AP PreCalculus Cover Sheet

Content Area: **Mathematics**
Course(s):
Time Period:
Length: **Full Year**
Status: **Published**

Course Overview

AP Precalculus is the fourth course in the honors college preparatory sequence that centers on functions modeling dynamic phenomena. This course is designed for students who have the appropriate background to understand the concepts and techniques in advanced college preparatory mathematics. The topics covered include logarithms, trigonometric functions, and identities, solving trigonometric equations, applications involving triangles, inverse trigonometric functions, trigonometric addition formulas, advanced graphing techniques, polar coordinates, vectors, sequences and series, and limits. This course is intended to prepare students for post-secondary education. It emphasizes higher-level mathematical thinking necessary to pursue the study of Advanced Placement Calculus and/or Statistics.

Course Name, Length, Date of Revision and Curriculum Writer

AP Precalculus Curriculum, Entire Year, 02/16/24, Manmeet Sachar and Melissa Prignoli

Unit 1 Polynomial and Rational Functions

Content Area: **Mathematics**
Course(s): **AP PreCalculus**
Time Period: **1st Marking Period**
Length: **7 weeks**
Status: **Published**

Section Title

Unit 1: Polynomial and Rational Functions

Enduring Understandings

- Identify functions, and find their domain and range.
- Use function notation and evaluate functions abstractly and graphically.
- Use functions to model and solve real-life problems.
- Find the zeros of a function
- Analyze functions by determining intervals on which they are increasing, decreasing, or constant and determining their relative minimum and relative maximum values
- Determine the average rate of change for linear, quadratic, and other polynomial functions
- Identify key characteristics of polynomial functions related to rates of change
- Analyze polynomial functions by finding all zeros including complex ones using the Fundamental Theorem of Algebra, factoring, and graphing them, both manually and using technology.
- Identify even and odd functions.
- Analyze polynomial and rational functions by finding all zeros, and asymptotes (horizontal, vertical, and slanted) and graphing them, both manually and using technology.
- Rewrite polynomial and rational expressions in equivalent forms
- Use long division and synthetic division to divide polynomials.
- Use the Binomial Theorem and Pascal's Triangle to find binomial coefficients and write binomial expansions

- Recognize graphs of common functions, and use vertical, and horizontal shifts, reflections, and non-rigid transformations to graph them
- Use functions to model and solve real life problems
- Use polynomial and rational functions to model and solve real-life problems including ones with minimum and maximum values.

Summary of the Unit

In Unit 1, students develop an understanding of two key function concepts while exploring polynomial and rational functions. The first concept is covariation, or how output values change in tandem with changing input values. The second concept is rates of change, including average rate of change, rate of change at a point, and changing rates of change. The central idea of a function as a rule for relating two simultaneously changing sets of values provides students with a vital tool that has many applications, in nature, human society, and business and industry. For example, the idea of crop yield increasing but at a decreasing rate or the efficacy of medicine decreasing but at an increasing rate are important understandings that inform critical decisions.

Essential Questions

- How can you determine if a relation is a function?
- How are functions and their graphs related?
- How can technology be used to investigate the properties of families of functions and their graphs?
- What does the degree of a polynomial tell you about its related polynomial function?
- For a polynomial function, how are factors, zeros, and x-intercepts related?
- For a polynomial function, how are factors and roots related?
- How do the characteristics of graphs relate to their corresponding equations?
- How can algebra help us get information about a graph from an equation?
- How can I identify the characteristics of a rational function?
- How are rational functions related to each other and to inverse functions?
- What does the degree of a polynomial tell you about its related polynomial function?
- How do you divide a polynomial by another polynomial and use polynomial division to find the rational and real zeros of polynomials?
- How can you rewrite polynomial and rational exponents and write in equivalent form?
- How can you use the Binomial Theorem to write binomial expansions?
- Given a verbal description, how would you sketch a graph of this function?
- How do you write equations and draw graphs for the simple transformations of functions?

- How do rational functions model real-world problems and their solutions?
- How do polynomial functions model real-world problems and their solutions?

Summative Assessment and/or Summative Criteria

Required District/State Assessments

- SGO Pre Assessment
- SGO Post Assessment

Suggested Formative/Summative Classroom Assessments

- Describe Learning Vertically
- Identify Key Building Blocks
- Make Connections (between and among crucial building blocks)
- Short/Extended Constructed Response Items
- Multiple-Choice Items (where multiple answer choices may be correct)
- Drag and Drop Items
- Use of Equation Editor
- Quizzes
- Journal Entries/Reflections/Quick-Writes
- Accountable talk
- Projects
- Portfolio
- Observation
- Graphic Organizers/ Concept Mapping
- Presentations
- Teacher-Student and Student-Student Conferencing
- AP Classroom – Assessments created with specific sections outlined.
- WebAssign Problem Sets WebAssign Instructor Help

- Homework
- Students will take formal assessments, such as tests and quizzes, to assess knowledge of concepts learned throughout the unit.
- Students will also demonstrate mastery through various assessment criteria included in the unit such as do nows, exit slips, graded classwork activities and assignments, and/or projects.

Resources

- AP Daily Videos: Section 1.1 - 1.2
- AP Daily Videos: Section 1.2 - 1.4
- AP Daily Videos: Section 1.5
- AP Daily Videos: Section 1.6-1.10
- AP Daily Videos: Section 1.11
- AP Daily Videos: Section 1.12
- AP Daily Videos: Section 1.13-1.14
- AP Precalculus Course Overview: https://apcentral.collegeboard.org/courses/ap-precalculus/course
- AP Precalculus Course and Exam Description: https://apcentral.collegeboard.org/media/pdf/ap-precalculus-course-and-exam-description.pdf
- AP Precalculus Practice Exam: https://apcentral.collegeboard.org/media/pdf/ap-precalculus-practice-exam-multiple-choice-section.pdf
- AP Precalculus Classroom Resources: https://apcentral.collegeboard.org/courses/ap-precalculus/classroom-resources
- Classpad.net (Casio): https://classpad.net/us/
- Desmos: https://www.desmos.com/
- Geogebra: https://www.geogebra.org/?lang=en
- Math Open Reference: https://www.mathopenref.com/
- TI Education (Texas Instruments): https://education.ti.com/en
- WolframAlpha: https://www.wolframalpha.com/

- Wolfram MathWorld: https://mathworld.wolfram.com/
- Khan Academy: https://www.khanacademy.org/math/precalculus
- Digital Mathematics Word Wall: http://www.mathwords.com/index_adv_alg_precal.htm
- Extra Notes for Pre-Calculus Content: https://sites.google.com/a/evergreenps.org/ms-griffin-s-math-classes/updates
- Review Documents for Pre-Calculus: https://sites.google.com/site/dgrahamcalculus/trigpre-calculus/trig-pre-calculus-worksheets
- Pre-Calculus IXL Topics and Resources: https://www.ixl.com/math/precalculus
- Classroom Challenges to Support Teachers in Formative Assessments: http://map.mathshell.org/materials/lessons.php?gradeid=24
- Applications of Function Models: https://www.ck12.org/algebra/Applications-of-Function-Models/lesson/Applications-of-Function-Models-BSC-ALG/?referrer=featured_content
- Statistics Education Web (STEW). http://www.amstat.org/education/STEW/
- The Data and Story Library (DASL). http://lib.stat.cmu.edu/DASL/
- WebAssign Instructor Resources
- WebAssign Student Resources
- Cengage Learning: PreCalculus with Limits - A Graphing Approach, Sixth Edition

Unit Plan

Topic/Selection Timeframe	General Objectives	Instructional Activities	Benchmarks/Assessments	Standards
1.1 - Change in Tandem (2 days)	SWBAT describes how the input and output values of a function vary together by comparing function values. SWBAT constructs a	Students will analyze how the input and the output values of a function, expressed in different forms, vary according to its function rule.	Circulate and monitor student progress as they are working on classwork. Have students complete problems at the board.	2.B 3.A

	graph representing two quantities that vary with respect to each other.	Students will determine intervals of increase and decrease over an interval of its domain. Students will determine if a function is concave up or down by analyzing the graph. Students will identify the zeros of function both with and without a graphing calculator.	Through questioning students will be able to identify information from graphical, numerical, analytical, and verbal representations or construct a model with and without technology Closure	
1.2 - Rates of Change (1 day)	SWBAT compare the rates of change at two points using average rate of change. SWBAT describe how two quantities vary together at different points and over different intervals of a function.	Students will recall that the slope of a line is the rate of change of the function. If the graph of a function is not a straight line, students will use the terminology average rate of change. Students will calculate the average rate of change from graphical, numerical, analytical, and verbal representations.	Circulate and monitor student progress as they are working on classwork. Have students complete problems at the board. Through questioning students will be able to calculate average rate of change from graphical, numerical, analytical, and verbal representations or construct a model with and without technology. Closure	2.A

1.3 - Rates of Change in Linear and Quadratic Functions (2 days)	SWBAT determine the average rates of change for linear and quadratic sequences and functions. SWBAT determine the change of average rates of change for linear and quadratic functions ,	Students will find the average rate of change over a closed interval. Students will find the slope of the secant line over a closed interval [a, b] from the point (a, f(a)) to (b, f(b)). Students will use average rates of change to determine concavity. Students will write a linear equation for consecutive equal length intervals that represent the average rate of change of a quadratic function.	Circulate and monitor student progress as they are working on classwork. Have students complete problems at the board. Through questioning students will be able to calculate average rate of change from graphical, numerical, analytical, and verbal representations or construct a model with and without technology. Closure	3.B 3.C
1.4 - Polynomial Functions and Rates of Change (3 days)	SWBAT identify key characteristics of polynomial functions related to rates of change.	Students will identify the turning points of a polynomial. Students will find relative and absolute extrema of a polynomial. Students will describe end behavior of a polynomial related to even/odd functions.	Circulate and monitor student progress as they are working on classwork. Have students complete problems at the board. Through questioning students will be able to identify key characteristics of polynomials related to rates of change.	2.A 3.A

		Students will find points of inflection by using key characteristics.	Closure Quiz topics 1.1 - 1.4	
1.5 - Polynomial and Complex Zeros (2 days)	SWBAT to identify key characteristics of a polynomial function related to its zeros when suitable factorizations are available or with technology. SWBAT determine if a polynomial function is even or odd .	Students will use long division and synthetic division to find zeros of polynomials. Students will factor polynomials over real and complex numbers. Students will understand that if a linear factor (x - a) is repeated n times, the corresponding zero of the polynomial has multiplicity n. Students will use the Conjugate Root Theorem to find the zeros of polynomials. Students will find the degree of a polynomial using successive differences of output values over equal interval input values.	Circulate and monitor student progress as they are working on classwork. Have students complete problems on the board. Through questioning students will be able to identify key characteristics of polynomials related to its zeros when factorizations are suitable or with technology and determine if a function is even or odd through symmetry tests. Closure	1.B 2.B

		Students will determine whether a polynomial is even or odd using algebraic symmetry tests.		
1.6 Recognize graphs of common functions, and use vertical, and horizontal shifts, reflections, and non-rigid transformations to graph them. (2 days)	SWBAT Analyze the effect of the coefficients on the graph of a function. Identify horizontal and vertical shifts. Identify reflections and non-rigid transformations to the graph.	Students will be assessed on objectives learned in sections 1.5 and 1.6	quiz 1.5 - 1.6	1.B 2.B
1.7 Rational Functions and End Behaviors (3 days)	SWBAT Express functions, equations, or expressions in analytically equivalent forms that are useful in a given mathematical or applied context.	Each student is given cards containing different rational functions in analytical representations. Have students use a calculator to graph the function and then	Circulate and monitor student progress as they are working on classwork. Have students complete problems at the board.	1.B 3.A

	SWBAT Describe the characteristics of a function with varying levels of precision, depending on the function representation and available mathematical tools.	record the intercepts on the card as well as limit expressions to describe the function’s end behavior and behavior at each vertical or horizontal asymptote (e.g., lim f x( ) = −∞, lim f x( ) = + x→3 x→∞ 2). In pairs, students take turns reading their limit statements to each other. Without seeing the actual rational function and without using a calculator, students will try to sketch the function’s graph and then check and discuss. Have students rotate to form new pairs and repeat.	Through questioning students will be able to identify information from graphical, numerical, analytical, and verbal representations or construct a model with and without technology Closure	
1.8 Rational Functions and Zeros (2 days)	SWBAT Solve equations and inequalities represented analytically, with and without technology.		Circulate and monitor student progress as they are working on classwork. Have students complete problems at the board. Through questioning students will be able to identify information from graphical, numerical, analytical, and verbal representations or construct a model with and without technology	1.A

			Closure	
1.9 Rational Functions and Vertical Asymptotes (2 days)	SWBAT Identify information from graphical, numerical, analytical, and verbal representations to answer a question or construct a model, with and without technology.		Circulate and monitor student progress as they are working on classwork. Have students complete problems at the board. Through questioning students will be able to identify information from graphical, numerical, analytical, and verbal representations or construct a model with and without technology Closure	2.A 3.C
1.10 Rational Functions and Holes (2 days)	SWBAT Support conclusions or choices with a logical rationale or appropriate data		Circulate and monitor student progress as they are working on classwork. Have students complete problems at the board. Through questioning students will be able to identify information from graphical, numerical, analytical, and verbal representations or construct a model with and without technology Closure	3.C

1.11 Equivalent Representations of Polynomial and Rational Expressions (2 days)	SWBAT 1Express functions, equations, or expressions in analytically equivalent forms that are useful in a given mathematical or applied context. SWBAT Apply numerical results in a given mathematical or applied context.	Students are presented with a nonconstant polynomial or rational function in analytical representations, and they then translate the expression into a variety of representations: constructing a graph, writing the expression as a product of linear factors (x a −) when possible, and verbally describing characteristics such as real zeros, x-intercepts, asymptotes, and holes. Then have students check their graphs using technology	Circulate and monitor student progress as they are working on classwork. Have students complete problems at the board. Through questioning students will be able to identify information from graphical, numerical, analytical, and verbal representations or construct a model with and without technology Closure	1.B 3.B
1.12 Transformations of Functions (3 days)	SWBAT Construct new functions, using transformations, compositions, inverses, or regressions, that may be useful in modeling contexts, criteria, or data, with and without technology. SWBAT Describe the characteristics of a function with varying levels of precision, depending on the function representation and available mathematical tools.	Students are given graphs of polynomial and rational functions. Students are then asked to graph a transformation of one of the provided graphs, such as a vertical dilation by a factor of 3 and a horizontal translation of 2 units. Students will then switch with a peer and try to write the new expression for the function transformation. Students then have time to discuss the new function	Circulate and monitor student progress as they are working on classwork. Have students complete problems at the board. Through questioning students will be able to identify information from graphical, numerical, analytical, and verbal representations or construct a model with and without technology	1.C 3.A

		expressions and adjust as needed.	Closure	
1.13 Function Model Selection and Assumption Articulation (2 days)	SWBAT Identify information from graphical, numerical, analytical, and verbal representations to answer a question or construct a model, with and without technology. SWBAT supports conclusions or choices with a logical rationale or appropriate data.	Students are given graphs of polynomial and rational functions. Students are then asked to graph a transformation of one of the provided graphs, such as a vertical dilation by a factor of 3 and a horizontal translation of 2 units. Students will then switch with a peer and try to write the new expression for the function transformation. Students then have time to discuss the new function expressions and adjust as needed.	Circulate and monitor student progress as they are working on classwork. Have students complete problems at the board. Through questioning students will be able to identify information from graphical, numerical, analytical, and verbal representations or construct a model with and without technology Closure	2.A 3.C
1.14 Function Model Construction and Application (3 days)	SWBAT Construct new functions, using transformations, compositions, inverses, or regressions, that may be useful in modeling contexts, criteria, or data, with and without technology. SWBAT Apply numerical results in a given mathematical or applied context.	Students are given graphs of polynomial and rational functions. Students are then asked to graph a transformation of one of the provided graphs, such as a vertical dilation by a factor of 3 and a horizontal translation of 2 units. Students will then switch with a peer and try to write the new expression for the function transformation. Students then have time to discuss the	Circulate and monitor student progress as they are working on classwork. Have students complete problems at the board. Through questioning students will be able to identify information from graphical, numerical, analytical, and verbal representations or construct a model with and without technology	1.C 3.B

