Overview of PAUSD Math

The PAUSD secondary math program, spanning grades 6 through 12, accommodates varying rates of growth among students. Courses are designed to meet the California Common Core State Standards in Mathematics (CCSS-M) for every student in the District. PAUSD offers three types of high school math coursework through flexible lanes of challenge: grade-level courses, accelerated and advanced courses, and courses that cater to students’ interests, allowing them to work beyond typical expectations. The CCSS-M standards consist of two types: eight Mathematical Practice Standards and the Math Content Standards (see Table 1).

Overall, the PAUSD math program aims to help students succeed at their own pace while meeting graduation requirements, state standards, and minimum admission requirements for California State University (CSU) and University of California (UC). PAUSD provides a range of math courses (all of which fulfill graduation requirements, state standards, and CSU/UC requirements) to ensure that students can find a suitably challenging math course in which they can learn effectively. Both parents/guardians and math teachers will offer insights and advice to assist students in selecting the appropriate courses for high school.

All students in PAUSD take either Algebra 1 or Geometry in Grade 8. The Mathematics Placement Act of 2015 (SB359) was enacted to enable all students, especially historically underrepresented students, to reach calculus by Grade 12. Consistent with this goal, both math pathways lead to calculus in Grade 11 or 12.

Table 1: CCSS-M Practices and Standards

<table>
<thead>
<tr>
<th>The Eight Mathematical Practices</th>
<th>Math Content Standards</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Make sense of problems and persevere in solving them</td>
<td>The content standards describe the expectations of content (topics, skills, and concepts) that build mathematical understanding, hone skills, develop and deepen knowledge, and work toward expertise throughout the years K through 12. The content standards for the different grade levels have been developed to coincide with brain development, keeping both concrete and abstract stages of development in consideration. The general areas of Math Content Standards are:</td>
</tr>
<tr>
<td>2. Reason abstractly and quantitatively</td>
<td>● Number, Quantity, and Operation</td>
</tr>
<tr>
<td>3. Construct viable arguments and critique the reasoning of others</td>
<td>● Algebraic Thinking and Algebra</td>
</tr>
<tr>
<td>4. Model with mathematics</td>
<td>● Functions</td>
</tr>
<tr>
<td>5. Use appropriate tools strategically</td>
<td></td>
</tr>
<tr>
<td>6. Attend to precision</td>
<td></td>
</tr>
<tr>
<td>7. Look for and make use of structure</td>
<td></td>
</tr>
<tr>
<td>8. Look for and express regularity in repeated reasoning</td>
<td></td>
</tr>
</tbody>
</table>

Insight into the mathematical practices as well as the content standards are available through the California Department of Education: [https://www.cde.ca.gov/re/cc/mathresources.asp](https://www.cde.ca.gov/re/cc/mathresources.asp)
Middle School Math

PAUSD offers a compacted sequence of courses in which students complete four years of Common Core Math in three years. Supports are in place to help each student find success in this compacted program. In addition, for more advanced math students, there is the opportunity to accelerate an additional year during middle school through a math validation process.

Foundational Math – Grade 6
The content of this course is a combination of California’s Common Core State Standards from Math 6, Math 7, and Math 8. The areas of content focus are: number system, expressions and equations, ratios and proportions, two-dimensional geometry, and data analysis and statistics.
Topics covered in Foundational Math are:
- Operations with Whole Numbers
- Integers and their Operations
- Operations with Rational Numbers
- Fractions and Decimals
- Expressions
- Equations and Inequalities
- Ratios, Rates, and Percents
- Data and Statistics
- Area of a 2D Figure

Concepts in Mathematics – Grade 7
Concepts in Mathematics is the second year in a three-year compacted math course sequence preparing students for high school Algebra in 8th grade. The content of this course is a combination of California’s Common Core State Standards from Math 6, Math 7 and Math 8.
Topics covered in Concepts in Math:
- Expressions, Equations and Inequalities
- Proportional Reasoning
- Exponents
- Pythagorean Theorem
- Data and Statistics

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Algebra 1 – Grade 8
The fundamental objectives of this Common Core-aligned course are to formalize and extend the mathematics students learned in their previous two courses in middle school. The first part of the course is focused on linear functions. The second part of the course is focused on nonlinear functions. The course will prepare students for one of the Geometry courses offered, when entering high school.

