Craig City School District



K-12
Mathematics Curriculum
April 2017

TABLE OF CONTENTS

ACKNOWLEDGEMENTS
PHILOSOPHY
ALASKA STANDARDS FOR MATHEMATICAL PRACTICE
ELEMENTARY K-5
SECTION CONTENTS
KINDERGARTEN
FIRST GRADE
SECOND GRADE
THIRD GRADE
FOURTH GRADE
FIFTH GRADE
MIDDLE SCHOOL 6-8
SECTION CONTENTS
COURSE SEQUENCES
SIXTH GRADE
SEVENTH GRADE
SEVENTH/EIGHTH GRADE
EIGHTH GRADE
EIGHTH/ALGEBRA I GRADE
HIGH SCHOOL 9-12
SECTION CONTENTS

(COURSE SEQUENCES
P	RE-ALGEBRA
A	LGEBRA I
(EOMETRY
	LGEBRA II
P	RE-CALCULUS
A	DVANCED PLACEMENT CALCULUS
F	INANCIAL ALGEBRA OR TRADES MATH
A	PPENDIX
	☐ ALASKA CONTENT STANDARDS
	☐ ALASKA CULTURAL STANDARDS

ACKOWLEDGEMENTS

CURRICULUM COMMITTEE

Deanna Claus – Craig High School
Josh Andrews – Craig Middle School
Lorraine Pierce – Craig Elementary School
Aly Howell – Craig Elementary School
Vanessa James – Craig Elementary School
Christy House – Craig Elementary School
Ginger Bird – Parent Committee Member
Kim Brookshire – Parent Committee Member

WE WOULD ALSO LIKE TO RECOGNIZE

The Craig City School District Board of Education and the many teachers, administrators, parents and community members who have so willingly given of their time and expertise in the planning and revision process of this document.

PHILOSOPHY

It is the philosophy of the Craig City School District that all students can succeed at the highest level of mathematics. The goal of mathematics instruction is to produce learners that are problem solvers and thinkers who not only have a grasp of basic facts and operations but also are also capable of generating their own questions and answers to mathematical problems. Students will be taught to see beyond the procedure of mathematics and develop fluency in concepts and gain an understanding of the applications of mathematical principles.

Math instruction involves the teaching of multiple approaches to problem solving. Teamwork and collaboration are encouraged. Mistakes are valued and are an important step in the acquisition of mathematical skills and the development of mathematical intuition. Mathematics is a multi-faceted discipline with the goal of building a mathematical mindset in which the student can apply operations inside the classroom and in daily life.

This curriculum document will reflect an understanding of the standards that instructors and the district hold for the development of student mathematical skill. The document is centered on the Alaska Standards for Mathematical Practice and will act as a framework for teachers. Multiple strategies and resources are contained in the document for the development of mathematical vocabulary and a mindset of mathematics. This will remain a living document in order to facilitate growth as new and more effective techniques are developed over time.

ELEMENTARY MATHEMATICS CURRICULUM

Grades K-5

SECTION CONTENTS

KINDERGARTEN	7
FIRST GRADE	11
SECOND GRADE	16
THIRD GRADE	23
FOURTH GRADE	29
FIFTH GRADE	32

The Alaska Mathematics Standards for Mathematical Practice are expected to be integrated into every mathematics lesson for all students Grades K-12. Below is a grade level analysis of these Practices as well as a few examples of how these Practice may be integrated into tasks that students accomplish.