		new function expressions and adjust as needed.	Closure	

MA.9-12.2.1.A.1	A sequence is a function from the whole numbers to the real numbers. Consequently, the graph of a sequence consists of discrete points instead of a curve.
MA.9-12.2.1.A.2	Successive terms in an arithmetic sequence have a common difference, or constant rate of change.
MA.9-12.2.1.A.3	The general term of an arithmetic sequence with a common difference d is denoted by a_n and is given by $a_n = a_0 + dn$, where a_0 is the initial value, or by $a_n = a_k + d(n - k)$, where a_k is the kth term of the sequence.
MA.9-12.2.1.B.1	Successive terms in a geometric sequence have a common ratio, or constant proportional change.
MA.9-12.2.1.B.2	The general term of a geometric sequence with a common ratio r is denoted by g_n and is given by $g_n = g_0 r^n$, where g_0 is the initial value, or by $g_n = g_k r^{(n - k)}$ where g_k is the kth term of the sequence.
MA.9-12.2.1.B.3	Increasing arithmetic sequences increase equally with each step, whereas increasing geometric sequences increase by a larger amount with each successive step.
MA.9-12.2.2.A.1	Linear functions of the form $f(x) = b + mx$ are similar to arithmetic sequences of the form $a_n = a_0 + dn$, as both can be expressed as an initial value (b or a_0) plus repeated addition of a constant rate of change, the slope (m or d).
MA.9-12.2.2.A.2	Similar to arithmetic sequences of the form $a_n = a_k + d(n - k)$, which are based on a known difference, d, and a kth term, linear functions can be expressed in the form $f(x) = y_i + m(x - x_i)$ based on a known slope, m, and a point, (x_i, y_i).
MA.9-12.2.2.A.3	Exponential functions of the form $f(x) = ab^x$ are similar to geometric sequences of the form $g_n = g_0 r^n$, as both can be expressed as an initial value (a or g_0) times repeated multiplication by a constant proportion (b or r).
MA.9-12.2.2.A.4	Similar to geometric sequences of the form $g_n = g_k r^{(n - k)}$, which are based on a known ratio, r, and a kth term, exponential functions can be expressed in the form $f(x) = y_i r^{(x - x_i)}$ based on a known ratio, r, and a point, (x_i, y_i).
MA.9-12.2.2.A.5	Sequences and their corresponding functions may have different domains.
MA.9-12.2.2.B.1	Over equal-length input-value intervals, if the output values of a function

	change at constant rate, then the function is linear; if the output values of a function change proportionally, then the function is exponential.
MA.9-12.2.2.B.2	Linear functions of the form $f(x) = b + mx$ and exponential functions of the form $f(x) = ab^x$ can both be expressed analytically in terms of an initial value and a constant involved with change. However, linear functions are based on addition, while exponential functions are based on multiplication.
MA.9-12.2.2.B.3	Arithmetic sequences, linear functions, geometric sequences, and exponential functions all have the property that they can be determined by two distinct sequence or function values.
MA.9-12.2.3.A.1	The general form of an exponential function is $f(x) = ab^x$, with the initial value a, where $a \neq 0$, and the base b, where $b > 0$, and $b \neq 1$. When $a > 0$ and $b > 1$, the exponential function is said to demonstrate exponential growth. When $a > 0$ and $0 < b < 1$, the exponential function is said to demonstrate exponential decay.
MA.9-12.2.3.A.2	When the natural numbers are input values in an exponential function, the input value specifies the number of factors of the base to be applied to the function's initial value. The domain of an exponential function is all real numbers.
MA.9-12.2.3.A.3	Because the output values of exponential functions in general form are proportional over equal-length input-value intervals, exponential functions are always increasing or always decreasing, and their graphs are always concave up or always concave down. Consequently, exponential functions do not have extrema except on a closed interval, and their graphs do not have points of inflection.
MA.9-12.2.3.A.4	If the values of the additive transformation function $g(x) = f(x) + k$ of any function f are proportional over equal-length input-value intervals, then f is exponential.
MA.9-12.2.3.A.5	For an exponential function in general form, as the input values increase or decrease without bound, the output values will increase or decrease without bound or will get arbitrarily close to zero. That is, for an exponential function in general form, $lim\ [x \to \pm\infty]\ ab^x = \infty$, $lim\ [x \to \pm\infty]\ ab^x = -\infty$, $lim\ [x \to \pm\infty]\ ab^x = 0$.
MA.9-12.2.4.A.1	The product property for exponents states that $b^m b^n = b^{(m+n)}$. Graphically, this property implies that every horizontal translation of an exponential function, $f(x) = b^{(x+k)}$, is equivalent to a vertical dilation, $f(x) = b^{(x+k)} = b^x b^k = ab^x$, where $a = b^k$.
MA.9-12.2.4.A.2	The power property for exponents states that $(b^m)^n = b^{(mn)}$. Graphically, this property implies that every horizontal dilation of an exponential function, $f(x) = b^{(cx)}$, is equivalent to a change of the base of an exponential function, $f(x) = (b^c)^x$, where b^c is a constant and $c \neq 0$.
MA.9-12.2.4.A.3	The negative exponent property states that $b^{-n} = 1/b^n$.
MA.9-12.2.4.A.4	The value of an exponential expression involving an exponential unit fraction, such as $b^{(1/k)}$ where k is a natural number, is the kth root of b, when

	it exists.
MA.9-12.2.5.A.1	Exponential functions model growth patterns where successive output values over equal-length input-value intervals are proportional. When the input values are whole numbers, exponential functions model situations of repeated multiplication of a constant to an initial value.
MA.9-12.2.5.A.2	A constant may need to be added to the dependent variable values of a data set to reveal a proportional growth pattern.
MA.9-12.2.5.A.3	An exponential function model can be constructed from an appropriate ratio and initial value or from two input-output pairs. The initial value and the base can be found by solving a system of equations resulting from the two input-output pairs.
MA.9-12.2.5.A.4	Exponential function models can be constructed by applying transformations to $f(x) = ab^x$ based on characteristics of a contextual scenario or data set.
MA.9-12.2.5.A.5	Exponential function models can be constructed for a data set with technology using exponential regressions.
MA.9-12.2.5.A.6	The natural base e, which is approximately 2.718, is often used as the base in exponential functions that model contextual scenarios.
MA.9-12.2.5.B.1	For an exponential model in general form $f(x) = ab^x$, the base of the exponent, b, can be understood as a growth factor in successive unit changes in the input values and is related to a percent change in context.
MA.9-12.2.5.B.2	Equivalent forms of an exponential function can reveal different properties of the function. For example, if d represents number of days, then the base of $f(d) = 2^d$ indicates that the quantity increases by a factor of 2 every day, but the equivalent form $f(d) = 2^{7(d7)}$ indicates that the quantity increases by a factor of 2^7 every week.
MA.9-12.2.5.B.3	Exponential models can be used to predict values for the dependent variable, depending on the contextual constraints on the domain.
MA.9-12.2.6.A.1	Two variables in a data set that demonstrate a slightly changing rate of change can be modeled by linear, quadratic, and exponential function models.
MA.9-12.2.6.A.2	Models can be compared based on contextual clues and applicability to determine which model is most appropriate.
MA.9-12.2.6.B.1	A model is justified as appropriate for a data set if the graph of the residuals of a regression, the residual plot, appear without pattern.
MA.9-12.2.6.B.2	The difference between the predicted and actual values is the error in the model. Depending on the data set and context, it may be more appropriate to have an underestimate or overestimate for any given interval.
MA.9-12.2.7.A.1	If f and g are functions, the composite function $f \circ g$ maps a set of input values to a set of output values such that the output values of g are used as input values of f. For this reason, the domain of the composite function is

	restricted to those input values of g for which the corresponding output value is in the domain of f. $(f \circ g)(x)$ can also be represented as $f(g(x))$.
MA.9-12.2.7.A.2	Values for the composite function $f \circ g$ can be calculated or estimated from the graphical, numerical, analytical, or verbal representations of f and g by using output values from g as input values for f.
MA.9-12.2.7.A.3	The composition of functions is not commutative; that is, $f \circ g$ and $g \circ f$ are typically different functions; therefore, $f(g(x))$ and $g(f(x))$ are typically different values.
MA.9-12.2.7.A.4	If the function $f(x) = x$ is composed with any function g, the resulting composite function is the same as g; that is, $g(f(x)) = f(g(x)) = g(x)$. The function $f(x) = x$ is called the identity function. When composing two functions, the identity function acts in the same way as 0, the additive identity, when adding two numbers and 1, the multiplicative identity, when multiplying two numbers.
MA.9-12.2.7.B.1	Function composition is useful for relating two quantities that are not directly related by an existing formula.
MA.9-12.2.7.B.2	When analytic representations of the functions f and g are available, an analytic representation of $f(g(x))$ can be constructed by substituting $g(x)$ for every instance of x in f.
MA.9-12.2.7.B.3	A numerical or graphical representation of $f \circ g$ can often be constructed by calculating or estimating values for $(x, f(g(x)))$.
MA.9-12.2.7.C.1	Functions given analytically can often be decomposed into less complicated functions. When properly decomposed, the variable in one function should replace each instance of the function with which it was composed.
MA.9-12.2.7.C.2	An additive transformation of a function, f, that results in vertical and horizontal translations can be understood as the composition of $g(x) = x + k$ with f.
MA.9-12.2.7.C.3	A multiplicative transformation of a function, f, that results in vertical and horizontal dilations can be understood as the composition of $g(x) = kx$ with f.
MA.9-12.2.8.A.1	On a specified domain, a function, f, has an inverse function, or is invertible, if each output value of f is mapped from a unique input value. The domain of a function may be restricted in many ways to make the function invertible.
MA.9-12.2.8.A.2	An inverse function can be thought of as a reverse mapping of the function. An inverse function, f^{-1}, maps the output values of a function, f, on its invertible domain to their corresponding input values; that is, if $f(a) = b$, then $f^{-1}(b) = a$. Alternately, on its invertible domain, if a function consists of input-output pairs (a, b), then the inverse function consists of input-output pairs (b, a).
MA.9-12.2.8.B.1	The composition of a function, f, and its inverse function, f^{-1}, is the identity function; that is, $f(f^{-1}(x)) = f^{-1}(f(x)) = x$.

MA.9-12.2.8.B.2	On a function's invertible domain, the function's range and domain are the inverse function's domain and range, respectively. The inverse of the table of values of $y = f(x)$ can be found by reversing the input-output pairs; that is, (a, b) corresponds to (b, a).
MA.9-12.2.8.B.3	The inverse of the graph of the function $y = f(x)$ can be found by reversing the roles of the x- and y-axes; that is, by reflecting the graph of the function over the graph of the identity function $h(x) = x$.
MA.9-12.2.8.B.4	The inverse of the function can be found by determining the inverse operations to reverse the mapping. One method for finding the inverse of the function f is reversing the roles of x and y in the equation $y = f(x)$, then solving for $y = f^{-1}(x)$.
MA.9-12.2.8.B.5	In addition to limiting the domain of a function to obtain an inverse function, contextual restrictions may also limit the applicability of an inverse function.
MA.9-12.2.9.A.1	The logarithmic expression $log\_b\ c$ is equal to, or represents, the value that the base b must be exponentially raised to in order to obtain the value c. That is, $log\_b\ c = a$ if and only if $b^a = c$, where a and c are constants, $b > 0$, and $b \neq 1$. (when the base of a logarithmic expression is not specified, it is understood as the common logarithm with a base of 10)
MA.9-12.2.9.A.2	The values of some logarithmic expressions are readily accessible through basic arithmetic while other values can be estimated through the use of technology.
MA.9-12.2.9.A.3	On a logarithmic scale, each unit represents a multiplicative change of the base of the logarithm. For example, on a standard scale, the units might be 0, 1, 2, …, while on a logarithmic scale, using logarithm base 10, the units might be 10^0, 10^1, 10^2, ….
MA.9-12.2.10.A.1	The general form of a logarithmic function is $f(x) = a\ log\_b\ x$, with base b, where $b > 0$, $b \neq 1$, and $a \neq 0$.
MA.9-12.2.10.A.2	The way in which input and output values vary together have an inverse relationship in exponential and logarithmic functions. Output values of general-form exponential functions change proportionately as input values increase in equal-length intervals. However, input values of general-form logarithmic functions change proportionately as output values increase in equal-length intervals. Alternately, exponential growth is characterized by output values changing multiplicatively as input values change additively, whereas logarithmic growth is characterized by output values changing additively as input values change multiplicatively.
MA.9-12.2.10.A.3	$f(x) = log\_b\ x$ and $g(x) = b^x$, where $b > 0$ and $b \neq 1$, are inverse functions. That is, $g(f(x)) = f(g(x)) = x$.
MA.9-12.2.10.A.4	The graph of the logarithmic function $f(x) = log\_b\ x$, where $b > 0$ and $b \neq 1$, is a reflection of the graph of the exponential function $g(x) = b^x$, where $b > 0$ and $b \neq 1$, over the graph of the identity function $h(x) = x$.
MA.9-12.2.10.A.5	If (s, t) is an ordered pair of the exponential function $g(x) = b^x$, where $b > 0$

and $b \neq 1$, then (t, s) is an ordered pair of the logarithmic function $f(x) = log\_b\ x$, where $b > 0$ and $b \neq 1$.

MA.9-12.2.11.A.1	The domain of a logarithmic function in general form is any real number greater than zero, and its range is all real numbers.
MA.9-12.2.11.A.2	Because logarithmic functions are inverses of exponential functions, logarithmic functions are also always increasing or always decreasing, and their graphs are either always concave up or always concave down. Consequently, logarithmic functions do not have extrema except on a closed interval, and their graphs do not have points of inflection.
MA.9-12.2.11.A.3	The additive transformation function $g(x) = f(x + k)$, where $k \neq 0$, of a logarithmic function f in general form does not have input values that are proportional over equal-length output-value intervals. However, if the input values of the additive transformation function, $g(x) = f(x + k)$, of any function f are proportional over equal-length output value intervals, then f is logarithmic.
MA.9-12.2.11.A.4	With their limited domain, logarithmic functions in general form are vertically asymptotic to $x = 0$, with an end behavior that is unbounded. That is, for a logarithmic function in general form, $lim\ [x \to 0^+]\ a\ log\_b\ x = \pm\infty$ and $lim\ [x \to \infty]\ a\ log\_b\ x = \pm\infty$.
MA.9-12.2.12.A.1	The product property for logarithms states that $log\_b\ (xy) = log\_b\ x + log\_b\ y$. Graphically, this property implies that every horizontal dilation of a logarithmic function, $f(x) = log\_b\ (kx)$, is equivalent to a vertical translation, $f(x) = log\_b\ (kx) = log\_b\ k + log\_b\ x = a + log\_b\ x$, where $a = log\_b\ k$.
MA.9-12.2.12.A.2	The power property for logarithms states that $log\_b\ x^n = n\ log\_b\ x$. Graphically, this property implies that raising the input of a logarithmic function to a power, $f(x) = log\_b\ x^k$, results in a vertical dilation, $f(x) = log\_b\ x^k = k\ log\_b\ x$.
MA.9-12.2.12.A.3	The change of base property for logarithms states that $log\_b\ x = (log_a\ x)/(log_a\ b)$, where $a > 0$ and $a \neq 1$. This implies that all logarithmic functions are vertical dilations of each other.
MA.9-12.2.12.A.4	The function $f(x) = \ln x$ is a logarithmic function with the natural base e; that is, $\ln x = \log_e x$.
MA.9-12.2.13.A.1	Properties of exponents, properties of logarithms, and the inverse relationship between exponential and logarithmic functions can be used to solve equations and inequalities involving exponents and logarithms.
MA.9-12.2.13.A.2	When solving exponential and logarithmic equations found through analytical or graphical methods, the results should be examined for extraneous solutions precluded by the mathematical or contextual limitations.
MA.9-12.2.13.A.3	Logarithms can be used to rewrite expressions involving exponential functions in different ways that may reveal helpful information. Specifically, $b^x = c\text{^}(log\_c\ b)(x)$.

MA.9-12.2.13.B.1	The function $f(x) = ab^{(x - h)} + k$ is a combination of additive transformations of an exponential function in general form. The inverse of $y = f(x)$ can be found by determining the inverse operations to reverse the mapping.
MA.9-12.2.13.B.2	The function $f(x) = a\ log\_b\ (x - h) + k$ is a combination of additive transformations of a logarithmic function in general form. The inverse of $y = f(x)$ can be found by determining the inverse operations to reverse the mapping.
MA.9-12.2.14.A.1	Logarithmic functions are inverses of exponential functions and can be used to model situations involving proportional growth, or repeated multiplication, where the input values change proportionally over equal-length output-value intervals. Alternately, if the output value is a whole number, it indicates how many times the initial value has been multiplied by the proportion.
MA.9-12.2.14.A.2	A logarithmic function model can be constructed from an appropriate proportion and a real zero or from two input-output pairs.
MA.9-12.2.14.A.3	Logarithmic function models can be constructed by applying transformations to $f(x) = a\ log\_b\ x$ based on characteristics of a context or data set.
MA.9-12.2.14.A.4	Logarithmic function models can be constructed for a data set with technology using logarithmic regressions.
MA.9-12.2.14.A.5	The natural logarithm function is often useful in modeling real-world phenomena.
MA.9-12.2.14.A.6	Logarithmic function models can be used to predict values for the dependent variable.
MA.9-12.2.15.A.1	In a semi-log plot, one of the axes is logarithmically scaled. When the y-axis of a semi-log plot is logarithmically scaled, data or functions that demonstrate exponential characteristics will appear linear.
MA.9-12.2.15.A.2	An advantage of semi-log plots is that a constant never needs to be added to the dependent variable values to reveal that an exponential model is appropriate.
MA.9-12.2.15.B.1	Techniques used to model linear functions can be applied to a semi-log graph.
MA.9-12.2.15.B.2	For an exponential model of the form $y = ab^x$, the corresponding linear model for the semi-log plot is $y = (log_n\ b)x + log_n\ a$, where $n > 0$ and $n \neq 1$. Specifically, the linear rate of change is $log_n\ b$, and the initial linear value is $log_n\ a$.
MA.9-12.I	Functions, Graphs, and Limits

Suggested Modifications for Special Education, ELL and Gifted Students

- Anchor charts to model strategies.