Topics covered in Algebra 1:
- Functions
- Equations and Inequalities
- Linear Functions
- Systems of Equations and Inequalities
- Exponential Functions
- Quadratic Functions

Acceleration in Middle School Mathematics
Students who wish to accelerate beyond this already compacted program may take part in the Math Validation Process and, in effect, cover five years of math in three years. Students successful in the Math Validation Process may only accelerate one additional year in the PAUSD middle school pathway. Students may participate more than once if they do not pass the assessments in a previous validation process.

For Spring 2024, rising sixth, seventh, and eighth-grade students who wish to accelerate beyond the already compacted middle school math program may participate in the Math Validation Process.

The updated Math Validation Process will now consist of one paper-and-pencil assessment instead of the previous two-part test. This assessment will evaluate the readiness for the grade-level math course the student would like to challenge, in addition to the student's knowledge of the topics and skills covered in the course.

These assessments are aligned to the state-adopted mathematics content standards. They will consist of multiple-choice (MC) and free-response questions (FRQ). The updated assessments are being developed in partnership with WestEd, a national leader in research, development, and service, alongside PAUSD Math Instructional Leaders.

- Participating in the math validation process is optional.
- The test will cover the PAUSD grade-specific standards and concepts for mastery.
Grade 6 Validation Process
Grade 5 students who attempt but do not pass the validation test will be enrolled in Grade 6 Foundational Math as described in the sequence. Students who pass the test will be enrolled in 7th-grade Math Concepts.

Grade 7 Validation Process
Grade 6 students who attempt but do not pass the validation test will be enrolled in Grade 7 Math Concepts as described in the sequence. Concepts in Math is the appropriate math placement for most students for their 7th-grade year. Students who pass the test will be enrolled in 8th grade Algebra 1 (with 8th grade students).

Grade 8 Validation Process
Grade 7 students who Students who attempt but do not pass the validation test will be enrolled in Algebra 1. Algebra 1 is the appropriate math placement for most students for their 8th-grade year. Students who pass the test will be enrolled in Geometry Honors.

High School Math

The high school math program is a choice program. It is important for students and their parents/guardians, with the advice of teachers and guidance personnel, to choose their math course lane based on finding the appropriate balance of challenge and success in the student’s day. Students should also be aware of the impact of the transcript grade in the college application process. It is recommended that students choose the most rigorous course of study available in which they can earn an A or a B on their own.

The workload and pace of the A-lane (advanced courses) are more demanding than the grade-level/college-prep courses. The workload and pace of the H-lane (honors courses) are more demanding than the A-lane courses. In deference to students' natural growth, PAUSD offers these flexible lanes rather than rigid tracks of math. If a student wishes to move to more advanced or deeper coursework, the student may need to complete summer work. If a student wishes to move to less challenging or more basic coursework, the student can most likely move at the end of a semester without any additional work.

Math Placement Protocols

Before entering high school, teachers or guidance personnel shall meet with students and parents to advise them on the importance of accurate mathematics course placement and its impact on future college eligibility so students may take each course in the mathematics course sequence leading to calculus by Grade 12. To ensure accurate Grade 9 mathematics course placement, students, parents/guardians, and school personnel will consider:

- Success in their current math course; test scores and overall grade
• Ability to work and persevere independently
• Readiness for the next level course as measured by the critical levels met in end-of-year-assessments
• MDTP results
• California Assessment of Student Performance and Progress (CAASPP) assessment results
• Formative and summative assessments
• Time commitments for academic and extracurricular activities; work-life balance

Students who achieve a passing grade of at least a “C” or better or are testing at proficient or advanced proficiency on state assessments will not repeat 8th-grade mathematics coursework and will advance to the next course in the recommended mathematics course sequence.