ALASKA MATHEMATICAL PRACTICE GRADES K-12	CES ANALYSIS OF MATHEMATICAL PRACTICES AND EXAMPLES (K)
1. Make sense of problems and persevere in solving them.	1. Mathematically proficient students in Kindergarten can make sense of problems and find a way to start the task. Kindergarteners monitor and evaluate their progress and change course if necessary, as they persevere. Kindergarten students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems and they continually ask themselves, "Does this make sense?"
2. Reason abstractly and quantitatively.	2. Mathematically proficient Kindergarten students make sense of quantities and their relationships in solving tasks. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols—and the ability to contextualize, to pause as needed during the manipulation process in order to probe for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities; and knowing and flexibly using different properties of operations and objects.
3. Construct viable arguments and critique the reasoning of others.	3. Mathematically proficient kindergarten students use grade appropriate vocabulary, and previously established results in constructing arguments. They are able to analyze situations by using a variety of strategies and use counter examples. They justify their conclusions, communicate them to others, and respond to the arguments of others. Kindergarten students can construct arguments using concrete references such as objects, drawings, diagrams, and actions. Kindergarten Students can listen to the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics.	4. Mathematically proficient students in kindergarten use concrete manipulatives and pictorial representations to provide further explanations of real-life mathematical situations. In Kindergarten, this might be as simple as using manipulatives to show an addition equation that solves problems arising in everyday life. Kindergarten students interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.
5. Use appropriate tools strategically.	5. Mathematically proficient students in kindergarten have access to and use tools appropriately. These tools may include snap cubes, counters, number lines, concrete geometric shapes, puzzles, games, and concrete models. During classroom instruction, kindergarteners have access to various mathematical tools as well as paper, and determine which tools are the most appropriate to use. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. They are able to use technological tools to explore and deepen their understanding of concepts.
6. Attend to precision.	6. Mathematically proficient students in kindergarten try to communicate precisely to others. They try to use grade appropriate vocabulary in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. In kindergarten, students give clear explanations to each other. They consider if their answer is reasonable and check their work to ensure accuracy of solutions.
7. Look for and make use of structure.	7. Mathematically proficient students in kindergarten look closely to discern a pattern or structure in many areas of mathematics. For example, kindergarteners might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have.
8. Look for and express regularity in repeated reasoning.	8. Mathematically proficient students in kindergarten notice if color, shape, size, or color patterns or numerals are repeated, and look for regularity in problem structures when solving mathematical tasks. Kindergarten students check for the reasonableness of their solutions during and after completing the task.

Kindergarten Critical Areas - the critical areas are designed to bring focus to the standards at each grade by describing the big ideas that educators can use to build their curriculum and to guide instruction.

1. Representing, relating, and operating on whole numbers, initially with sets of objects

• Students use numbers, including written numerals, to represent quantities and to solve quantitative problems, such as counting objects in a set; counting out a given number of objects; comparing sets or numerals; and modeling simple joining and separating situations with sets of objects, or eventually with equations such as 5+2=7 and 7-2=5. (Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required.) Students choose, combine, and apply effective strategies for answering quantitative questions, including quickly recognizing the cardinalities of small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets, or counting the number of objects that remain in a set after some are taken away.

2. Describing shapes and space

• Students describe their physical world using geometric ideas(e.g., shape, orientation, spatial relations) and vocabulary. They identify, name, and describe basic two-dimensional shapes, such as squares, triangles, circles, rectangles, and hexagons, presented in a variety of ways (e.g., with different sizes and orientations), as well as three-dimensional shapes such as cubes, cones, cylinders, and spheres. They use basic shapes and spatial reasoning to model objects in their environment and to construct more complex shapes.

Alaska Mathematical Domains Grades K-8	Alaska Mathematical Content Standards (K)	CES Interpretation of Standards (K)
Counting and Cardinality (CC)	K.CC.1 - K.CC.7	What knowledge and skills will students need in order to fully meet the standard?
Know number names and the count sequence.	K.CC.1. Count to 100 by ones and by tens. K.CC.2. Count forward beginning from a given number within the known sequence. K.CC.3. Write numbers from 0 to 20. Represent a number of objects with a written numeral 0 - 20 (with 0 representing a count of no objects).	 K.CC.1 - Students will verbally count from 1 to 100 by ones and tens. K.CC.2 - students need to know numbers 1-100 and be able to count on from a given number K.CC.3 - demonstrate one-to-one correspondence and recognize and write the numerals.
Count to tell the number	K.CC.4. Understand the relationship	K.CC.4 - demonstrate one-to-one

Milaci gai teli			
of objects.	between numbers and quantities; connect counting to cardinality. a. When counting objects, say the number names in standard order, pairing each object with one and only one number name and each number name with one and only one object. b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. c. Understand that each successive number name refers to a quantity that is one larger. K.CC.5. Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array or a circle, or as many as 10 things in a scattered configuration; given a number from 1-20, count out that many objects.	K.CC.5 - identify how many objects are in a set by strategies to help them with one-to-one correspondence and keep track of objects that have been previously counted to correctly answer how many.	
Compare numbers.	K.CC.6. Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group (e.g., by using matching, counting, or estimating strategies).	K.CC.6 - use one-to-one correspondence to compare sets of objects and determine which set is greater than, less than, or equal to the original set up to 10. They will use this by lining them up and matching, counting, grouping and comparing.	
	K.CC.7. Compare and order two numbers between 1 and 10 presented as written numerals.	K.CC.7 - compare two numerals, 1-10, and understand that one numerals represents one set, while the other numeral represents the other set. Students will need to have a complete understanding of ordering numbers while using manipulatives before using only numerals.	