- Review Algebra concepts to ensure students have the information needed to progress in understanding.

- Pre-teach pertinent vocabulary.

- Provide reference sheets that list formulas, step-by-step procedures, theorems, and modeling of strategies.

- Word wall with visual representations of mathematical terms.

- Teacher modeling of thinking processes involved in solving, graphing, and writing equations.

- Introduce concepts embedded in real-life context to help students relate to the mathematics involved.

- Record formulas, processes, and mathematical rules in reference notebooks.

- Graphing calculator to assist with computations and graphing of trigonometric functions.

- Utilize technology through interactive sites to represent nonlinear data.

- Graphic organizers to help students interpret the meaning of terms in an expression or equation in context.

- Translation dictionary.

- Sentence stems to provide additional language support for ELL students.

Suggested Technological Innovations/Use

Students find and maximize the productive value of existing and new technology to accomplish workplace tasks and solve workplace problems. They are flexible and adaptive in acquiring new technology. They are proficient with ubiquitous technology applications. They understand the inherent risks of personal and organizational technology applications, and they take action to prevent or mitigate these risks.

Example: Students will be introduced to the math instructional program, Larson Math, as well as digital platforms such as Google Classroom, Meet, and Jamboard. Students will make sound judgments about the use of specific tools, such as Graphing Calculators and Geogebra to explore and deepen their understanding of the concepts related to trigonometric ratios and functions, probability, data analysis, and statistics.

Cross Curricular/21st Century Connections

Model interdisciplinary thinking to expose students to other disciplines.

Social Studies and ELA Literacy Connection:

Name of Task: Americans' spending: NJSLS: **6.1.12.HistoryCC.16.b, 6.2.12.EconGE.5.a**

From July 1998 to July 1999, Americans' spending rose from 5.82 trillion dollars to 6.20 trillion dollars

a. Let $x = 0$ represent July 1998, $x = 1$ represent August 1998, ..., and $x = 12$ represent July 1999. Write a linear equation for Americans' spending in terms of the month x

b. Use the equation in (a) to predict Americans' spending in July 2002.

c. Based on the model created in (a) when would the aggregate expenditure exceed 10 trillion dollars?

d. What part of the US GDP is spent by the Americans in 2013?

Name of Task: Publishing Cost:

A publishing company estimates that the average cost (in dollars) for one copy of a new scenic calendar it plans to produce can be approximated by the function

C (x) = (2.25x + 275)/x

2.25 x+

Where x is the number of calendars printed.

a. Find the average cost per calendar when the company prints 100 calendars.

b. Identify the domain and range of this function.

c. After analyzing the function, Alex said that this company should not be allowed to publish zero calendars. As a result, the company has no option to shut down and go out of business. Write an argument to support or reject Alex's conclusion.

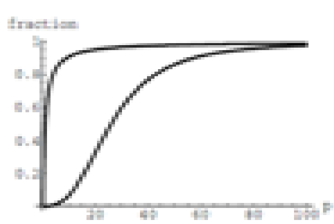
Science Connection:

Name of Task: Myoglobin and Hemoglobin: **NJSLS: HS-LS1-2; HS-LS1-4**

Myoglobin and hemoglobin are oxygen-carrying molecules in the human body. Hemoglobin is found inside red blood cells, which flow from the lungs to the muscles through the bloodstream. Myoglobin is found in muscle cells. The function y = M(p) = p/(1 + p) calculates the fraction of myoglobin saturated with oxygen at a given presure p Torrs. For example, at a pressure of 1 Torr, $M(1) = 0.5$, which means half of the myoglobin (i.e. 50%) is oxygen saturated. (Note: More precisely, you need to use something called the "partial pressure", but the distinction is not important for this problem.) Likewise, the function calculates the fraction of hemoglobin saturated with oxygen at a given pressure p. [UW]

a. The graphs of M(P) and H(P) are given here on the domain

$0 \leq p \leq 100$



Which is which?

b. If the pressure in the lungs is 100 Torrs, what is the level of oxygen saturation of the hemoglobin in the lungs?

c. The pressure in an active muscle is 20 Torrs. What is the level of oxygen saturation of myoglobin in an active muscle? What is the level of hemoglobin in an active muscle?

d. Define the efficiency of oxygen transport at a given pressure *p* to be M(p) – H(p) . What is the oxygen transport efficiency at 20 Torrs? At 40 Torrs? At 60 Torrs? Sketch the graph of M(p) – H(p) ; are there conditions under which transport efficiency is maximized (explain)?

Business Connection :
Name of Task: Minimize the metal in a can: **NJSLS: 9.1.12.A.4; W.11-12.1**
A manufacturer wants to manufacture a metal can that holds 1000 cm^3 of oil. The can is in the shape of a right cylinder with a radius *r* and height *h*. Assume the thickness of the material used to make the metal can is negligible.

For each question, include correct units of measurement and round your answers to the nearest tenth. Using your knowledge of volume and surface area of a right cylinder, write a function S(r) that represents the surface area of the cylindrical can in terms of the radius, r, of its base. Show in detail your algebraic thinking.

1. Sketch the graph of S(r) and show key features of the graph. State any restriction on the value of r so that it represents the physical model of the can.
2. What dimensions will minimize the quantity of metal needed to manufacture the cylindrical can? Show in detail your mathematical solution.
3. Calculate the minimum value of the function S(r) and interpret the result in the context of the physical model. Show the mathematical steps you used to obtain the answer.

Name of Task: Chemco Manufacturing**: NJSLS: 9.1.12.A.4; W.11-12.1**
Chemco Manufacturing estimates that its profit *P* in hundreds of dollars is P = $-4x^2$+10-30 *x* is the number of units produced in thousands.

a. How many units must be produced to obtain the maximum profit?
b. Graph the profit function and identify its vertex.
c. An increase in productivity increased profit by $7 at each quantity sold. What kind of a transformation would model this situation? Show your work graphically and algebraically.
d. A decrease in marginal cost lead to a 4 units increase in the optimum level of production. What kind of a transformation would model this situation? Show your work graphically and algebraically.

Table of Contents

Unit 2 Exponential and Logarithmic Functions

Content Area: **Mathematics**
Course(s): **AP PreCalculus**
Time Period: **2nd Marking Period**
Length: **7 weeks**
Status: **Not Published**

Section Title

Unit 2: Exponential and Logarithmic Functions

Enduring Understandings

- o Recognize, write, and find nth terms of arithmetic and geometric sequences.
- o Use arithmetic and geometric sequences to model real world problems.
- o Describe similarities and differences between linear and exponential functions.
- o Construct functions that are comparable to arithmetic and geometric sequences.

- ○ Solve real world applications using arithmetic and geometric sequences.
- ○ Identify key characteristics of exponential functions.
- ○ Use properties of exponents to simplify and evaluate expressions.
- ○ Apply properties of logarithms to evaluate expressions and graph functions.
- ○ Understand the characteristics and effects of constants in the exponential model y = ab^x.
- ○ Apply exponential models about data sets in contextual scenarios and construct a model.
- ○ Construct exponential function models using regression equations and technology.
- ○ Evaluate the composition of two or more functions for given values.
- ○ Rewrite a given function as a composition of two ore more functions
- ○ Determine the inverse of a function on an invertible domain.
- ○ Identify key characteristics of logarithmic functions
- ○ Rewrite logarithmic expressions in equivalent forms.
- ○ Solve exponential and logarithmic equations and inequalities.
- ○ Construct the inverse of exponential functions.

Summary of the Unit

In Unit 2, students will build an understanding of exponential and logarithmic functions. Exponential and logarithmic function models are widespread in the natural and social sciences. Exponential functions are key to modeling population growth, radioactive decay, interest rates, and the amount of medication in a patient. Logarithmic functions are useful in modeling sound intensity and frequency, the magnitude of earthquakes, the pH scale in chemistry, and the working memory in humans. The study of these two function types touches careers in business, medicine, chemistry, physics, education, and geography.

Essential Questions

- How can I make a single model that merges the interest I earn from my bank with the taxes that are due so I can know how much I will have in the end?
- How can we adjust the scale of distance for a model of planets in the solar system so the relationships among the planets are easier to see?
- If different functions can be used to model data, how do we pick which one is best?
- Why is the number e important?
- What are the key characteristics of an exponential function?
- What is a composite function?
- How do we create the composition of two or more functions?
- What do we know about the domain of composite functions?
- How do we decompose a function?
- How can you find the inverse of a relation or function?
- How can you determine whether the inverse of a function is a function?

- How do I use exponential functions to model exponential behavior?
- What is the relationship between an exponential expression and a logarithm?
- How can the properties of exponents be used to rewrite exponential expressions?

Summative Assessment and/or Summative Criteria

Required District/State Assessments

- SGO Pre Assessment
- SGO Post Assessment

Suggested Formative/Summative Classroom Assessments

- Describe Learning Vertically
- Identify Key Building Blocks
- Make Connections (between and among crucial building blocks)
- Short/Extended Constructed Response Items
- Multiple-Choice Items (where multiple answer choices may be correct)
- Drag and Drop Items
- Use of Equation Editor
- Quizzes
- Journal Entries/Reflections/Quick-Writes
- Accountable talk
- Projects
- Portfolio
- Observation
- Graphic Organizers/ Concept Mapping

- Presentations
- Teacher-Student and Student-Student Conferencing
- AP Classroom – Assessments created with specific sections outlined.
- WebAssign Problem Sets WebAssign Instructor Help
- Homework
- Students will take formal assessments, such as tests and quizzes, to assess knowledge of concepts learned throughout the unit.
- Students will also demonstrate mastery through various assessment criteria included in the unit such as do nows, exit slips, graded classwork activities and assignments, and/or projects.

Resources

- AP Daily Videos: Section 2.1 - 2.2
- AP Daily Videos: Section 2.3 - 2.5
- AP Daily Videos: Section 2.6
- AP Daily Videos: Section 2.7 - 2.8
- AP Daily Videos: Section 2.9 - 2.12
- AP Daily Videos: Section 2.13
- AP Daily Videos: Section 2.14-2.15
- AP Precalculus Course Overview: https://apcentral.collegeboard.org/courses/ap-precalculus/course
- AP Precalculus Course and Exam Description: https://apcentral.collegeboard.org/media/pdf/ap-precalculus-course-and-exam-description.pdf
- AP Precalculus Practice Exam: https://apcentral.collegeboard.org/media/pdf/ap-precalculus-practice-exam-multiple-choice-section.pdf
- AP Precalculus Classroom Resources: https://apcentral.collegeboard.org/courses/ap-precalculus/classroom-resources
- Classpad.net (Casio): https://classpad.net/us/

- Desmos: https://www.desmos.com/
- Geogebra: https://www.geogebra.org/?lang=en
- Math Open Reference: https://www.mathopenref.com/
- TI Education (Texas Instruments): https://education.ti.com/en
- WolframAlpha: https://www.wolframalpha.com/
- Wolfram MathWorld: https://mathworld.wolfram.com/
- Khan Academy: https://www.khanacademy.org/math/precalculus
- Digital Mathematics Word Wall: http://www.mathwords.com/index_adv_alg_precal.htm
- Extra Notes for Pre-Calculus Content: https://sites.google.com/a/evergreenps.org/ms-griffin-s-math-classes/updates
- Review Documents for Pre-Calculus: https://sites.google.com/site/dgrahamcalculus/trigpre-calculus/trig-pre-calculus-worksheets
- Pre-Calculus IXL Topics and Resources: https://www.ixl.com/math/precalculus
- Classroom Challenges to Support Teachers in Formative Assessments: http://map.mathshell.org/materials/lessons.php?gradeid=24
- Applications of Function Models: https://www.ck12.org/algebra/Applications-of-Function-Models/lesson/Applications-of-Function-Models-BSC-ALG/?referrer=featured_content
- Statistics Education Web (STEW). http://www.amstat.org/education/STEW/
- The Data and Story Library (DASL). http://lib.stat.cmu.edu/DASL/
- WebAssign Instructor Resources
- WebAssign Student Resources
- Cengage Learning: PreCalculus with Limits - A Graphing Approach, Sixth Edition

Unit Plan

Topic/Selection Timeframe	General Objectives	Instructional Activities	Benchmarks/Assessments	Standards
2.1 - Changes in Arithmetic and Geometric Sequences (2 days)	SWBAT recognize, write, and find nth terms of arithmetic and geometric sequences. SWBAT use arithmetic and geometric sequences to model real-world problems.	Students will identify the common difference and first term of arithmetic sequences written in a variety of ways and use that to write a rule. Students will model arithmetic sequences from contextual and mathematical scenarios. Students will identify the common ratio and the first term of geometric sequences written in a variety of ways and use that information to write a rule. Students will model geometric sequences from contextual and mathematical scenarios.	Circulate and monitor student progress as they are working on classwork. Have students complete problems at the board. Through questioning students will be able to write rules for arithmetic and geometric sequences and construct a model with and without technology Closure	2.1.A & 2.1.B

2.2 - Changes in Linear and Exponential Functions (2 days)	SWBAT describes similarities and differences between linear and exponential functions. SWBAT constructs functions that are comparable to arithmetic and geometric sequences. SWBAT to solve real world applications using arithmetic and geometric sequences.	Students will write linear functions and understand how it is similar to an explicit rule of an arithmetic sequence. Students will write rules for exponential functions and understand how it is similar to an explicit rule of geometric sequences. Students will understand the sequences and their corresponding functions may have different domains. Students will model geometric and arithmetic sequences from mathematical and contextual scenarios.	Circulate and monitor student progress as they are working on classwork. Have students complete problems on the board. Through questioning students will be able to identify similarities and differences between arithmetic sequences and linear functions and geometric sequences and exponential functions. Students will also be able to construct a model with and without technology Closure Quiz 2.1 - 2.2	2.2A & 2.2.B

2.3 - Exponential Functions (2 days)	SWBAT to identify key characteristics of exponential functions.	Students will investigate the basic behaviors of exponential functions and their graphs. Students will analyze exponential graphs for key characteristics, such as domain, range, end behavior, continuity, symmetry, asymptotes, and intervals of increase and decrease.	Circulate and monitor student progress as they are working on classwork. Have students complete problems on the board. Through questioning students will be able to identify key characteristics of exponential functions. Closure	2.3.A
2.4 - Exponential Function Manipulation (2 days)	SWBAT use properties of exponents to simplify and evaluate expressions. SWBAT apply properties of logarithms to evaluate expressions and graph functions. SWBAT understand the characteristics and effects of constants in the exponential model y = ab^x.	Students will simplify functions using rules of exponents. Students will apply transformations of functions to exponential functions. Students will graph these functions and identify key characteristics.	Circulate and monitor student progress as they are working on classwork. Have students complete problems on the board. Through questioning students will be able to apply transformations to graph exponential functions and analyze key characteristics. Closure	2.4.A
2.5 - Exponential Function Context and	SWBAT construct a model for scenarios involving	Students will identify if a situation or equation represents exponential growth or	Circulate and monitor student progress as they are working on classwork.	2.5.A & 2.5.B

Data Modeling (2 days)	proportional output values related to real life scenarios. SWBAT apply exponential models about data sets in contextual scenarios. SWBAT construct exponential function models using regression equations and technology.	decay. Students will write equations for exponential models from an appropriate ratio and initial value or from two input output pairs by solving a system of equations. Students will construct exponential function models for a data set with technology using exponential regressions. Students will apply exponential models to answer questions about a data set or contextual scenario.	Have students complete problems at the board. Through questioning students will be able to construct a model for scenarios involving exponential functions, apply exponential models about data sets ,and construct exponential function models with and without technology. Closure	
2.6 - Competing Function Model Validation (5 days)	SWBAT use contextual clues to determine an appropriate model and find an appropriate regression model for a data set. SWBAT recognize a correct regression model using the graph of residuals.	Students will find appropriate residuals given a scenario. Students will construct regression equations given a scenario.	Circulate and monitor student progress as they are working on classwork. Have students complete problems at the board. Through questioning students will be able to identify information from graphical, numerical, analytical, and verbal representations to answer a question or construct a model, with or without	2.6.A & 2.6.B