Grade 9 Placement Checkpoints (California Senate Bill 359)

During the first month of high school, all Grade 9 math students are assessed for proper placement. Instructors formatively assess students by administering the appropriate CCSS-aligned test for the initially assigned math course (Mathematics Diagnostic Testing Project/MDTP). For students who score proficient or higher on a readiness assessment (as determined by the test provider), staff will administer the next higher-level readiness assessment to determine the appropriate math placement. The placement testing will continue until the student reaches the highest level of readiness.

For students enrolled in Algebra, this may involve moving into Geometry or Geometry Honors. Students enrolled in Geometry might move into Geometry Honors. All students must take a geometry course to meet graduation requirements and University of California minimum admission requirements. For students in Algebra 2/Trigonometry A and Algebra 2/Trigonometry H, possible course changes include Algebra 2/Trigonometry H, Introduction to Analysis and Calculus, or Analysis H.

If after the placement checkpoint, a placement change appears to be appropriate, teachers and/or the Math Instructional Leader will once again meet with the student and their parents/guardians to finalize proper placement.

Changing Math Placement

Math Placement Appeal

Students and their parents/guardians who feel they are not appropriately placed should meet with their teacher and counselor to determine if a schedule correction is needed (BP 6152.1). Within 10 school days of an initial placement decision, or a placement decision upon reevaluation, a student and parent/guardian may appeal the decision to the Superintendent or designee. The criteria used to help determine placement may include:
- Grade in previous course
- MDTP readiness assessment
- Grades on in-class assessments taken during the current school year
- CAASPP scores, if available
- Recommendation of teacher or counselor who has personnel knowledge of the students academic ability
- Other assessment results, as appropriate and available

**Challenge by Examination**

PAUSD Board Policy *(BP 6155)* allows high school students to challenge courses by examination. This examination is aligned to state-adopted content standards in mathematics taught in the course being challenged. If a student successfully challenges a course, a grade based on exam results and five units per semester covered by exam, will be added to the transcript with a note indicating that the requirement was met by examination.

**Off-Campus Courses (non-PAUSD)**

High school students may take a geometry course during the summer through a pre-approved third-party provider *(Sample of Approved Outside Institutions for Non-District Courses)*. It is important for students to receive prior approval before taking the course to make sure that it qualifies as a course for PAUSD and fits in with the student's academic plans. [Please note that an off-campus geometry course may not fully prepare students for the highest lane of Algebra 2 (Algebra 2/Trig H).]

All other specific course graduation requirements are to be taken on a PAUSD campus. Students wishing to take off-campus courses (non-PAUSD) must have prior approval. Please see the Gunn High School and Palo Alto High School websites for more detailed information regarding off-campus courses. Students who do not receive prior approval for off-campus courses will not have those courses placed on their transcript. *This may impact graduation status.* Please confer with your guidance counselor.

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High School Math Courses

When reviewing the high school courses below, please note that “Successful completion” means a grade of C- or above. Students who earn below a C- should repeat the course to move forward in a math pathway and for the course to be considered for college admission.

Algebra 1
Students study the applications of linear and quadratic functions. Students learn properties of real numbers, solve linear equations and inequalities, graph linear equations, equations and inequalities with absolute value, solve systems of linear equations and inequalities, simplify exponential expressions, graph and solve quadratic equations and use factoring and the quadratic formula. Students will summarize, represent and interpret data for single count variables, and analyze scatter plots for two quantitative variables. Students learn how to simplify irrational expressions, and solve equations with square roots.

• Prerequisite(s): Successful completion of a Pre-Algebra course
• Students self-report spending an average of 1.5 hours a week outside of school on this course.
• This is a college prep course.

Geometry
Students will formalize the geometric concepts students learned in middle school, and expand their knowledge of Euclidean geometry. Students deepen their understanding of geometric relationships and prove them in different ways. Topics include: congruence, similarity, right triangle trigonometry, transformations, area and volume. This course includes a comprehensive review of Algebra to prepare students for Advanced Algebra and Trigonometry. Students can still access the honors level in future years from this course and are recommended to begin here if they have mastered fewer than 5 of the 7 Algebra strands as assessed in their algebra class.