Alaska Mathematical Domains Grades K-8	Alaska Mathematical Content Standards (K)	CES Interpretation of Standards (K)
Operations and Algebraic Thinking (OA)	K.OA.1 - K.OA.6	What knowledge and skills will students need in order to fully meet the standard?
Understand addition as putting together and adding to, and understand subtraction as taking apart and	K.OA.1. Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps) acting out situations, verbal explanations, expressions, or equations.	K.OA.1 - The students will understand the concept of addition and subtraction by joining and separating objects in various ways.
taking from.	K.OA.2. Add or subtract whole numbers to 10 (e.g., by using objects or drawings to solve word problems).	K.OA.2 - they will use objects, actions or drawings to represent the problem and correctly solve it.
	K.OA.3. Decompose numbers less than or equal to 10 into pairs in more than one way (e.g., by using objects or drawings, and record each decomposition by a drawing or equation). For example, $5 = 2 + 3$ and $5 = 4 + 1$.	K.OA.3 - Develop an understanding of a part/whole relationship and know that numbers can be broken into smaller groups and still remain the same number (5=4+1 and 5=2+3)
	K.OA.4. For any number from 1 - 4, find the number that makes 5 when added to the given number and, for any number from 1 - 9, find the number that makes 10 when added to the given number (e.g., by using objects, drawings or 10 frames) and	 K.OA.4 - the students will understand that numbers can be decomposed into parts that equal 10 and be able to find the missing part through numerous concrete experiences. K.OA.5 - recognize and master
	record the answer with a drawing or equation. K.OA.5. Fluently add and subtract numbers up to 5.	subtraction and addition sentences that equal up to 5. They need to be able to break apart a set to recognize that subsets make that set through using manipulatives
Identify and continue patterns	K.OA.6. Recognize, identify and continue simple patterns of color,	K.OA.6 - students will use objects to recognize and extend patterns

The Craig City School District is dedicated to providing a meaningful, comprehensive, and engaging education to all students so they participate responsibly in the global society.

1	shape, and size.	of color shape and size.

Alaska Mathematical Domains Grades K-8	Alaska Mathematical Content Standards (K)	CES Interpretation of Standards (K)
Number and Operations in Base Ten (NBT)	K.NBT.1	What knowledge and skills will students need in order to fully meet the standard?
Work with numbers 11- 19 to gain foundations for place value.	K.NBT.1. Compose and decompose numbers from 11 to 19 into ten ones and some further ones (e.g., by using objects or drawings) and record each composition and decomposition by a drawing or equation (e.g., $18 = 10 + 8$); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight or nine ones.	K.NBT.1 - The student will recognize that a set (14) is a group of ten (10) and a group of extras (4). The student will also know that 10 ones equals ten. They can record it by using objects, drawings or equations to compose and decompose the number up to 19.

Alaska Mathematical Domains Grades K-8	Alaska Mathematical Content Standards (K)	CES Interpretation of Standards (K)
Measurement and Data (MD)	K.MD.1 - K.MD.6	What knowledge and skills will students need in order to fully meet the standard?
Describe and compare measurable attributes.	K.MD.1. Describe measurable attributes of objects (e.g., length or weight). Match measuring tools to attribute (e.g., ruler to length). Describe several measurable attributes of a single object.	K.MD.1 - students will be able to describe an object using numerous measurable attributes and be able to compare that object to another object with different attributes and be able to discuss differences and similarities.
	K.MD.2. Make comparisons between two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. For example, directly compare the heights of two children and describe one child as	K.MD.2 - The students will compare two objects side by side, with each end lined up and note the similarities and differences between the two using "more of" or "less than", "longer" or "shorter" or any other descriptors.

	taller/shorter.	
Classify objects and count the number of objects in each category.	K.MD.3. Classify objects into given categories (attributes). Count the number of objects in each category (limit category counts to be less than or equal to 10).	K.MD.3 - the students will identify similarities and differences between objects and use the identified attribute to sort objects and be able to count how many in each set. The students will need to be able to sort the sets by the number in each set.
Work with time and money.	K.MD.4. Name in sequence the days of the week. K.MD.5. Tell time to the hour using both analog and digital clocks. K.MD.6. Identify coins by name.	 K.MD.4 - The students will be able to name in sequence the days of the week K.MD.5 - The students will be able to recognize the different hands on the clock and be able to read time to the hour on both analog and digital clocks. K.MD.6 - the students will be able to identify the four types of coins.