	SWBAT find the residual errors of a model and analyze the findings.		technology. Closure Quiz 2.3 - 2.6	
2.7 - Composition of Functions (3 days)	SWBAT evaluate the composition of two or more functions for given values. SWBAT construct a representation of the composition of two or more functions. SWBAT rewrite a given function as a composition of two ore more functions	Students will perform function operations. Students will find a composition of one function with another function. Students will rewrite a given function as a composite of two or more functions. Students will recognize that the composition of functions is not commutative.	Circulate and monitor student progress as they are working on classwork. Have students complete problems at the board. Through questioning students will be able to construct new functions using transformations and compositions that may be useful in modeling contexts, criteria, or data, with and without technology. Closure	2.7.A & 2.7.B

2.8 - Inverse Functions (5 days)	SWBAT to determine the input - output pairs of the inverse of a function. SWBAT determine the inverse of a function on an invertible domain.	Students will verify that two functions are inverses both algebraically and graphically. Students will use the horizontal line test to determine if the functions are one to one. Students will find inverse functions both informally and algebraically. Students will use function notation to describe real world scenarios. Students will analyze and choose the appropriate method to find an inverse of a relation or function.	Circulate and monitor student progress as they are working on classwork. Have students complete problems at the board. Through questioning students will be able to solve equations and inequalities represented analytically with and without technology, Closure Quiz 2.7 - 2.8	2.8.A & 2.8.B
2.9 - Logarithmic Expressions (2 days)	SWBAT to evaluate logarithmic expressions.	Students will apply the properties of logarithms to evaluate expressions with and without technology. Students will use properties of exponents and logarithms to simplify and evaluate	Circulate and monitor student progress as they are working on classwork. Have students complete problems at the board. Through questioning students will be able to express functions,	2.9.A

		expressions. Students will understand that each unit of the logarithmic scale is a multiplicative change of base 10,	equations, or expressions in analytically equivalent forms. Closure	
2.10 - Inverses of Exponential Functions (2 days)	SWBAT construct representations of the inverse of an exponential function with an initial value of 1.	Students will understand the way in which input and output values vary together have an inverse relationship in exponential and logarithmic functions. Students will show the compositions of exponential functions and log functions are inverses. Students will show the graph of an exponential function reflected over the line y = x is the graph of a logarithmic function.	Circulate and monitor student progress as they are working on classwork. Have students complete problems at the board. Through questioning students will be able to construct new functions using transformations, compositions, inverses, or regressions, with and without technology. Closure	2.10.A

2.11 - Logarithmic Functions (1 day)	SWBAT identify key characteristics of logarithmic functions.	Students will understand the domain of logarithmic functions is any real number greater than zero. Students will understand that just like exponential functions, logarithmic functions are always increasing or decreasing and are either always concave up or concave down. Students will understand how to apply transformations to logarithmic graphs.	Circulate and monitor student progress as they are working on classwork. Have students complete problems at the board. Through questioning students will be able to describe the characteristics of a function with varying levels of precision. Closure	2.11.A
2.12 - Logarithmic Function Manipulation (2 days)	SWBAT rewrite logarithmic expressions in equivalent forms.	Students will apply the properties of logarithms to evaluate expressions and graph functions. Students will use properties of exponents and logarithms to simplify and evaluate expressions Students will recognize graphic translations of logarithms related to the product, power, and change of base property.	Circulate and monitor student progress as they are working on classwork. Have students complete problems at the board. Through questioning students will be able to express functions, equations, or expressions in analytically equivalent forms. Closure	2.12.A

2.13 - Exponential and Logarithmic Equations and Inequalities (6 days)	SWBAT solve exponential and logarithmic equations and inequalities. SWBAT construct the inverse function for exponential and logarithmic functions.	Students will use properties of exponents and logarithms to solve equations and inequalities. When solving exponential and logarithmic equations students will analyze for extraneous solutions. Students will use inverse operations to write exponential and logarithmic equations that reverse the mapping.	Circulate and monitor student progress as they are working on classwork. Have students complete problems at the board. Through questioning students will be able to solve equations and inequalities represented analytically with and without technology. Closure Quiz 2.9 - 2.14	2.13.A & 2.13.B
2.14 - Logarithmic Function Context and Data Modeling (3 days)	SWBAT construct a logarithmic function model.	Students will write a logarithmic function given two ordered pairs of a proportion and a real zero. Students will use logarithms to model proportional growth. Students will use transformations based on context or data set to model logarithmic behavior. Students will use logarithmic regression and technology given a	Circulate and monitor student progress as they are working on classwork. Have students complete problems at the board. Through questioning students will be able to construct new functions using transformations, compositions, inverses, or regressions, that may be useful in modeling contexts, criteria or data with and without technology. Closure	2.14. A

		data set. Students will use the natural log function to model phenomena, Students will use logarithmic models to predict values for dependent variables.		
2.15 - Semi- log Plots (3 days)	SWBAT determine if an exponential model is appropriate by examining a semi-log plot of a data set. SWBAT construct the linearization of exponential data.	Students will use semi-log plots with the y axis logarithmically scaled that show linear behavior from exponential characteristics. Students will apply techniques used to model linear behavior to semi-log graphs. Students will construct the linearization of exponential data.	Circulate and monitor student progress as they are working on classwork. Have students complete problems at the board. Through questioning students will construct equivalent, graphical, numerical, analytical, and verbal representations of functions that are useful in a given mathematical or applied context with or without technology. Closure Quiz 2.14 - 2.15	2.15.A & 2.15. B

MA.9-12.1.2.A.1	The average rate of change of a function over an interval of the function's domain is the constant rate of change that yields the same change in the output values as the function yielded on that interval of the function's domain. It is the ratio of the change in the output values to the change in input values over that interval.
MA.9-12.1.2.A.2	The rate of change of a function at a point quantifies the rate at which output values would change were the input values to change at that point. The rate of change at a point can be approximated by the average rates of change of the function over small intervals containing the point, if such values exist.
MA.9-12.2.1.A.1	A sequence is a function from the whole numbers to the real numbers. Consequently, the graph of a sequence consists of discrete points instead of a curve.
MA.9-12.2.1.A.2	Successive terms in an arithmetic sequence have a common difference, or constant rate of change.
MA.9-12.2.1.A.3	The general term of an arithmetic sequence with a common difference d is denoted by a_n and is given by $a_n = a_0 + dn$, where a_0 is the initial value, or by $a_n = a_k + d(n - k)$, where a_k is the kth term of the sequence.
MA.9-12.2.1.B.1	Successive terms in a geometric sequence have a common ratio, or constant proportional change.
MA.9-12.2.1.B.2	The general term of a geometric sequence with a common ratio r is denoted by g_n and is given by $g_n = g_0 r^n$, where g_0 is the initial value, or by $g_n = g_k r^{(n-k)}$ where g_k is the kth term of the sequence.
MA.9-12.2.1.B.3	Increasing arithmetic sequences increase equally with each step, whereas increasing geometric sequences increase by a larger amount with each successive step.
MA.9-12.2.2.A.1	Linear functions of the form $f(x) = b + mx$ are similar to arithmetic sequences of the form $a_n = a_0 + dn$, as both can be expressed as an initial value (b or a_0) plus repeated addition of a constant rate of change, the slope (m or d).
MA.9-12.2.2.A.2	Similar to arithmetic sequences of the form $a_n = a_k + d(n - k)$, which are based on a known difference, d, and a kth term, linear functions can be expressed in the form $f(x) = y_i + m(x - x_i)$ based on a known slope, m, and a point, (x_i, y_i).
MA.9-12.2.2.A.3	Exponential functions of the form $f(x) = ab^x$ are similar to geometric sequences of the form $g_n = g_0 r^n$, as both can be expressed as an initial value (a or g_0) times repeated multiplication by a constant proportion (b or r).
MA.9-12.2.2.A.4	Similar to geometric sequences of the form $g_n = g_k r^{(n-k)}$, which are based on a known ratio, r, and a kth term, exponential functions can be expressed in the form $f(x) = y_i r^{(x - x_i)}$ based on a known ratio, r, and a point, (x_i, y_i).
MA.9-12.2.2.A.5	Sequences and their corresponding functions may have different domains.
MA.9-12.2.2.B.1	Over equal-length input-value intervals, if the output values of a function change at constant rate, then the function is linear; if the output values of a

	function change proportionally, then the function is exponential.
MA.9-12.2.2.B.2	Linear functions of the form $f(x) = b + mx$ and exponential functions of the form $f(x) = ab^x$ can both be expressed analytically in terms of an initial value and a constant involved with change. However, linear functions are based on addition, while exponential functions are based on multiplication.
MA.9-12.2.2.B.3	Arithmetic sequences, linear functions, geometric sequences, and exponential functions all have the property that they can be determined by two distinct sequence or function values.
MA.9-12.2.3.A.1	The general form of an exponential function is $f(x) = ab^x$, with the initial value a, where $a \neq 0$, and the base b, where $b > 0$, and $b \neq 1$. When $a > 0$ and $b > 1$, the exponential function is said to demonstrate exponential growth. When $a > 0$ and $0 < b < 1$, the exponential function is said to demonstrate exponential decay.
MA.9-12.2.3.A.2	When the natural numbers are input values in an exponential function, the input value specifies the number of factors of the base to be applied to the function's initial value. The domain of an exponential function is all real numbers.
MA.9-12.2.3.A.3	Because the output values of exponential functions in general form are proportional over equal-length input-value intervals, exponential functions are always increasing or always decreasing, and their graphs are always concave up or always concave down. Consequently, exponential functions do not have extrema except on a closed interval, and their graphs do not have points of inflection.
MA.9-12.2.3.A.4	If the values of the additive transformation function $g(x) = f(x) + k$ of any function f are proportional over equal-length input-value intervals, then f is exponential.
MA.9-12.2.3.A.5	For an exponential function in general form, as the input values increase or decrease without bound, the output values will increase or decrease without bound or will get arbitrarily close to zero. That is, for an exponential function in general form, $lim\ [x \rightarrow \pm\infty]\ ab^x = \infty$, $lim\ [x \rightarrow \pm\infty]\ ab^x = -\infty$, $lim\ [x \rightarrow \pm\infty]\ ab^x = 0$.
MA.9-12.2.4.A.1	The product property for exponents states that $b^m b^n = b^{(m+n)}$. Graphically, this property implies that every horizontal translation of an exponential function, $f(x) = b^{(x+k)}$, is equivalent to a vertical dilation, $f(x) = b^{(x+k)} = b^x b^k = ab^x$, where $a = b^k$.
MA.9-12.2.4.A.2	The power property for exponents states that $(b^m)^n = b^{(mn)}$. Graphically, this property implies that every horizontal dilation of an exponential function, $f(x) = b^{(cx)}$, is equivalent to a change of the base of an exponential function, $f(x) = (b^c)^x$, where b^c is a constant and $c \neq 0$.
MA.9-12.2.4.A.3	The negative exponent property states that $b^{-n} = 1/b^n$.
MA.9-12.2.4.A.4	The value of an exponential expression involving an exponential unit fraction, such as $b^{(1/k)}$ where k is a natural number, is the kth root of b, when it exists.

MA.9-12.2.5.A.1	Exponential functions model growth patterns where successive output values over equal-length input-value intervals are proportional. When the input values are whole numbers, exponential functions model situations of repeated multiplication of a constant to an initial value.
MA.9-12.2.5.A.2	A constant may need to be added to the dependent variable values of a data set to reveal a proportional growth pattern.
MA.9-12.2.5.A.3	An exponential function model can be constructed from an appropriate ratio and initial value or from two input-output pairs. The initial value and the base can be found by solving a system of equations resulting from the two input-output pairs.
MA.9-12.2.5.A.4	Exponential function models can be constructed by applying transformations to $f(x) = ab^x$ based on characteristics of a contextual scenario or data set.
MA.9-12.2.5.A.5	Exponential function models can be constructed for a data set with technology using exponential regressions.
MA.9-12.2.5.A.6	The natural base e, which is approximately 2.718, is often used as the base in exponential functions that model contextual scenarios.
MA.9-12.2.5.B.1	For an exponential model in general form $f(x) = ab^x$, the base of the exponent, b, can be understood as a growth factor in successive unit changes in the input values and is related to a percent change in context.
MA.9-12.2.5.B.2	Equivalent forms of an exponential function can reveal different properties of the function. For example, if d represents number of days, then the base of $f(d) = 2^d$ indicates that the quantity increases by a factor of 2 every day, but the equivalent form $f(d) = 2^{7(d7)}$ indicates that the quantity increases by a factor of 2^7 every week.
MA.9-12.2.5.B.3	Exponential models can be used to predict values for the dependent variable, depending on the contextual constraints on the domain.
MA.9-12.2.6.A.1	Two variables in a data set that demonstrate a slightly changing rate of change can be modeled by linear, quadratic, and exponential function models.
MA.9-12.2.6.A.2	Models can be compared based on contextual clues and applicability to determine which model is most appropriate.
MA.9-12.2.6.B.1	A model is justified as appropriate for a data set if the graph of the residuals of a regression, the residual plot, appear without pattern.
MA.9-12.2.6.B.2	The difference between the predicted and actual values is the error in the model. Depending on the data set and context, it may be more appropriate to have an underestimate or overestimate for any given interval.
MA.9-12.2.7.A.1	If f and g are functions, the composite function $f \circ g$ maps a set of input values to a set of output values such that the output values of g are used as input values of f. For this reason, the domain of the composite function is restricted to those input values of g for which the corresponding output value is in the domain of f. $(f \circ g)(x)$ can also be represented as $f(g(x))$.

MA.9-12.2.7.A.2	Values for the composite function $f \circ g$ can be calculated or estimated from the graphical, numerical, analytical, or verbal representations of f and g by using output values from g as input values for f.
MA.9-12.2.7.A.3	The composition of functions is not commutative; that is, $f \circ g$ and $g \circ f$ are typically different functions; therefore, $f(g(x))$ and $g(f(x))$ are typically different values.
MA.9-12.2.7.A.4	If the function $f(x) = x$ is composed with any function g, the resulting composite function is the same as g; that is, $g(f(x)) = f(g(x)) = g(x)$. The function $f(x) = x$ is called the identity function. When composing two functions, the identity function acts in the same way as 0, the additive identity, when adding two numbers and 1, the multiplicative identity, when multiplying two numbers.
MA.9-12.2.7.B.1	Function composition is useful for relating two quantities that are not directly related by an existing formula.
MA.9-12.2.7.B.2	When analytic representations of the functions f and g are available, an analytic representation of $f(g(x))$ can be constructed by substituting $g(x)$ for every instance of x in f.
MA.9-12.2.7.B.3	A numerical or graphical representation of $f \circ g$ can often be constructed by calculating or estimating values for $(x, f(g(x)))$.
MA.9-12.2.7.C.1	Functions given analytically can often be decomposed into less complicated functions. When properly decomposed, the variable in one function should replace each instance of the function with which it was composed.
MA.9-12.2.7.C.2	An additive transformation of a function, f, that results in vertical and horizontal translations can be understood as the composition of $g(x) = x + k$ with f.
MA.9-12.2.7.C.3	A multiplicative transformation of a function, f, that results in vertical and horizontal dilations can be understood as the composition of $g(x) = kx$ with f.
MA.9-12.2.8.A.1	On a specified domain, a function, f, has an inverse function, or is invertible, if each output value of f is mapped from a unique input value. The domain of a function may be restricted in many ways to make the function invertible.
MA.9-12.2.8.A.2	An inverse function can be thought of as a reverse mapping of the function. An inverse function, f^{-1}, maps the output values of a function, f, on its invertible domain to their corresponding input values; that is, if $f(a) = b$, then $f^{-1}(b) = a$. Alternately, on its invertible domain, if a function consists of input-output pairs (a, b), then the inverse function consists of input-output pairs (b, a).
MA.9-12.2.8.B.1	The composition of a function, f, and its inverse function, f^{-1}, is the identity function; that is, $f(f^{-1}(x)) = f^{-1}(f(x)) = x$.
MA.9-12.2.8.B.2	On a function's invertible domain, the function's range and domain are the inverse function's domain and range, respectively. The inverse of the table

	of values of $y = f(x)$ can be found by reversing the input-output pairs; that is, (a, b) corresponds to (b, a).
MA.9-12.2.8.B.3	The inverse of the graph of the function $y = f(x)$ can be found by reversing the roles of the x- and y-axes; that is, by reflecting the graph of the function over the graph of the identity function $h(x) = x$.
MA.9-12.2.8.B.4	The inverse of the function can be found by determining the inverse operations to reverse the mapping. One method for finding the inverse of the function f is reversing the roles of x and y in the equation $y = f(x)$, then solving for $y = f^{-1}(x)$.
MA.9-12.2.8.B.5	In addition to limiting the domain of a function to obtain an inverse function, contextual restrictions may also limit the applicability of an inverse function.
MA.9-12.2.9.A.1	The logarithmic expression $log\_b\ c$ is equal to, or represents, the value that the base b must be exponentially raised to in order to obtain the value c. That is, $log\_b\ c = a$ if and only if $b^a = c$, where a and c are constants, $b > 0$, and $b \neq 1$. (when the base of a logarithmic expression is not specified, it is understood as the common logarithm with a base of 10)
MA.9-12.2.9.A.2	The values of some logarithmic expressions are readily accessible through basic arithmetic while other values can be estimated through the use of technology.
MA.9-12.2.9.A.3	On a logarithmic scale, each unit represents a multiplicative change of the base of the logarithm. For example, on a standard scale, the units might be 0, 1, 2, …, while on a logarithmic scale, using logarithm base 10, the units might be 10^0, 10^1, 10^2, ….
MA.9-12.2.10.A.1	The general form of a logarithmic function is $f(x) = a\ log\_b\ x$, with base b, where $b > 0$, $b \neq 1$, and $a \neq 0$.
MA.9-12.2.10.A.2	The way in which input and output values vary together have an inverse relationship in exponential and logarithmic functions. Output values of general-form exponential functions change proportionately as input values increase in equal-length intervals. However, input values of general-form logarithmic functions change proportionately as output values increase in equal-length intervals. Alternately, exponential growth is characterized by output values changing multiplicatively as input values change additively, whereas logarithmic growth is characterized by output values changing additively as input values change multiplicatively.
MA.9-12.2.10.A.3	$f(x) = log\_b\ x$ and $g(x) = b^x$, where $b > 0$ and $b \neq 1$, are inverse functions. That is, $g(f(x)) = f(g(x)) = x$.
MA.9-12.2.10.A.4	The graph of the logarithmic function $f(x) = log\_b\ x$, where $b > 0$ and $b \neq 1$, is a reflection of the graph of the exponential function $g(x) = b^x$, where $b > 0$ and $b \neq 1$, over the graph of the identity function $h(x) = x$.
MA.9-12.2.10.A.5	If (s, t) is an ordered pair of the exponential function $g(x) = b^x$, where $b > 0$ and $b \neq 1$, then (t, s) is an ordered pair of the logarithmic function $f(x) = log\_b\ x$, where $b > 0$ and $b \neq 1$.

