^{-1} x, \cos^{-1} x \ and \ \tan^{-1} x$.

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Functions-Trigonometric Functions	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Prove and apply trigonometric identities.	
Standards	**Clarifications**
KY.HS.F.20 (+) Proving identities and formulas within the context of trigonometry. a. Prove the Pythagorean identity and use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle. b. Prove the addition and subtraction formulas for sine, cosine and tangent and use them to solve problems. **MP.3**, **MP.7**	In the unit circle, the x-value is the cosine and the y-value represents the sine. Since the hypotenuse of any right triangle on the unit circle is 1, the Pythagorean relationship of $x^2 + y^2 = 1$ holds. Students connect the Pythagorean Theorem in geometry and the study of trigonometry to understand this relationship.

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