• Prerequisite(s): Successful completion of both semesters Algebra
• Students self-report spending an average of 1.5 hours a week outside of school on this course.
• This is a college prep course.

Geometry H
This course covers the same content as Geometry, but at a much higher level of rigor. Geometry H is designed for students who have mastered at least five out of seven Algebra strands, as assessed by their Algebra teacher. Only students who are interested in rigorous problem solving and reaching our highest math course (BC Calculus AP) should
choose Geo H. Extensive proof and problem solving are themes. This class will involve some Algebra review, but not to the extent of the non-honors Geometry course.

• Prerequisite(s): Successful completion of an Algebra course
• Students self-report spending an average of 2.5 hours a week outside of school on this course.
• This is an honors course.

**Algebra 2**

Students study families of functions, including: linear, quadratic, polynomial, exponential, logarithmic, rational, and radical functions. Students extend the domain of trigonometric functions using the unit circle and model periodic phenomena with trigonometric functions. Other topics include: complex numbers, interpreting categorical/quantitative data and using regression to interpret data.

• Prerequisite(s): Successful completion of an Algebra course and a Geometry course
• Students self-report spending an average of 1 hour a week outside of school on this course.
• This is a college prep course.

**Algebra 2/Trigonometry A**

In addition to the objectives of the Algebra 2 curriculum, students study circular trigonometry in greater depth, including trig identities, trigonometric applications, and solving trigonometric equations. Basic counting principles are introduced.

• Prerequisite(s): Successful completion of an Algebra course and a Geometry course
• Students self-report spending an average of 2 hours a week outside of school on this course.
• This is an advanced course.

**Algebra 2/Trigonometry Honors**

This course covers content similar to Alg2/Trig A, but with a greater emphasis on proof and problem solving. In each unit, students are expected to apply their understanding of the content to solve problems that are similar, but not identical to problems solved in class. Additional content includes an in-depth study of conic sections, as well as arithmetic and geometric sequences and series, law of sines and cosines, and vectors.

• Prerequisite(s): Successful completion of an introductory Algebra course and a PAUSD Geometry course
• Students self-report spending an average of 2.5 hours a week outside of school on this course.
• This is an honors course.
Precalculus
Students review exponential, logarithmic, and rational functions and study trigonometry, including trigonometric identities, law of sines and cosines, trigonometric applications, and solving trigonometric equations. Additional topics include vectors, parametric and polar functions, matrices, sequence and series, and an introduction to limits and calculus. This class is meant to prepare a student for a formal class in Calculus.

• Prerequisite(s): Successful completion of Algebra 2
• Students self-report spending an average of 1.5 hours a week outside of school on this course.
• This is a college prep course.

Introduction to Data Science
In this course students will learn to understand, ask questions of, and represent data through project-based units. The units will give students opportunities to be data explorers through active engagement, developing their understanding of data analysis, sampling, correlation/causeation, bias and uncertainty, modeling with data, making and evaluating data-based arguments, and the importance of data in society.

● Prerequisite(s): Successful completion of a Geometry and Algebra 2 course
● Students self-report spending an average of 1-2 hours a week outside of school on this course.
● This is a college prep course.
● Honors students should select AP Statistics instead of Data Science.

Introduction to Analysis & Calculus (IAC)
Students will build on their knowledge of functions and explore new topics including: vectors, polar functions, logistic functions, parametric functions, probability, combinatorics, matrices, sequences, series, and statistics. The fourth quarter of the year will focus on limits and an introduction of calculus including the derivative.

● Prerequisite(s): Successful completion of Algebra 2 and a complete Trigonometry course
● Students self-report spending an average of 1-2 hours a week outside of school on this course.
● This is an advanced course.

Analysis H
In addition to the objectives of IAC course, students study Group Theory, Algebraic Problem Solving, quadric surfaces, and more applications of matrices, such as transformations and Markov Chains. The course concludes with the “A” portion of calculus including limits, difference quotients, derivatives, and an introduction to the indefinite integral.