MA.9-12.2.11.A.1	The domain of a logarithmic function in general form is any real number greater than zero, and its range is all real numbers.
MA.9-12.2.11.A.2	Because logarithmic functions are inverses of exponential functions, logarithmic functions are also always increasing or always decreasing, and their graphs are either always concave up or always concave down. Consequently, logarithmic functions do not have extrema except on a closed interval, and their graphs do not have points of inflection.
MA.9-12.2.11.A.3	The additive transformation function $g(x) = f(x + k)$, where $k \neq 0$, of a logarithmic function f in general form does not have input values that are proportional over equal-length output-value intervals. However, if the input values of the additive transformation function, $g(x) = f(x + k)$, of any function f are proportional over equal-length output value intervals, then f is logarithmic.
MA.9-12.2.11.A.4	With their limited domain, logarithmic functions in general form are vertically asymptotic to $x = 0$, with an end behavior that is unbounded. That is, for a logarithmic function in general form, $lim\ [x \to 0^+]\ a\ log\_b\ x = \pm\infty$ and $lim\ [x \to \infty]\ a\ log\_b\ x = \pm\infty$.
MA.9-12.2.12.A.1	The product property for logarithms states that $log\_b\ (xy) = log\_b\ x + log\_b\ y$. Graphically, this property implies that every horizontal dilation of a logarithmic function, $f(x) = log\_b\ (kx)$, is equivalent to a vertical translation, $f(x) = log\_b\ (kx) = log\_b\ k + log\_b\ x = a + log\_b\ x$, where $a = log\_b\ k$.
MA.9-12.2.12.A.2	The power property for logarithms states that $log\_b\ x^n = n\ log\_b\ x$. Graphically, this property implies that raising the input of a logarithmic function to a power, $f(x) = log\_b\ x^k$, results in a vertical dilation, $f(x) = log\_b\ x^k = k\ log\_b\ x$.
MA.9-12.2.12.A.3	The change of base property for logarithms states that $log\_b\ x = (log_a\ x)/(log_a\ b)$, where $a > 0$ and $a \neq 1$. This implies that all logarithmic functions are vertical dilations of each other.
MA.9-12.2.12.A.4	The function $f(x) = \ln x$ is a logarithmic function with the natural base e; that is, $\ln x = \log_e x$.
MA.9-12.2.13.A.1	Properties of exponents, properties of logarithms, and the inverse relationship between exponential and logarithmic functions can be used to solve equations and inequalities involving exponents and logarithms.
MA.9-12.2.13.A.2	When solving exponential and logarithmic equations found through analytical or graphical methods, the results should be examined for extraneous solutions precluded by the mathematical or contextual limitations.
MA.9-12.2.13.A.3	Logarithms can be used to rewrite expressions involving exponential functions in different ways that may reveal helpful information. Specifically, $b^x = c\hat{}(log\_c\ b)(x)$.
MA.9-12.2.13.B.1	The function $f(x) = ab^{(x - h)} + k$ is a combination of additive transformations of an exponential function in general form. The inverse of $y = f(x)$ can be

	found by determining the inverse operations to reverse the mapping.
MA.9-12.2.13.B.2	The function $f(x) = a\ log\_b\ (x - h) + k$ is a combination of additive transformations of a logarithmic function in general form. The inverse of $y = f(x)$ can be found by determining the inverse operations to reverse the mapping.
MA.9-12.2.14.A.1	Logarithmic functions are inverses of exponential functions and can be used to model situations involving proportional growth, or repeated multiplication, where the input values change proportionally over equal-length output-value intervals. Alternately, if the output value is a whole number, it indicates how many times the initial value has been multiplied by the proportion.
MA.9-12.2.14.A.2	A logarithmic function model can be constructed from an appropriate proportion and a real zero or from two input-output pairs.
MA.9-12.2.14.A.3	Logarithmic function models can be constructed by applying transformations to $f(x) = a\ log\_b\ x$ based on characteristics of a context or data set.
MA.9-12.2.14.A.4	Logarithmic function models can be constructed for a data set with technology using logarithmic regressions.
MA.9-12.2.14.A.5	The natural logarithm function is often useful in modeling real-world phenomena.
MA.9-12.2.14.A.6	Logarithmic function models can be used to predict values for the dependent variable.
MA.9-12.2.15.A.1	In a semi-log plot, one of the axes is logarithmically scaled. When the y-axis of a semi-log plot is logarithmically scaled, data or functions that demonstrate exponential characteristics will appear linear.
MA.9-12.2.15.A.2	An advantage of semi-log plots is that a constant never needs to be added to the dependent variable values to reveal that an exponential model is appropriate.
MA.9-12.2.15.B.1	Techniques used to model linear functions can be applied to a semi-log graph.
MA.9-12.2.15.B.2	For an exponential model of the form $y = ab^x$, the corresponding linear model for the semi-log plot is $y = (log_n\ b)x + log_n\ a$, where $n > 0$ and $n \neq 1$. Specifically, the linear rate of change is $log_n\ b$, and the initial linear value is $log_n\ a$.

Suggested Modifications for Special Education, ELL and Gifted Students

- Anchor charts to model strategies.
- Review Algebra concepts to ensure students have the information needed to progress in understanding.

- Pre-teach pertinent vocabulary.
- Provide reference sheets that list formulas, step-by-step procedures, theorems, and modeling of strategies.
- Word wall with visual representations of mathematical terms.
- Teacher modeling of thinking processes involved in solving, graphing, and writing equations.
- Introduce concepts embedded in real-life context to help students relate to the mathematics involved.
- Record formulas, processes, and mathematical rules in reference notebooks.
- Graphing calculator to assist with computations and graphing of trigonometric functions.
- Utilize technology through interactive sites to represent nonlinear data.
- Graphic organizers to help students interpret the meaning of terms in an expression or equation in context.
- Translation dictionary.
- Sentence stems to provide additional language support for ELL students.

Suggested Technological Innovations/Use

Cross Curricular/21st Century Connections

Model interdisciplinary thinking to expose students to other disciplines.

Social Studies and ELA Literacy Connection:

Name of Task: Americans' spending: NJSLS: **6.1.12.HistoryCC.16.b, 6.2.12.EconGE.5.a**

From July 1998 to July 1999, Americans' spending rose from 5.82 trillion dollars to 6.20 trillion dollars

a. Let $x = 0$ represent July 1998, $x = 1$ represent August 1998, ..., and $x = 12$ represent July 1999. Write a linear equation for Americans' spending in terms of the month x

b. Use the equation in (a) to predict Americans' spending in July 2002.

c. Based on the model created in (a) when would the aggregate expenditure exceed 10 trillion dollars?

d. What part of the US GDP is spent by the Americans in 2013?

Name of Task: Publishing Cost:

A publishing company estimates that the average cost (in dollars) for one copy of a new scenic calendar it

plans to produce can be approximated by the function
C (x) = (2.25x + 275)/x

2.25 x+

Where x is the number of calendars printed.

a. Find the average cost per calendar when the company prints 100 calendars.
b. Identify the domain and range of this function.
c. After analyzing the function, Alex said that this company should not be allowed to publish zero calendars. As a result, the company has no option to shut down and go out of business. Write an argument to support or reject Alex's conclusion.

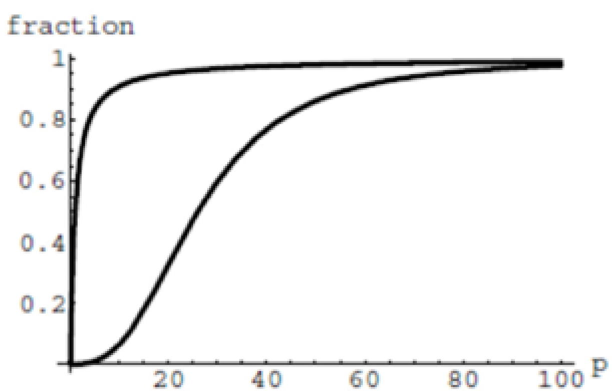
Science Connection:

Name of Task: Myoglobin and Hemoglobin: **NJSLS: HS-LS1-2; HS-LS1-4**

Myoglobin and hemoglobin are oxygen-carrying molecules in the human body. Hemoglobin is found inside red blood cells, which flow from the lungs to the muscles through the bloodstream. Myoglobin is found in muscle cells. The function y = M(p) = p/(1 + p) calculates the fraction of myoglobin saturated with oxygen at a given presure p Torrs. For example, at a pressure of 1 Torr, $M(1) = 0.5$, which means half of the myoglobin (i.e. 50%) is oxygen saturated. (Note: More precisely, you need to use something called the "partial pressure", but the distinction is not important for this problem.) Likewise, the function calculates the fraction of hemoglobin saturated with oxygen at a given pressure p. [UW]

a. The graphs of M(P) and H(P) are given here on the domain

$0 \leq p \leq 100$



Which is which?

b. If the pressure in the lungs is 100 Torrs, what is the level of oxygen saturation of the hemoglobin in the lungs?

c. The pressure in an active muscle is 20 Torrs. What is the level of oxygen saturation of myoglobin in an active muscle? What is the level of hemoglobin in an active muscle?

d. Define the efficiency of oxygen transport at a given pressure p to be M(p) – H(p) . What is the oxygen transport efficiency at 20 Torrs? At 40 Torrs? At 60 Torrs? Sketch the graph of M(p) – H(p) ; are there conditions under which transport efficiency is maximized (explain)?

Business Connection :

Name of Task: Minimize the metal in a can: **NJSLS: 9.1.12.A.4; W.11-12.1**

A manufacturer wants to manufacture a metal can that holds 1000 cm^3 of oil. The can is in the shape of a right cylinder with a radius *r* and height *h*. Assume the thickness of the material used to make the metal can is negligible.

For each question, include correct units of measurement and round your answers to the nearest tenth. Using your knowledge of volume and surface area of a right cylinder, write a function S(r) that represents the surface area of the cylindrical can in terms of the radius, r, of its base. Show in detail your algebraic thinking.

1. Sketch the graph of S(r) and show key features of the graph. State any restriction on the value of r so that it represents the physical model of the can.
2. What dimensions will minimize the quantity of metal needed to manufacture the cylindrical can? Show in detail your mathematical solution.
3. Calculate the minimum value of the function S(r) and interpret the result in the context of the physical model. Show the mathematical steps you used to obtain the answer.

Name of Task: Chemco Manufacturing**: NJSLS: 9.1.12.A.4; W.11-12.1**

Chemco Manufacturing estimates that its profit *P* in hundreds of dollars is $P = -4x^2 + 40x + 3$ where *x* is the number of units produced in thousands.

a. How many units must be produced to obtain the maximum profit?
b. Graph the profit function and identify its vertex.
c. An increase in productivity increased profit by $7 at each quantity sold. What kind of a transformation would model this situation? Show your work graphically and algebraically.
d. A decrease in marginal cost lead to a 4 units increase in the optimum level of production. What kind of a transformation would model this situation? Show your work graphically and algebraically.

Unit 3 Trigonometric and Polar Functions

Content Area: **Mathematics**
Course(s): **AP PreCalculus**
Time Period: **3rd Marking Period**
Length: **7 weeks**
Status: **Published**

Section Title

Trigonometric and Polar Functions

Enduring Understandings

Measure angles in standard position on the coordinate plane and their properties.

A radian is an angle measure with an arc length of one radius.

Label the angles on the unit circle in radians using proportional reasoning.

Use special right triangles to determine the coordinates at key points on the unit circle. Evaluate sine, cosine, and tangent for key angles on the unit circle.

Construct graphs of the sine and cosine functions by observing that as the input values, or angle measures, of the sine function increase, the output values of sine and cosine oscillate between −1 and 1.

Relate additive transformations with horizontal/vertical transformation and multiplicative transformation with dilation and use these transformations to graph transformed sine and cosine functions.

Identify information from graphical, numerical, analytical, and verbal representations to answer a question or construct a model, with and without technology

Summary of the Unit

Analyze, describe different types of angles. Analyze and describe angles and angular movement and relate this information to real life phenomena. Model and make predictions about periodic behavior using the graphs of trigonometric functions. Solve the equation using trigonometric relationships and algebraic techniques.

Essential Questions

- Since energy usage goes up and down through the year, how can I use trends in data to predict my monthly electricity bills when I get my first apartment?
- How do we model aspects of circular and spinning objects without using complex equations from the x-y rectangular-based coordinate system?
- How does right triangle trigonometry from geometry relate to trigonometric functions?

Summative Assessment and/or Summative Criteria

Required District/State Assessments

- SGO Pre Assessment
- SGO Post Assessment

Suggested Formative/Summative Classroom Assessments

- Describe Learning Vertically
- Identify Key Building Blocks
- Make Connections (between and among crucial building blocks)
- Short/Extended Constructed Response Items
- Multiple-Choice Items (where multiple answer choices may be correct)
- Drag and Drop Items
- Use of Equation Editor
- Quizzes
- Journal Entries/Reflections/Quick-Writes
- Accountable talk
- Projects

- Portfolio
- Observation
- Graphic Organizers/ Concept Mapping
- Presentations
- Teacher-Student and Student-Student Conferencing
- AP Classroom – Assessments created with specific sections outlined.
- WebAssign Problem Sets WebAssign Instructor Help
- Homework
- Students will take formal assessments, such as tests and quizzes, to assess knowledge of concepts learned throughout the unit.
- Students will also demonstrate mastery through various assessment criteria included in the unit such as do nows, exit slips, graded classwork activities and assignments, and/or projects.