Alaska Mathematical Domains Grades K-8	Alaska Mathematical Content Standards (K)	CES Interpretation of Standards (K)
Geometry (G)	K.G.1 - K.G.6 (shapes to include - squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, & spheres)	What knowledge and skills will students need in order to fully meet the standard?
Identify and describe shapes.	K.G.1. Describe objects in the environment using names of shapes and describe their relative positions (e.g., above, below, beside, in front of, behind, next to). K.G.2. Name shapes regardless of their orientation or overall size.	 K.OA.1 - The students will be able to locate and identify shapes in their environment and then be able to correctly describe objects and their orientation to a fixed object using positional words. K.OA.2 - the students will correctly identify shapes using certain attributes after

	K.G.3. Identify shapes as two-dimensional (flat) or three-dimensional (solid).	manipulation and exploration of these shapes in different orientation and sizes. K.OA.3 - the students will recognize the difference between two-dimensional and three-dimensional shapes through exploration and discussion of the properties of various shapes.
Analyze, compare, create, and compose shapes.	K.G.4. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices), and other attributes (e.g., having sides of equal lengths). K.G.5. Build shapes (e.g., using sticks and clay) and draw shapes.	K.G.4 - Students relate one shape to another as they note similarities and differences between 2-D and 3-D shapes using informal language to discuss attributes of the objects. K.G.5 - the students will manipulate objects such as clay and sticks to create a shape and be able to draw their finished product.
	K.G.6. Put together two-dimensional shapes to form larger shapes (e.g., join two triangles with full sides touching to make a rectangle).	K.G.6 - the students will manipulate shapes to add to other shapes to create new shapes.

The Alaska Mathematics Standards for Mathematical Practice are expected to be integrated into every mathematics lesson for all students Grades K-12. Below is a grade level analysis of these Practices as well as a few examples of how these Practice may be integrated into tasks that students accomplish.

ALASKA MATHEMATICAL PRACTICE GRADES K-12	CES ANALYSIS OF MATHEMATICAL PRACTICES AND EXAMPLES (1st)
1. Make sense of problems and persevere in solving them.	Awaiting Teacher Submission 1.
2. Reason abstractly and quantitatively.	2.
3. Construct viable arguments and critique the reasoning of others.	3.
4. Model with mathematics.	4.
5. Use appropriate tools strategically.	5.
6. Attend to precision.	6.
7. Look for and make use of structure.	7.
8. Look for and express regularity in repeated reasoning.	8.

First Grade Critical Areas - the critical areas are designed to bring focus to the standards at each grade by describing the big ideas that educators can use to build their curriculum and to quide instruction.

1. Developing understanding of addition, subtraction, and strategies for addition and subtraction within 20

• Students develop strategies for adding and subtracting whole numbers based on their prior work with small numbers. They use a variety of models, including discrete objects and length-based models (e.g., cubes connected to form lengths), to model add-too, take-from, put-together, take-apart, and compare situation to develop meaning for the operations of addition and subtraction, and to develop strategies to solve arithmetic problems with these operations. Students understand connections between counting and addition and subtraction (e.g., adding two is the same as counting on two). They use properties of addition to add whole numbers and to create and use increasingly sophisticated strategies based on these properties (e.g., making tens) to solve addition and subtraction problems within 20. By comparing a variety of solution strategies, children build their understanding of the relationship between addition and subtraction.

2. Developing understanding of whole number relationships and place value, including grouping in tens and ones

• Students develop, discuss, and use efficient, accurate, and generalizable methods to add within 100 and subtract multiples of 10. They compare whole numbers (at least to 100) to develop understanding of and solve problems involving their relative sizes. They think of whole numbers between 10 and 100 in terms of tens and ones (especially recognizing the numbers 11 to 19 as composed of a ten and some ones). Through activities that build number sense, they understand the order of the counting numbers and their relative magnitudes.