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Calculus
This course will introduce calculus topics to enable success with college level mathematics. Topics include reinforcement of foundational skills necessary for success in a calculus course and basic calculus topics such as limit, derivative, and anti-derivative. This course is designed for students who want to continue in mathematics, but do not want to enroll in an Advanced Placement calculus mathematics course.

- Prerequisites: Successful completion of a Precalculus course.
- Estimated Time Spent on Homework: 2-3 hours per week

AP AB Calculus
Students study the equivalent of the first semester of a yearlong college calculus course. Students develop their understanding of limit, derivative, definite integral, and the indefinite integral. Students represent their work algebraically, graphically, numerically and verbally.

- Prerequisite(s): Successful completion of Pre-Calculus, Introductory Analysis and Calculus, or Analysis Honors
- Students self-report spending an average of 1.5 hours a week outside of school on this course.
- This is an advanced placement course.

AP BC Calculus
Students complete the study of a yearlong college calculus course that was started in Analysis H. The course content includes: functions and limits, derivatives of algebraic functions, the definite and indefinite integral, area, volumes of revolution, differentiation of exponential, logarithmic, trigonometric and inverse trigonometric functions, methods of integration, infinite series, simple differential equations, vectors, and applications.

- Prerequisite(s): Successful completion of Analysis Honors
- Students self-report spending an average of 2.5 hours a week outside of school on this course.
- This is an advanced placement course.
High School Math Electives and Where They Fit into the Student’s Schedule

Any math course beyond the Algebra 1-Geometry-Algebra 2 PAUSD graduation and CSU/UC entrance requirements is defined as an elective math course. Certain courses fit into the linear stream of work arriving at calculus: Pre-Calculus, IAC, Analysis H, Calculus (Paly), AP Calculus AB, and AP Calculus BC, as demonstrated in the flow charts. There are additional courses that students may take to deepen their experiences in particular areas of mathematics. The offerings vary somewhat at each high school site. These courses also earn credits in mathematics in the elective category.

<table>
<thead>
<tr>
<th>Course</th>
<th>Prerequisite</th>
<th>Duration</th>
<th>Course</th>
<th>Prerequisite</th>
<th>Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Henry M Gunn High School</strong></td>
<td></td>
<td></td>
<td><strong>Palo Alto Senior High School</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Applied Math</td>
<td>Successful completion of Algebra 2 Grades 11-12</td>
<td>Semester (UC Approved/UC Weighted)</td>
<td>Stats Applications</td>
<td>Successful completion of Algebra 2 Grades 10-12</td>
<td>Semester (UC Approved)</td>
</tr>
<tr>
<td>AP Statistics</td>
<td>Successful completion of Algebra 2 Grades 11-12</td>
<td>Year (UC Approved/UC Weighted)</td>
<td>AP Statistics</td>
<td>Successful completion of Algebra 2 Grades 11-12</td>
<td>Year (UC Approved/UC Weighted)</td>
</tr>
<tr>
<td>Advanced Problem Solving</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Four Semesters are Available. (not UC Approved)</td>
</tr>
</tbody>
</table>

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### Standard Pathway

#### Middle School

<table>
<thead>
<tr>
<th>Grade 6</th>
<th>Grade 7</th>
<th>Grade 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Foundational Math</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*The content of this course is a combination of California’s Common Core State Standards from Grade 6, Grade 7, and Algebra. The areas of content focus are: number system, expressions and equations, ratios and proportions, two-dimensional geometry, and data analysis and statistics.*

| Concepts in Math |

*The content of this course is a combination of California’s Common Core State Standards from Grade 6, Grade 7 and Math 8. The areas of content focus are: number system, expressions and equations, ratios and proportions, three-dimensional geometry, and probability and bivariate data.*

| Algebra 1 |

*The content of this course encompasses California’s Common Core State Standards for Algebra 1. The first part of the course is focused on linear functions. The second part of the course is focused on nonlinear functions.*

- Middle school follows compacted program (4 years of math in 3 years).
- Algebra 1 leads to Geometry or Geometry H in Grade 9.