Resources

- AP Daily Videos: Section 3.1 - 3.3
- AP Daily Videos: Section 3.4-3.6
- AP Daily Videos: Section 3.7
- AP Daily Videos: Section 3.8
- AP Daily Videos: Section 3.9 - 3.11
- AP Daily Videos: Section 3.13 - 3.15
- AP Precalculus Course Overview: https://apcentral.collegeboard.org/courses/ap-precalculus/course
- AP Precalculus Course and Exam Description: https://apcentral.collegeboard.org/media/pdf/ap-precalculus-course-and-exam-description.pdf
- AP Precalculus Practice Exam: https://apcentral.collegeboard.org/media/pdf/ap-precalculus-practice-exam-multiple-choice-section.pdf
- AP Precalculus Classroom Resources: https://apcentral.collegeboard.org/courses/ap-precalculus/classroom-resources

- Classpad.net (Casio): https://classpad.net/us/
- Desmos: https://www.desmos.com/
- Geogebra: https://www.geogebra.org/?lang=en
- Math Open Reference: https://www.mathopenref.com/
- TI Education (Texas Instruments): https://education.ti.com/en
- WolframAlpha: https://www.wolframalpha.com/
- Wolfram MathWorld: https://mathworld.wolfram.com/
- Khan Academy: https://www.khanacademy.org/math/precalculus
- Digital Mathematics Word Wall: http://www.mathwords.com/index_adv_alg_precal.htm
- Extra Notes for Pre-Calculus Content: https://sites.google.com/a/evergreenps.org/ms-griffin-s-math-classes/updates
- Review Documents for Pre-Calculus: https://sites.google.com/site/dgrahamcalculus/trigpre-calculus/trig-pre-calculus-worksheets
- Pre-Calculus IXL Topics and Resources: https://www.ixl.com/math/precalculus
- Classroom Challenges to Support Teachers in Formative Assessments: http://map.mathshell.org/materials/lessons.php?gradeid=24
- Applications of Function Models: https://www.ck12.org/algebra/Applications-of-Function-Models/lesson/Applications-of-Function-Models-BSC-ALG/?referrer=featured_content
- Statistics Education Web (STEW). http://www.amstat.org/education/STEW/
- The Data and Story Library (DASL). http://lib.stat.cmu.edu/DASL/
- WebAssign Instructor Resources
- WebAssign Student Resources
- Cengage Learning: PreCalculus with Limits - A Graphing Approach, Sixth Edition

Topic/Selection Timeframe	General Objectives	Instructional Activities	Benchmarks/Assessments	Standards
3.1 Periodic Phenomena (2 days)	SWBAT Construct graphs of periodic relationships based on verbal representations. SWBAT Describe key characteristics of a periodic function based on a verbal representation.	Students will determine how to measure angles in standard position on the coordinate plane and their properties. Students will determine that a radian is an angle measure with an arc length of one radius. Students will label the angles on the unit circle in radians using proportional reasoning.	Circulate and monitor student progress as they are working on classwork. Have students complete problems at the board. Through questioning students will be able to determine coordinates of unit circle. Closure	3.1.A.1 3.1.A.2 3.1.B.1 3.1.B.2 3.1.B.3

3.2 Sine, Cosine, and Tangent (3 days)	SWBAT Determine the sine, cosine, and tangent of an angle using the unit circle.	Students will use special right triangles to determine the coordinates at key points on the unit circle. Students will evaluate sine, cosine, and tangent for key angles on the unit circle. Students will find coordinates of points on circles where $r \neq 1$.	Circulate and monitor student progress as they are working on classwork. Have students complete problems at the board. Through questioning students will be able to evaluate sine, cosine, and tangent for key angles on the unit circle. Closure	3.2.A.1 3.2.A.2 3.2.A.3 3.2.A.4 3.2.A.5

3.3 Sine and Cosine Function Values (3 days)	SWBAT Determine coordinates of points on a circle centered at the origin.	Students will determine that in a unit circle, the sine and cosine ratios correspond to the y-value and x-value, respectively, of the point where the terminal ray intersects the circle. Students then will be able to use symmetry to identify relationships between the sine and cosine values of angles in all four quadrants.	Circulate and monitor student progress as they are working on classwork. Have students complete problems at the board. Through questioning students will be able to determine values of any angle in the unit circle. Closure	3.3.A.1 3.3.A.2
3.4 Sine and Cosine Function Graphs (3 days)	Construct representations of the sine and cosine functions using the unit circle.	Students would be able to construct graphs of the sine and cosine functions by observing that as the input values, or angle measures, of the sine function increase, the output values of sine and and cosine oscillate between −1 and 1, taking every value in between and tracking the vertical distance of points on the unit circle from the x-axis.	Circulate and monitor student progress as they are working on classwork. Have students complete problems at the board. Through questioning students will be able to construct graphs of the sine and cosine functions using values from the unit circle. Closure	3.4.A.1 3.4.A.2 3.4.A.3 3.4.A.4

3.5 Sinusoidal Functions (3 days)	SWBAT Identify key characteristics of the sine and cosine functions.	Students would be able to determine that a sinusoidal function is any function that involves additive and multiplicative transformations of sin x. The sine and cosine functions are both sinusoidal functions. The period and frequency of a sinusoidal function are reciprocals. The amplitude of a sinusoidal function is half the difference between its maximum and minimum values. The midline of the graph of a sinusoidal function is determined by the average, or arithmetic mean, of the maximum and minimum values of the function. The midline of the graphs of y = sinθ and y = cosθ is y = 0. As input values increase, the graphs of sinusoidal functions oscillate between concave down and concave up. The graph of y = sinx has rotational symmetry about the origin and is therefore an odd function. The graph of y = cosx has reflective symmetry over the y-axis and is therefore an even function.	Circulate and monitor student progress as they are working on classwork. Have students complete problems at the board. Through questioning students will be able to Identify key characteristics for the parent functions y=sin x and y=cos x, including domain, amplitude, midline, period, and symmetry. Closure	3.5.A.1 3.5.A.2 3.5.A.3 3.5.A.4 3.5.A.5

3.6 Sinusoidal Function Transformations (3 days)	SWBAT Identify the amplitude, vertical shift, period, and phase shift of a sinusoidal function.	Students will determine how the amplitude, period, domain, range, and midline of sinusoidal functions are affected by transformations. Students would be able relate additive transformations with horizontal/vertical transformation and and multiplicative transformation with dilation and use these transformations to graph transformed sine and cosine functions.	Circulate and monitor student progress as they are working on classwork. Have students complete problems at the board. Through questioning students will be able to graph transformed sine and cosine functions given an equation . Closure	3.6.A.1 3.6.A.2 3.6.A.3 3.6.A.4 3.6.A.5 3.6.A.6
3.7 Sinusoidal Function Context and Data Modeling (3 days)	SWBAT Construct sinusoidal function models of periodic phenomena.	Students will determine how to use trigonometric ratios and angle measured relative to the north-south line to solve problems using navigations. Students will Identify information from graphical, numerical, analytical, and verbal representations to answer a question or construct a model, with and without technology. Students would be able to describe the characteristics of a function with varying levels of precision, depending on the	Circulate and monitor student progress as they are working on classwork. Have students complete problems at the board. Through questioning students will be able to Interpret a sinusoidal function's period, amplitude, midline, and range in context be able to construct a trigonometric model based on data points and key features. Closure	3.7.A.1 3.7.A.2 3.7.A.3 3.7.A.4 3.7.A.5

		function representation and available mathematical tools.		
3.8 The Tangent Function (2 days)	SWBAT Construct representations of the tangent function using the unit circle. SWBAT Describe key characteristics of the tangent function. SWBAT Describe additive and multiplicative transformations involving the tangent function.	Students would be able to construct graphs of the tangent functions by observing that as the input values, or angle measures, of the tangent function increase. The asymptote equations are found when the function in the denominator is zero.	Circulate and monitor student progress as they are working on classwork. Have students complete problems at the board. Through questioning students will be able to evaluate tangent for key angles on the unit circle. Closure	3.8.A.1 3.8.A.2 3.8.B.1 3.8.B.2 3.8.B.3
3.9 Inverse Trigonometric Functions (3 days)	SWBAT Construct analytical and graphical representations of the inverse of the sine, cosine, and tangent functions over a restricted domain.	SWBAT understand that inverse trigonometric functions input ratios and output angles. The input and output values are switched from their corresponding trigonometric functions. SWBAT understand through graphical representation why and how the domains of sine, cosine, and tangent must be restricted to create an inverse function. SWBAT evaluate inverse trig	Circulate and monitor student progress as they are working on classwork. Have students complete problems at the board. Through questioning students will be able to evaluate inverse of sine, cosine, and tangent for key points on the unit circle. Closure	3.9.A.1 3.9.A.2 3.9.A.3

		expressions.		
3.10 Trigonometric Equations and Inequalities (4 days)	SWBAT Solve equations and inequalities involving trigonometric functions.	SWBAT extend the process of inverse operations to trigonometric equations and inequalities. SWBAT understand that using the unit circle will give infinite solutions to a trigonometric equation which may need to be restricted based on context and that an inverse trig function gives only one solution that may need to be expanded using symmetry.	Circulate and monitor student progress as they are working on classwork. Have students complete problems at the board. Through questioning students will be able to find solutions given trigonometric equations. Closure	3.10.A.1 3.10.A.2 3.10.A.3
3.11 The Secant, Cosecant, and Cotangent Functions (3 days)	SWBAT Identify key characteristics of functions that involve quotients of the sine and cosine functions.	SWBAT define the secant, cosecant, and cotangent functions as the reciprocal of the cosine, sine, and tangent functions, respectively. SWBAT understand how the zeros, vertical asymptotes, and range are related for a trigonometric function and its reciprocal function.	Circulate and monitor student progress as they are working on classwork. Have students complete problems at the board. Through questioning students will be able to Identify key characteristics for the parent functions y=csc x and y=sec x and cotangent including domain, period, asymptotes and symmetry. Closure	3.11.A.1 3.11.A.2 3.11.A.3 3.11.A.4 3.11.A.5
3.12 Equivalent Representations of	SWBAT Rewrite trigonometric expressions in	SWBAT explore relationships between all six trigonometric	Circulate and monitor student progress as they are	3.12.A.1

Trigonometric Functions (3 days)	equivalent forms with the Pythagorean identity. SWBAT Rewrite trigonometric expressions in equivalent forms with sine and cosine sum identities. SWBAT Solve equations using equivalent analytic representations of trigonometric functions.	functions, including the Pythagorean identities. SWBAT use identities to establish and verify other trigonometric relationships and solve trigonometric equations	working on classwork. Have students complete problems at the board. Through questioning students will be able to verify trigonometric relationships. Closure	3.12.A.2 3.12.B.1 3.12.B.2 3.12.B.3 3.12.B.4
3.13 Trigonometry and Polar Coordinates (3 days)	SWBAT Determine the location of a point in the plane using both rectangular and polar coordinates.	SWBAT understand that polar coordinates give an alternate method for locating points using a distance from the origin and an angle from the positive x-axis. SWBAT use coterminal angles and reflected radii to name polar points in multiple ways. SWBAT convert between polar and rectangular coordinates.	Circulate and monitor student progress as they are working on classwork. Have students complete problems at the board. Through questioning students will be able to plot coordinates, and convert between polar and rectangular coordinates. Closure	3.13.A.1 3.13.A.2 3.13.A.3 3.13.A.4
3.14 Polar Function Graphs (3 days)	SWBAT Construct graphs of polar functions.	SWBAT understand that polar functions input angle measures and output radii and point-by-point	Circulate and monitor student progress as they are working on classwork.	3.14.A.1 3.14.A.2

		graphing can be used to construct their graphs.	Have students complete problems at the board. Through questioning students will be able to graph polar graphs using key features. Closure	3.14.A.3
3.15 Rates of Change in Polar Functions (3 days)	SWBAT Describe characteristics of the graph of a polar function.	SWBAT identify the number, length, and location of petals of a polar rose from the values of the parameters, a and n. SWBAT identify special types of limacons by comparing values of the parameters, a and b. SWBAT describe key features of the graphs of circles, roses and limacons including symmetry, intercepts, domain, and range, and maximum and minimum values.	Circulate and monitor student progress as they are working on classwork. Have students complete problems at the board. Through questioning students will be able to graph and identify special polar curves. Closure	3.15.A.1 3.15.A.2 3.15.A.3 3.15.A.4 3.15.A.5

MA.9-12.3.1.A.1	A periodic relationship can be identified between two aspects of a context if, as the input values increase, the output values demonstrate a repeating pattern over successive equal-length intervals.
MA.9-12.3.1.A.2	The graph of a periodic relationship can be constructed from the graph of a

	single cycle of the relationship.
MA.9-12.3.1.B.1	The period of the function is the smallest positive value k such that $f(x + k) = f(x)$ for all x in the domain. Consequently, the behavior of a periodic function is completely determined by any interval of width k.
MA.9-12.3.1.B.2	The period can be estimated by investigating successive equal-length output values and finding where the pattern begins to repeat.
MA.9-12.3.1.B.3	Periodic functions take on characteristics of other functions, such as intervals of increase and decrease, different concavities, and various rates of change. However, with periodic functions, all characteristics found in one period of the function will be in every period of the function.
MA.9-12.3.2.A.1	In the coordinate plane, an angle is in standard position when the vertex coincides with the origin and one ray coincides with the positive x-axis. The other ray is called the terminal ray. Positive and negative angle measures indicate rotations from the positive x-axis in the counterclockwise and clockwise direction, respectively. Angles in standard position that share a terminal ray differ by an integer number of revolutions.
MA.9-12.3.2.A.2	The radian measure of an angle in standard position is the ratio of the length of the arc of a circle centered at the origin subtended by the angle to the radius of that same circle. For a unit circle, which has radius 1, the radian measure is the same as the length of the subtended arc.
MA.9-12.3.2.A.3	Given an angle in standard position and a circle centered at the origin, there is a point, P, where the terminal ray intersects the circle. The sine of the angle is the ratio of the vertical displacement of P from the x-axis to the distance between the origin and point P. Therefore, for a unit circle, the sine of the angle is the y-coordinate of point P.
MA.9-12.3.2.A.4	Given an angle in standard position and a circle centered at the origin, there is a point, P, where the terminal ray intersects the circle. The cosine of the angle is the ratio of the horizontal displacement of P from the y-axis to the distance between the origin and point P. Therefore, for a unit circle, the cosine of the angle is the x-coordinate of point P.
MA.9-12.3.2.A.5	Given an angle in standard position, the tangent of the angle is the slope, if it exists, of the terminal ray. Because the slope of the terminal ray is the ratio of the vertical displacement to the horizontal displacement over any interval, the tangent of the angle is the ratio of the y-coordinate to the x-coordinate of the point at which the terminal ray intersects the unit circle; alternately, it is the ratio of the angle's sine to its cosine.
MA.9-12.3.3.A.1	Given an angle of measure θ in standard position and a circle with radius r centered at the origin, there is a point, P, where the terminal ray intersects the circle. The coordinates of point P are $(r \cos \theta, r \sin \theta)$.
MA.9-12.3.3.A.2	The geometry of isosceles right and equilateral triangles, while attending to the signs of the values based on the quadrant of the angle, can be used to find exact values for the cosine and sine of angles that are multiples of $\pi/4$ and $\pi/6$ radians and whose terminal rays do not lie on an axis.

MA.9-12.3.4.A.1	Given an angle of measure θ in standard position and a unit circle centered at the origin, there is a point, P, where the terminal ray intersects the circle. The sine function, $f(\theta) = \sin\theta$, gives the y-coordinate, or vertical displacement from the x-axis, of point P. The domain of the sine function is all real numbers.
MA.9-12.3.4.A.2	As the input values, or angle measures, of the sine function increase, the output values oscillate between –1 and 1, taking every value in between and tracking the vertical distance of points on the unit circle from the x-axis.
MA.9-12.3.4.A.3	Given an angle of measure θ in standard position and a unit circle centered at the origin, there is a point, P, where the terminal ray intersects the circle. The cosine function, $f(\theta) = \cos\theta$, gives the x-coordinate, or horizontal displacement from the y-axis, of point P. The domain of the cosine function is all real numbers.
MA.9-12.3.4.A.4	As the input values, or angle measures, of the cosine function increase, the output values oscillate between –1 and 1, taking every value in between and tracking the horizontal distance of points on the unit circle from the y-axis.
MA.9-12.3.5.A.1	A sinusoidal function is any function that involves additive and multiplicative transformations of $f(\theta) = \sin\theta$. The sine and cosine functions are both sinusoidal functions, with $\cos\theta = \sin(\theta + \pi/2)$.
MA.9-12.3.5.A.2	The period and frequency of a sinusoidal function are reciprocals. The period of $f(\theta) = \sin\theta$ and $g(\theta) = \cos\theta$ is 2π, and the frequency is $1/2\pi$.
MA.9-12.3.5.A.3	The amplitude of a sinusoidal function is half the difference between its maximum and minimum values. The amplitude of $f(\theta) = \sin\theta$ and $g(\theta) = \cos\theta$ is 1.
MA.9-12.3.5.A.4	The midline of the graph of a sinusoidal function is determined by the average, or arithmetic mean, of the maximum and minimum values of the function. The midline of the graphs of $y = \sin\theta$ and $y = \cos\theta$ is $y = 0$.
MA.9-12.3.5.A.5	As input values increase, the graphs of sinusoidal functions oscillate between concave down and concave up.
MA.9-12.3.5.A.6	The graph of $y = \sin\theta$ has rotational symmetry about the origin and is therefore an odd function. The graph of $y = \cos\theta$ has reflective symmetry over the y-axis and is therefore an even function.
MA.9-12.3.6.A.1	Functions that can be written in the form $f(\theta) = a\sin(b(\theta + c)) + d$ or $g(\theta) = a\cos(b(\theta + c)) + d$, where a, b, c, and d are real numbers and $a \neq 0$, are sinusoidal functions and are transformations of the sine and cosine functions. Additive and multiplicative transformations are the same for both sine and cosine because the cosine function is a phase shift of the sine function by $-\pi/2$ units.
MA.9-12.3.6.A.2	The graph of the additive transformation $g(\theta) = \sin\theta + d$ of the sine function $f(\theta) = \sin\theta$ is a vertical translation of the graph of f, including its midline, by d units. The same transformation of the cosine function yields the same result.