3. Developing understanding of linear measurement and measuring lengths as iterating length units

• Students develop an understanding of the meaning and processes of measurement, including underlying concepts such as iterating, (the mental activity of building up the length of an object with equal-sized units) and the transitivity principle for indirect measurement.

4. Reasoning about attributes of, and composing and decomposing geometric shapes

• Students compose and decompose plane or solid figures (e.g., put two triangles together to make a quadrilateral) and build understanding of part-whole relationships as well as the properties of the original and composite shapes. As they combine shapes, they recognize them from different perspectives and orientations, describe their geometric attributes, and determine how they are alike and different, to develop the background for measurement and for initial understanding of properties such as congruence and symmetry.

Alaska Mathematical	Alaska Mathematical Content	CES Interpretation of
Domains Grades K-8	Standards (1st)	Standards (1st)
Counting and Cardinality (CC)	1.CC.1 - 1.CC.6	What knowledge and skills will students need in order to fully meet the standard?

	1.CC.1. Skip count by 2s and 5s. 1.CC.2. Use ordinal numbers correctly when identifying object position (e.g., first, second, third, etc.).	 1.CC.1 - the students will be able to skip count by 2s and 5s. 1.CC.2 - the students will know and understand ordinal numbers and be able to correctly identify placement of objects by their ordinal number.
	1.CC.3. Order numbers from 1-100. Demonstrate ability in counting forward and backward.	1.CC.3 - the students will be able to recognize numbers 1-100 and place them in the correct order. They will then be able to count forward and backwards from that given number.
Count to tell the number of objects.	1.CC.4 - Count a large quantity of objects by grouping into 10s and counting by 10s and 1s to find the quantity.	1.CC.4 - students will decompose a large quantity into groups of tens and will then count by tens and ones to compose the set
Compare numbers.	1.CC.5 - Use the symbols for greater than, less than or equal to when comparing two numbers or groups of objects.	1.CC.5 - The students will identify the larger or smaller numeral or groups of objects and place the correct symbol to show which is greater than, less than or equal to.
	1.CC.6 - Estimate how many and how much in a given set to 20 and then verify estimate by counting.	1.CC.6 - The students will be able to look at a set of objects and estimate how much is in the given set up to 20. They will then verify their estimation by counting the objects.

Alaska Mathematical Domains Grades K-8	Alaska Mathematical Content Standards (1st)	CES Interpretation of Standards (1st)
Operations and Algebraic Thinking (OA)	1.OA.1 - 1.OA.9	What knowledge and skills will students need in order to fully meet the standard?
Represent and solve	1.0A.1 - Use addition and subtraction	1.OA.1 - The students will

problems involving addition and subtraction.	strategies to solve word problems (using numbers up to 20), involving situations of adding to, taking from, putting together, taking apart and comparing, with unknowns in all positions, using a number line (e.g., by using objects, drawings and equations). Record and explain using equation symbols and a symbol for the unknown number to represent the problem.	decompose and compose numbers to solve word problems up to 20. They will also need to solve for the missing number by using a number line or other strategies. They will also need to recognize and use the correct symbols to create and record the equation.
	1.OA.2 - Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20 (e.g., by using objects, drawings and equations). Record and explain using equation symbols and a symbol for the unknown number to represent the problem.	1.OA.2 - The students will solve multistep problems by composing three numbers whose sum is less than or equal to 20 using a variety of mathematical representations (e.g., by using objects, drawing or equations).
Understand and apply properties of operations and the relationship between addition and subtraction.	Apply properties of operations as strategies to add and subtract. (Students need not know the name of the property.)	1.OA.3 -The students need to apply the properties of commutative and associative operations as strategies to compose and decompose. For example: If $8 + 3 = 11$ is known, then $3 + 8 = 11$ is also known (Commutative property of addition). To add $2 + 6 + 4$, the second two numbers can be added to make a ten, so $2 + 6 + 4 = 2 + 10 = 12$ (Associative property of addition). Demonstrate that when adding zero to any number, the quantity does not change (Identity property of addition).
	1.OA.4 - Understand subtraction as an unknown-addend problem. For example, subtract 10 - 8 by finding the number that makes 10 when added to 8	1.OA.4 - The students will understand the relationship between addition and subtraction so they may use various strategies