### High School

<table>
<thead>
<tr>
<th>Grade 9</th>
<th>Grade 10</th>
<th>Grade 11</th>
<th>Grade 12</th>
</tr>
</thead>
<tbody>
<tr>
<td>● Geometry H</td>
<td>● Algebra 2/Trig H</td>
<td>● Analysis H</td>
<td>● AP Calculus BC</td>
</tr>
<tr>
<td>● Geometry</td>
<td>● Algebra 2/Trig A</td>
<td>● Intro to Analysis &amp; Calculus (IAC)</td>
<td>● AP Calculus AB</td>
</tr>
<tr>
<td>● Algebra 1</td>
<td>● Algebra 2</td>
<td>● Precalculus</td>
<td>● Calculus</td>
</tr>
<tr>
<td>● Geometry</td>
<td>● Geometry</td>
<td>● Algebra 2</td>
<td>● Precalculus</td>
</tr>
<tr>
<td>● Statistics</td>
<td>● Statistics</td>
<td>● AP Stats</td>
<td>● AP Stats</td>
</tr>
<tr>
<td>Applications</td>
<td>Applications</td>
<td>● Intro to Data Science</td>
<td>● Intro to Data Science</td>
</tr>
<tr>
<td></td>
<td></td>
<td>● Statistics</td>
<td>● Statistics</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Applications</td>
<td>Applications</td>
</tr>
<tr>
<td></td>
<td></td>
<td>● Applied Math H</td>
<td>● Applied Math H</td>
</tr>
</tbody>
</table>
**Acceleration Pathway**

**Middle School**

<table>
<thead>
<tr>
<th>Grade 6</th>
<th>Grade 7</th>
<th>Grade 8</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Validation tests passed in May of Grade 5:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Concepts in Math</td>
<td>Algebra 1</td>
<td>Geometry H</td>
</tr>
<tr>
<td><strong>Validation tests passed in May of 6th grade:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Foundational Math</td>
<td>Algebra 1</td>
<td>Geometry H</td>
</tr>
<tr>
<td><strong>Validation tests passed in May of 7th grade:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Foundational Math</td>
<td>Concepts in Math</td>
<td>Geometry H</td>
</tr>
</tbody>
</table>

**High School**

<table>
<thead>
<tr>
<th>Grade 9</th>
<th>Grade 10</th>
<th>Grade 11</th>
<th>Grade 12</th>
</tr>
</thead>
</table>
| ● Algebra 2/Trig H  
● Algebra 2/Trig A | ● Analysis H  
● Intro to Analysis & Calculus (IAC) | ● AP Calculus BC  
● AP Calculus AB  
● AP Stats  
● Intro to Data Science  
● Applied Math H | ● AP Stats  
● Intro to Data Science  
● Applied Math H  
● Community college courses |
Math Flow Charts
The three flowcharts below help show the various pathways available to students. The addition of an A or an H in the title of a course indicates an Advanced course or an Honors course, respectively.

The first flowchart depicts the common pathways (without the flexibility included) so that the basic configuration of the flow is more obvious.

[Flowchart image]

* indicates course meets all HS graduation requirements and the minimum for CSU/UC eligibility

Students may also take AP Statistics, Data Science and/or other electives after required courses are completed.

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The second flowchart illustrates the most common flexible pathways. Students wishing to move to a higher lane should check in with their teacher to determine what learning gaps they may need to address through summer work.
The third flowchart shows the path for students who participate in the middle school Math Validation Process. Common pathways show a natural, linear flow of concepts and skills from one level to another.

PAUSD Secondary Math Flow Chart: Pathways with Validation Process (End of 5th, 6th OR 7th grade)

* indicates course meets all HS graduation requirements and the minimum for CSU/UC eligibility

All courses that follow Algebra 2 or Algebra 2/Trig are UC Approved for math elective credit.

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