MA.9-12.3.6.A.3	The graph of the additive transformation $g(\theta) = \sin(\theta + c)$ of the sine function $f(\theta) = \sin\theta$ is a horizontal translation, or phase shift, of the graph of f by $-c$ units. The same transformation of the cosine function yields the same result.
MA.9-12.3.6.A.4	The graph of the multiplicative transformation $g(\theta) = a\sin\theta$ of the sine function $f(\theta) = \sin\theta$ is a vertical dilation of the graph of f and differs in amplitude by a factor of $\|a\|$. The same transformation of the cosine function yields the same result.
MA.9-12.3.6.A.5	The graph of the multiplicative transformation $g(\theta) = \sin(b\theta)$ of the sine function $f(\theta) = \sin\theta$ is a horizontal dilation of the graph of f and differs in period by a factor of $\|1/b\|$. The same transformation of the cosine function yields the same result.
MA.9-12.3.6.A.6	The graph of $y = f(\theta) = a\sin(b(\theta + c)) + d$ has an amplitude of $\|a\|$ units, a period of $\|1/b\|\, 2\pi$ units, a midline vertical shift of d units from $y = 0$, and a phase shift of $-c$ units. The same transformations of the cosine function yield the same results.
MA.9-12.3.7.A.1	The smallest interval of input values over which the maximum or minimum output values start to repeat, that is, the input-value interval between consecutive maxima or consecutive minima, can be used to determine or estimate the period and frequency for a sinusoidal function model.
MA.9-12.3.7.A.2	The maximum and minimum output values can be used to determine or estimate the amplitude and vertical shift for a sinusoidal function model.
MA.9-12.3.7.A.3	An actual pair of input-output values can be compared to pairs of input-output values produced by a sinusoidal function model to determine or estimate a phase shift for the model.
MA.9-12.3.7.A.4	Sinusoidal function models can be constructed for a data set with technology by estimating key values or using sinusoidal regressions.
MA.9-12.3.7.A.5	Sinusoidal functions that model a data set are frequently only useful over their contextual domain and can be used to predict values of the dependent variable from values of the independent variable.
MA.9-12.3.8.B.1	Because the slope values of the terminal ray repeat every one-half revolution of the circle, the tangent function has a period of π.
MA.9-12.3.8.B.2	The tangent function demonstrates periodic asymptotic behavior at input values $\theta = \pi/2 + k\pi$, for integer values of k, because $\cos\theta = 0$ at those values.
MA.9-12.3.8.B.3	The tangent function increases and its graph changes from concave down to concave up between consecutive asymptotes.
MA.9-12.3.8.C.1	The graph of the additive transformation $g(\theta) = \tan\theta + d$ of the tangent function $f(\theta) = \tan\theta$ is a vertical translation of the graph of f and the line containing its points of inflection by d units.
MA.9-12.3.8.C.2	The graph of the additive transformation $g(\theta) = \tan(\theta + c)$ of the tangent function $f(\theta) = \tan\theta$ is a horizontal translation, or phase shift, of the graph

	of f by $-c$ units.
MA.9-12.3.8.C.3	The graph of the multiplicative transformation $g(\theta) = a \tan \theta$ of the tangent function $f(\theta) = \tan \theta$ is a vertical dilation of the graph of f by a factor of $\lvert a \rvert$. If $a < 0$, the transformation involves a reflection over the x-axis.
MA.9-12.3.8.C.4	The graph of the multiplicative transformation $g(\theta) = \tan (b\theta)$ of the tangent function $f(\theta) = \tan \theta$ is a horizontal dilation of the graph of f and differs in period by a factor of $\lvert 1/b \rvert$. If $b < 0$, the transformation involves a reflection over the y-axis.
MA.9-12.3.8.C.5	The graph of $y = f(\theta) = a \tan (b(\theta + c)) + d$ is a vertical dilation of the graph of $y = \tan \theta$ by a factor of $\lvert a \rvert$, has a period of $\lvert 1/b \rvert \pi$ units, is a vertical shift of the line containing the points of inflection of the graph of $y = \tan \theta$ by d units, and is a phase shift of $-c$ units.
MA.9-12.3.9.A.1	For inverse trigonometric functions, the input and output values are switched from their corresponding trigonometric functions, so the output value of an inverse trigonometric function is often interpreted as an angle measure and the input is a value in the range of the corresponding trigonometric function.
MA.9-12.3.9.A.2	The inverse trigonometric functions are called arcsine, arccosine, and arctangent (also represented as $\sin^{-1}x$, $\cos^{-1}x$, and $\tan^{-1}x$). Because the corresponding trigonometric functions are periodic, they are only invertible if they have restricted domains.
MA.9-12.3.9.A.3	In order to define their respective inverse functions, the domain of the sine function is restricted to $[-\pi/2, \pi/2]$, the cosine function to $[0, \pi]$, and the tangent function to $(-\pi/2, \pi/2)$.
MA.9-12.3.10.A.1	Inverse trigonometric functions are useful in solving equations and inequalities involving trigonometric functions, but solutions may need to be modified due to domain restrictions.
MA.9-12.3.10.A.2	Because trigonometric functions are periodic, there are often infinitely many solutions to trigonometric equations.
MA.9-12.3.10.A.3	In trigonometric equations and inequalities arising from a contextual scenario, there is often a domain restriction that can be implied from the context, which limits the number of solutions.
MA.9-12.3.11.A.1	The secant function, $f(\theta) = \sec \theta$, is the reciprocal of the cosine function, where $\cos \theta \neq 0$.
MA.9-12.3.11.A.2	The cosecant function, $f(\theta) = \csc \theta$, is the reciprocal of the sine function, where $\sin \theta \neq 0$.
MA.9-12.3.11.A.3	The graphs of the secant and cosecant functions have vertical asymptotes where cosine and sine are zero, respectively, and have a range of $(-\infty, -1] \cup [1, \infty)$.
MA.9-12.3.11.A.4	The cotangent function, $f(\theta) = \cot \theta$, is the reciprocal of the tangent function, where $\tan \theta \neq 0$. Equivalently, $\cot \theta = \cos \theta/\sin \theta$, where $\sin \theta \neq 0$.

MA.9-12.3.11.A.5	The graph of the cotangent function has vertical asymptotes for domain values where tan θ = 0 and is decreasing between consecutive asymptotes.
MA.9-12.3.12.A.1	The Pythagorean Theorem can be applied to right triangles with points on the unit circle at coordinates (cos θ, sin θ), resulting in the Pythagorean identity: $\sin^2 \theta + \cos^2 \theta = 1$.
MA.9-12.3.12.A.2	The Pythagorean identity can be algebraically manipulated into other forms involving trigonometric functions, such as $\tan^2 \theta = \sec^2 \theta - 1$, and can be used to establish other trigonometric relationships, such as $\arcsin x = \arccos (\sqrt{1 - x^2})$, with appropriate domain restrictions.
MA.9-12.3.12.B.1	The sum identity for sine is $\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$.
MA.9-12.3.12.B.2	The sum identity for cosine is $\cos(\alpha + \beta) \cos \alpha \cos \beta - \sin \alpha \sin \beta$.
MA.9-12.3.12.B.3	The sum identities for sine and cosine can also be used as difference and double-angle identities.
MA.9-12.3.12.B.4	Properties of trigonometric functions, known trigonometric identities, and other algebraic properties can be used to verify additional trigonometric identities.
MA.9-12.3.12.C.1	A specific equivalent form involving trigonometric expressions can make information more accessible.
MA.9-12.3.12.C.2	Equivalent trigonometric forms may be useful in solving trigonometric equations and inequalities.
MA.9-12.3.13.A.1	The polar coordinate system is based on a grid of circles centered at the origin and on lines through the origin. Polar coordinates are defined as an ordered pair, (r, θ), such that $\lvert r \rvert$ represents the radius of the circle on which the point lies, and θ represents the measure of an angle in standard position whose terminal ray includes the point. In the polar coordinate system, the same point can be represented many ways.
MA.9-12.3.13.A.2	The coordinates of a point in the polar coordinate system, (r, θ), can be converted to coordinates in the rectangular coordinate system, (x, y), using $x = r \cos \theta$ and $y = r \sin \theta$.
MA.9-12.3.13.A.3	The coordinates of a point in the rectangular coordinate system, (x, y), can be converted to coordinates in the polar coordinate system, (r, θ), using $r = \sqrt{(x^2 + y^2)}$ and $\theta = \arctan (y/x)$ for $x > 0$ or $\theta = \arctan (y/x) + \pi$ for $x < 0$.
MA.9-12.3.13.A.4	A complex number can be understood as a point in the complex plane and can be determined by its corresponding rectangular or polar coordinates. When the complex number has the rectangular coordinates (a, b), it can be expressed as $a + bi$. When the complex number has polar coordinates (r, θ), it can be expressed as $(r \cos \theta) + i(r \sin \theta)$.
MA.9-12.3.14.A.1	The graph of the function $r = f(\theta)$ in polar coordinates consists of input-output pairs of values where the input values are angle measures and the output values are radii.
MA.9-12.3.14.A.2	The domain of the polar function $r = f(\theta)$, given graphically, can be

	restricted to a desired portion of the function by selecting endpoints corresponding to the desired angle and radius.
MA.9-12.3.14.A.3	When graphing polar functions in the form of $r = f(\theta)$, changes in input values correspond to changes in angle measure from the positive x-axis, and changes in output values correspond to changes in distance from the origin.
MA.9-12.3.15.A.1	If a polar function, $r = f(\theta)$, is positive and increasing or negative and decreasing, then the distance between $f(\theta)$ and the origin is increasing.
MA.9-12.3.15.A.2	If a polar function, $r = f(\theta)$, is positive and decreasing or negative and increasing, then the distance between $f(\theta)$ and the origin is decreasing.
MA.9-12.3.15.A.4	The average rate of change of r with respect to θ over an interval of θ is the ratio of the change in the radius values to the change in θ over an interval of θ. Graphically, the average rate of change indicates the rate at which the radius is changing per radian.
MA.9-12.3.15.A.5	The average rate of change of r with respect to θ over an interval of θ can be used to estimate values of the function within the interval.

Suggested Modifications for Special Education, ELL and Gifted Students

- Anchor charts to model strategies.
- Review Algebra concepts to ensure students have the information needed to progress in understanding.
- Pre-teach pertinent vocabulary.
- Provide reference sheets that list formulas, step-by-step procedures, theorems, and modeling of strategies.
- Word wall with visual representations of mathematical terms.
- Teacher modeling of thinking processes involved in solving, graphing, and writing equations.
- Introduce concepts embedded in real-life context to help students relate to the mathematics involved.
- Record formulas, processes, and mathematical rules in reference notebooks.
- Graphing calculator to assist with computations and graphing of trigonometric functions.
- Utilize technology through interactive sites to represent nonlinear data.
- Graphic organizers to help students interpret the meaning of terms in an expression or equation in context.
- Translation dictionary.

- Sentence stems to provide additional language support for ELL students.

Suggested Technological Innovations/Use

- TI-84 graphing calculator
- Desmos

Cross Curricular/21st Century Connections

Model interdisciplinary thinking to expose students to other disciplines.

Model interdisciplinary thinking to expose students to other disciplines.

Art connection:

Name of Task: Math Music NJSLS: 1.3B.12. Cr1a

A pure tone produces a sine wave when shown on an oscilloscope. When an instrument is played, the tone is not pure. For instance, when a guitar string or piano string vibrates it does not produce a simple sine wave. It does produce other, less distinguishable harmonious waves of higher pitch called harmonic waves. For instance, y =sin (2x) is called the second harmonic, y= sin (3x) is called the third harmonic, and y= sin (4x) is called the fourth harmonic.

Science Connection:

Name of Task: Rabbits, Rabbits Everywhere NJSLS: HS-LS4-2, HS-LS4-3, HS-LS4-5

The rabbit population in a national park rises and falls throughout the year. The population is at its approximate minimum of 6000 rabbits in December. As the weather gets warmer and food becomes more available, the population grows to its approximate maximum of 16,000 rabbits in June. The function describing the rabbit population is f(x)=

5000sin 6x-2+11,000 where x is the time in months and f(x) is the rabbit population.

Name of Task: Speed of CD-RW NJSLS: HS-PS2-1, HS-PS2-2

A CD-RW has a diameter of 120 millimeters. When playing audio, the angular speed varies to keep the linear speed constant where the disc is being read. When reading along the outer edge of the disc, the angular speed is about 200 RPM (revolutions per minute).

1. Find the linear speed.
2. What would the angular speed be when you reach half of the CD?
3. When being burned in this writable CD-R drive, the angular speed of the CD is often much faster than when playing audio, but the angular speed still varies to keep the linear speed constant where the disc is being written. When writing along the outer edge of the disc, the angular speed of one drive is about 4800 RPM (revolutions per minute). Find the linear speed.

Unit 4 Functions Involving Parameters, Vectors, and Matrices

Content Area: **Mathematics**
Course(s): **AP PreCalculus**
Time Period: 4^{th} Marking Period

Length: **2 weeks**
Status: **Published**

Section Title

Unit 4 Functions Involving Parameters, Vectors, and Matrices

Enduring Understandings

Students will be able to

- Evaluate sets of parametric equations for given values of the parameter.
- Graph curves that are represented by sets of parametric equations with and without graphing calculators.
- Rewrite sets of parametric equations as single rectangular equations by eliminating the parameters.
- Use time as a parameter in parametric equations.
- Find parametric equations for curves defined by rectangular equations.
- Represent vectors as directed line segments.
- Write the component forms of vectors.
- Perform basic vector operations and represent vectors graphically.
- Write vectors as linear combination of unit vectors.
- Find the direction angles of vectors.
- Use vectors to model and solve real-life problems.
- Find the dot product of two vectors and use the properties of the dot product.
- Find the angle between two vectors and determine whether two vectors are orthogonal.
- Write vectors as the sums of two vector components.
- Use vectors to find the work done by a force.
- Find the distance between two points in space.
- How do you use matrices to solve system of equations?

Summary of the Unit

Parametric equations can be used to model the path of an object. These objects can be represented by conics (ellipses, parabolas, and hyperbolas). Conics are applied in real-world situations, such as orbits of planets, flashlights, and satellites. Since conics are not always functions, they can be defined using parametric equations. The study of vectors is crucial in applied mathematics. Vectors relate to geometry, trigonometry, and physics.

Essential Questions

- How can we determine when the populations of species in an ecosystem will be relatively steady?
- How can we analyze the vertical and horizontal aspects of motion independently?
- How does high resolution computer generated imaging achieve smooth and realistic motion on screen with so many pixels?
- What is a vector in the plane?
- How do you represent and perform operations with vector quantities?
- How do you write a vector as a sum of two vector components?
- What is the dot product? How is it used to analyze vectors?
- What makes a vector orthogonal?
- What do vector quantities signify in real life situations?
- How do you locate points, and find distances in three dimensions?

Summative Assessment and/or Summative Criteria

Required District/State Assessments

- SGO Pre Assessment
- SGO Post Assessment

Suggested Formative/Summative Classroom Assessments

- Describe Learning Vertically
- Identify Key Building Blocks
- Make Connections (between and among crucial building blocks)
- Short/Extended Constructed Response Items
- Multiple-Choice Items (where multiple answer choices may be correct)
- Drag and Drop Items
- Use of Equation Editor

- Quizzes
- Journal Entries/Reflections/Quick-Writes
- Accountable talk
- Projects
- Portfolio
- Observation
- Graphic Organizers/ Concept Mapping
- Presentations
- Teacher-Student and Student-Student Conferencing
- AP Classroom – Assessments created with specific sections outlined.
- WebAssign Problem Sets WebAssign Instructor Help
- Homework
- Students will take formal assessments, such as tests and quizzes, to assess knowledge of concepts learned throughout the unit.
- Students will also demonstrate mastery through various assessment criteria included in the unit such as do nows, exit slips, graded classwork activities and assignments, and/or projects.

Resources

- AP Daily Videos: Section 4.1 - 4.2
- AP Daily Videos: Section 4.4 , 4.7
- AP Daily Videos: Section 4.8
- AP Daily Videos: Section 4.10- 4.11
- AP Precalculus Course Overview: https://apcentral.collegeboard.org/courses/ap-precalculus/course
- AP Precalculus Course and Exam Description: https://apcentral.collegeboard.org/media/pdf/ap-precalculus-course-and-exam-description.pdf
- AP Precalculus Practice Exam: https://apcentral.collegeboard.org/media/pdf/ap-precalculus-practice-exam-multiple-choice-section.pdf
- AP Precalculus Classroom Resources: https://apcentral.collegeboard.org/courses/ap-

precalculus/classroom-resources

- Classpad.net (Casio): https://classpad.net/us/
- Desmos: https://www.desmos.com/
- Geogebra: https://www.geogebra.org/?lang=en
- Math Open Reference: https://www.mathopenref.com/
- TI Education (Texas Instruments): https://education.ti.com/en
- WolframAlpha: https://www.wolframalpha.com/
- Wolfram MathWorld: https://mathworld.wolfram.com/
- Khan Academy: https://www.khanacademy.org/math/precalculus
- Digital Mathematics Word Wall: http://www.mathwords.com/index_adv_alg_precal.htm
- Extra Notes for Pre-Calculus Content: https://sites.google.com/a/evergreenps.org/ms-griffin-s-math-classes/updates
- Review Documents for Pre-Calculus: https://sites.google.com/site/dgrahamcalculus/trigpre-calculus/trig-pre-calculus-worksheets
- Pre-Calculus IXL Topics and Resources: https://www.ixl.com/math/precalculus
- Classroom Challenges to Support Teachers in Formative Assessments: http://map.mathshell.org/materials/lessons.php?gradeid=24
- Applications of Function Models: https://www.ck12.org/algebra/Applications-of-Function-Models/lesson/Applications-of-Function-Models-BSC-ALG/?referrer=featured_content
- Statistics Education Web (STEW). http://www.amstat.org/education/STEW/
- The Data and Story Library (DASL). http://lib.stat.cmu.edu/DASL/
- WebAssign Instructor Resources
- WebAssign Student Resources
- Cengage Learning: PreCalculus with Limits - A Graphing Approach, Sixth Edition

Unit Plan

Topic/Selection Timeframe	General Objectives	Instructional Activities	Benchmarks/Assessments	Standards
4.1 Parametric Functions (2 days)	SWBAT Construct a graph or table of values for a parametric function represented analytically.	Students would be able to determine when is it advantageous to define curves parametrically: Students will work in groups to determine the orientation of a curve, and thus the usefulness of parametric equations.	Circulate and monitor student progress as they are working on classwork. Have students complete problems at the board. Through questioning students will be able to describe how quantities change with respect to each other in a parametric function. Closure	4.1.A.1 4.1.A.2 4.1.A.3 4.1.A.4 4.1.A.5

4.2 Parametric Functions Modeling Planar Motion (2 days)	SWBAT Identify key characteristics of a parametric planar motion function that are related to position.	Students will determine to write equations to describe the motion of a point in a plane they need to introduce a third variable, or a parameter, parametric equations with those involving only x and y.	Circulate and monitor student progress as they are working on classwork. Have students complete problems at the board. Through questioning students will be able to determine how do you write equations to describe the motion of a point in a plane. Closure	4.2.A.1 4.2.A.2 4.2.A.3

4.4 Parametrically defined Circles and Lines (2 days)	SWBAT express motion around a circle or along a line segment parametrically	Students will determine a complete counterclockwise revolution around the unit circle that starts and ends at (1, 0) and is centered at the origin can be modeled by relating x and y to cost and sint with restricted domain.	Circulate and monitor student progress as they are working on classwork. Have students complete problems at the board. Through questioning students will be able to determine transformation of the parametric function (x, y) = (cost, sint) can be model any circular path transverse in the plane. Closure	4.4.A.1 4.4.A.2
4.7 Parametrization of Implicitly Defined Functions (4 days)	SWBAT represent a curve in the plane parametrically.	Students will determine parametrization for an implicitly defined function in terms of third variable will satisfy the corresponding equation for every value in the domain.	Circulate and monitor student progress as they are working on classwork. Have students complete problems at the board. Through questioning students will be able to determine how to determine the domain of the function after parametrization. Closure	4.7.A.1 4.7.A.2

4.8 Vectors (5 days)	SWBAT Identify characteristics of a vector. SWBAT Determine sums and products involving vectors. SWBAT Determine a unit vector for a given vector. SWBAT Determine angle measures between vectors and magnitudes of vectors involved in vector addition.	As a class discuss the properties of vectors algebraically and geometrically. Explain and model the definition of two-dimensional vectors. Model vector addition numerically and geometrically. Multiplying a Vector by a Scalar. Explain and model the definition of two-dimensional vectors. Geometric Interpretation of Dot Product: Applies the dot product to scenarios such as determining the angle between two vectors and using the dot product to prove if a vector is orthogonal.	Circulate and monitor student progress as they are working on classwork. Have students complete problems at the board. Through questioning students will be able to describe motion of an object using vectors. Closure Students will be given a unit test that assesses their understanding of the concepts and skills from the material on two-dimensional vectors and will contain open ended problem solving which includes both short constructed response and extended response questions.	4.8.A.1 4.8.A.2 4.8.A.3 4.8.A.4 4.8.B.1 4.8.B.2 4.8.B.3 4.8.C.1 4.8.C.2 4.8.D.1 4.8.D.2
4.10 Matrices (2 days)	SWBAT Determine the product of two matrices.	Students will discover that matrix multiplication computes the composition of two linear transformations. Two matrix can be multiplied if the	Circulate and monitor student progress as they are working on classwork. Have students complete problems at the board.	4.10.A.1 4.10.A.2

		number of columns in the first matrix equals the number of rows in the second matrix.	Through questioning students will be able to determine matrix multiplication computes the composition of two linear transformations. Closure	
4.11 The Inverse and Determinant of a Matrix (4 days)	SWBAT Determine the inverse of a 2 X 2 matrix.	Students will be able to determine how to represent system of equations in matrix form and by manipulating matrices to find solutions.	Circulate and monitor student progress as they are working on classwork. Have students complete problems at the board. Through questioning students will be able to determine the way a linear system matches up with a matrix. Closure	4.11.A.1 4.11.A.2 4.11.A.3

MA.9-12.4.1.A.1	A parametric function in $\mathbb{R}^2$, the set of all ordered pairs of two real numbers, consists of a set of two parametric equations in which two dependent variables, x and y, are dependent on a single independent variable, t, called the parameter.
MA.9-12.4.1.A.2	Because variables x and y are dependent on the independent variable, t, the coordinates (x_i, y_i) at time t_i can be written as functions of t and can be expressed as the single parametric function $f(t) = (x(t), y(t))$, where in this case x and y are names of two functions.
MA.9-12.4.1.A.3	A numerical table of values can be generated for the parametric function $f(t) = (x(t), y(t))$ by evaluating x_i and y_i at several values of t_i within the domain.
MA.9-12.4.1.A.4	A graph of a parametric function can be sketched by connecting several points from the numerical table of values in order of increasing value of t.

MA.9-12.4.1.A.5	The domain of the parametric function f is often restricted, which results in start and end points on the graph of f.
MA.9-12.4.2.A.1	A parametric function given by $f(t) = (x(t), y(t))$ can be used to model particle motion in the plane. The graph of this function indicates the position of a particle at time t.
MA.9-12.4.2.A.2	The horizontal and vertical extrema of a particle's motion can be determined by identifying the maximum and minimum values of the functions $x(t)$ and $y(t)$, respectively.
MA.9-12.4.2.A.3	The real zeros of the function $x(t)$ correspond to y-intercepts, and the real zeros of $y(t)$ correspond to x-intercepts.
MA.9-12.4.4.A.1	A complete counterclockwise revolution around the unit circle that starts and ends at (1, 0) and is centered at the origin can be modeled by $(x(t), y(t)) = (\cos t, \sin t)$ with domain $0 \leq t \leq 2\pi$.
MA.9-12.4.4.A.2	Transformations of the parametric function $(x(t), y(t)) = (\cos t, \sin t)$ can model any circular path traversed in the plane.
MA.9-12.4.4.A.3	A linear path along the line segment from the point (x_1, y_1) to the point (x_2, y_2) can be parametrized many ways, including using an initial position (x_1, y_1) and rates of change for x with respect to t and y with respect to t.
MA.9-12.4.7.A.1	A parametrization $(x(t), y(t))$ for an implicitly defined function will, when $x(t)$ and $y(t)$ are substituted for x and y, respectively, satisfy the corresponding equation for every value of t in the domain.
MA.9-12.4.7.A.2	If f is a function of x, then $y = f(x)$ can be parametrized as $(x(t), y(t)) = (t, f(t))$. If f is invertible, its inverse can be parametrized as $(x(t), y(t)) = (f(t), t)$ for an appropriate interval of t.
MA.9-12.4.7.B.1	A parabola can be parametrized in the same way that any equation that can be solved for x or y can be parametrized. Equations that can be solved for x can be parametrized as $(x(t), y(t)) = (f(t), t)$ by solving for x and replacing y with t. Equations that can be solved for y can be parametrized as $(x(t), y(t)) = (t, f(t))$ by solving for y and replacing x with t.
MA.9-12.4.7.B.2	An ellipse can be parametrized using the trigonometric functions $x(t) = h + a \cos t$ and $y(t) = k + b \sin t$ for $0 \leq t \leq 2\pi$.
MA.9-12.4.7.B.3	A hyperbola can be parametrized using trigonometric functions. For a hyperbola that opens left and right, the functions are $x(t) = h + a \sec t$ and $y(t) = k + b \tan t$ for $0 \leq t \leq 2\pi$. For a hyperbola that opens up and down, the functions are $x(t) = h + a \tan t$ and $y(t) = k + b \sec t$ for $0 \leq t \leq 2\pi$.
MA.9-12.4.8.A.1	A vector is a directed line segment. When a vector is placed in the plane, the point at the beginning of the line segment is called the tail, and the point at the end of the line segment is called the head. The length of the line segment is the magnitude of the vector.
MA.9-12.4.8.A.2	A vector $\overrightarrow{P_1P_2}$ with two components can be plotted in the xy-plane from $P_1 = (x_1, y_1)$ to $P_2 = (x_2, y_2)$. The vector is identified by a and b, where $a = x_2 -$

	x_1 and $b = y_2 - y_1$. The vector can be expressed as a, b. A zero vector ⟨0, 0⟩ is the trivial case when $P_1 = P_2$.
MA.9-12.4.8.A.3	The direction of the vector is parallel to the line segment from the origin to the point with coordinates (a, b). The magnitude of the vector is the square root of the sum of the squares of the components.
MA.9-12.4.8.A.4	For a vector represented geometrically in the plane, the components of the vector can be found using trigonometry.
MA.9-12.4.10.A.1	An $n \times m$ matrix is an array consisting of n rows and m columns.
MA.9-12.4.10.A.2	Two matrices can be multiplied if the number of columns in the first matrix equals the number of rows in the second matrix. The product of the matrices is a new matrix in which the component in the ith row and jth column is the dot product of the ith row of the first matrix and the jth column of the second matrix.
MA.9-12.4.11.A.1	The identity matrix, $\boldsymbol{I}$, is a square matrix consisting of 1s on the diagonal from the top left to bottom right and 0s everywhere else.
MA.9-12.4.11.A.2	Multiplying a square matrix by its corresponding identity matrix results in the original square matrix.
MA.9-12.4.11.A.3	The product of a square matrix and its inverse, when it exists, is the identity matrix of the same size.
MA.9-12.4.11.A.4	The inverse of a 2 × 2 matrix, when it exists, can be calculated with or without technology.
MA.9-12.4.11.B.1	The determinant of the matrix A = [a over b, c over d] is $ad - bc$. The determinant can be calculated with or without technology and is denoted $\det(A)$.
MA.9-12.4.11.B.2	If a 2 × 2 matrix consists of two column or row vectors from $\mathbb{R}^2$, then the nonzero absolute value of the determinant of the matrix is the area of the parallelogram spanned by the vectors represented in the columns or rows of the matrix. If the determinant equals 0, then the vectors are parallel.
MA.9-12.4.11.B.3	The square matrix A has an inverse if and only if $\det(A) \neq 0$.

Suggested Modifications for Special Education, ELL and Gifted Students

- Anchor charts to model strategies.
- Review Algebra concepts to ensure students have the information needed to progress in understanding.
- Pre-teach pertinent vocabulary.
- Provide reference sheets that list formulas, step-by-step procedures, theorems, and modeling of strategies.

- Word wall with visual representations of mathematical terms.
- Teacher modeling of thinking processes involved in solving, graphing, and writing equations.
- Introduce concepts embedded in real-life context to help students relate to the mathematics involved.
- Record formulas, processes, and mathematical rules in reference notebooks.
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- Utilize technology through interactive sites to represent nonlinear data.
- Graphic organizers to help students interpret the meaning of terms in an expression or equation in context.
- Translation dictionary.
- Sentence stems to provide additional language support for ELL students.

Suggested Technological Innovations/Use

4. TI-84 graphing calculator
5. Desmos

Cross Curricular/21st Century Connections

Model interdisciplinary thinking to expose students to other disciplines.

Social Studies and ELA Literacy Connection:

Name of Task: Americans' spending: NJSLS: **6.1.12.HistoryCC.16.b, 6.2.12.EconGE.5.a**

From July 1998 to July 1999, Americans' spending rose from 5.82 trillion dollars to 6.20 trillion dollars

a. Let $x = 0$ represent July 1998, $x = 1$ represent August 1998, ..., and $x = 12$ represent July 1999. Write a linear equation for Americans' spending in terms of the month x

b. Use the equation in (a) to predict Americans' spending in July 2002.

c. Based on the model created in (a) when would the aggregate expenditure exceed 10 trillion dollars?

d. What part of the US GDP is spent by the Americans in 2013?

Name of Task: Publishing Cost:

A publishing company estimates that the average cost (in dollars) for one copy of a new scenic calendar it plans to produce can be approximated by the function

$C(x) = (2.25x + 275)/x$

2.25 x+

Where *x* is the number of calendars printed.

a. Find the average cost per calendar when the company prints 100 calendars.
b. Identify the domain and range of this function.
c. After analyzing the function, Alex said that this company should not be allowed to publish zero calendars. As a result, the company has no option to shut down and go out of business. Write an argument to support or reject Alex's conclusion.

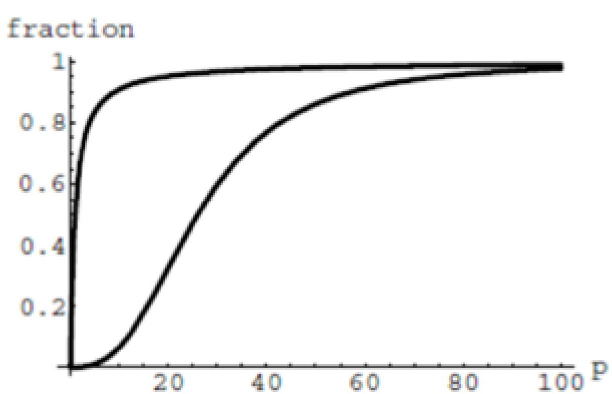
Science Connection:

Name of Task: Myoglobin and Hemoglobin: **NJSLS: HS-LS1-2; HS-LS1-4**

Myoglobin and hemoglobin are oxygen-carrying molecules in the human body. Hemoglobin is found inside red blood cells, which flow from the lungs to the muscles through the bloodstream. Myoglobin is found in muscle cells. The function $y = M(p) = p/(1 + p)$ calculates the fraction of myoglobin saturated with oxygen at a given presure *p* Torrs. For example, at a pressure of 1 Torr, $M(1) = 0.5$, which means half of the myoglobin (i.e. 50%) is oxygen saturated. (Note: More precisely, you need to use something called the "partial pressure", but the distinction is not important for this problem.) Likewise, the function calculates the fraction of hemoglobin saturated with oxygen at a given pressure *p*. [UW]

a. The graphs of M(P) and H(P) are given here on the domain

$0 \le p \le 100$



Which is which?

b. If the pressure in the lungs is 100 Torrs, what is the level of oxygen saturation of the hemoglobin in the lungs?

c. The pressure in an active muscle is 20 Torrs. What is the level of oxygen saturation of myoglobin in an active muscle? What is the level of hemoglobin in an active muscle?

d. Define the efficiency of oxygen transport at a given pressure *p* to be M(p) – H(p) . What is the oxygen transport efficiency at 20 Torrs? At 40 Torrs? At 60 Torrs? Sketch the graph of M(p) – H(p) ; are there conditions under which transport efficiency is maximized (explain)?

Business Connection :

Name of Task: Minimize the metal in a can: **NJSLS: 9.1.12.A.4; W.11-12.1**

A manufacturer wants to manufacture a metal can that holds 1000 cm^3 of oil. The can is in the shape of a right cylinder with a radius *r* and height *h*. Assume the thickness of the material used to make the metal can is negligible.

For each question, include correct units of measurement and round your answers to the nearest tenth. Using your knowledge of volume and surface area of a right cylinder, write a function S(r) that represents the surface area of the cylindrical can in terms of the radius, r, of its base. Show in detail your algebraic thinking.

1. Sketch the graph of S(r) and show key features of the graph. State any restriction on the value of r so that it represents the physical model of the can.
2. What dimensions will minimize the quantity of metal needed to manufacture the cylindrical can? Show in detail your mathematical solution.
3. Calculate the minimum value of the function S(r) and interpret the result in the context of the physical model. Show the mathematical steps you used to obtain the answer.

Name of Task: Chemco Manufacturing**: NJSLS: 9.1.12.A.4; W.11-12.1**

Chemco Manufacturing estimates that its profit *P* in hundreds of dollars is $P = -4x^2 + 40x + 3$ where *x* is the number of units produced in thousands.

a. How many units must be produced to obtain the maximum profit?
b. Graph the profit function and identify its vertex.
c. An increase in productivity increased profit by $7 at each quantity sold. What kind of a transformation would model this situation? Show your work graphically and algebraically.
d. A decrease in marginal cost lead to a 4 units increase in the optimum level of production. What kind of a transformation would model this situation? Show your work graphically and algebraically.