		to solve subtraction problems.
Add and subtract using numbers up to 20.	1.OA.5 - Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).	1.OA.5 - the students will use counting strategies such as counting on, counting all, and counting back to solve addition and subtractions problems.
	Add and subtract using numbers up to 20, demonstrating fluency for addition and subtraction up to 10.	1.OA.6 - the students will learn about and use various strategies to solve addition and subtraction problems up to 20. When they continually use such strategies they will internalize the facts and develop fluency for addition and subtraction up to 10. Use strategies such as counting on making ten $(8 + 6 = 8 + 2 + 4 = 10 + 4 = 14)$ decomposing a number leading to a ten $(13 - 4 = 13 - 3 - 1 = 10 - 1 = 9)$ using the relationship between addition and subtraction, such as fact families, $(8 + 4 = 12$ and $12 - 8 = 4)$ creating equivalent but easier or known sums (e.g., adding $6 + 7$ by creating the known equivalent $6 + 6 + 1 = 12 + 1 = 13$).
Work with addition and subtraction equations.	1.OA.7 - Understand the meaning of the equal sign (e.g., read equal sign as "same as") and determine if equations involving addition and subtraction are true or false.	1.0A.7 - the students will recognize and know that the equal sign means "same as" and use this knowledge to determine if the equation is true or false. For example, which of the following equations are true and which are false? $6 = 6$, $7 = 8 - 1$, $5 + 2 = 2 + 5$, $4 + 1 = 5 + 2$).
	1.OA.8 - Determine the unknown whole number in an addition or subtraction	1.0A.8 - The students will use their understanding of addition

,	equation.	and subtraction to solve for the unknown number. For example, determine the unknown number that makes the equation true in each of the equations $8 + ? = 11$, $6 + 6 = ?$, $5 = ? - 3$
Identify and continue patterns.	1.OA.9 - Identify, continue and label patterns. Create patterns using number, shape, size, rhythm or color.	1.OA.9 - The students will be able to identify and recognize patterns based on their attributes and be able to correctly continue the pattern. (e.g., aabb, abab)

Alaska Mathematical Domains Grades K-8	Alaska Mathematical Content Standards (1st)	CES Interpretation of Standards (1st)
Number and Operations in Base Ten (NBT)	1.NBT.1 - 1.NBT.6	What knowledge and skills will students need in order to fully meet the standard?
Extend the counting sequence.	1.NBT.1 - Count to 120. In this range, read, write and order numerals and represent a number of objects with a written numeral.	1.NBT.1 - the students will be able to count to 120 and understand that the counting sequence means that the number is one more than the number previously. They also need to recognize that a set of objects corresponds to the same numeral.
Understand place value.	Model and identify place value positions of two digit numbers. Include: a. 10 can be thought of as a bundle of ten ones, called a "ten". b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight or nine ones. c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90, refer to one, two, three, four, five, six, seven, eight or nine tens	a.The students will identify and model a bundle of ten ones to represent ten. b. the numbers 11-19 are composed of one ten and the extras. c. 10, 20, 30, 40, 50, 60, 70, 80, 90 are groups of tens with no extra numbers.

First Grade		
	(and 0 ones).	
	1.NBT.3 - Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, <.	1.NBT.3 - the students will compare two numbers by examining the amount of tens and ones in each number and use the comparison vocabulary.
Use place value understanding and properties of operations to add and subtract.	Add using numbers up to 100 including adding a two-digit number and a one-digit number and adding a two-digit number and a multiple of 10. Use: concrete models or drawings and strategies based on place value properties of operations and/or relationship between addition and subtraction. Relate the strategy to a written method and explain the reasoning used. Demonstrate in adding two-digit numbers, tens and tens are added, ones and ones are added and sometimes it is necessary to compose a ten from ten ones.	1.NBT.4 - students will use concrete materials, models, drawings and place value strategies to add to 100. Students will then be able to explain how they got their answer. Students will be able to show they can add 10's to 10's and ones to ones. Ten one's are sometimes composed to make a ten.
	1.NBT.5 - Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.	1.NBT.5 - the students will mentally add ten more or ten less than the given number by using their knowledge of counting by tens.
	 1.NBT.6 - Subtract multiples of 10 up to 100. Use: concrete models or drawings strategies based on place value properties of operations and/or the relationship between 	1.NBT.6 - The students will use concrete models, drawings and strategies to subtract multiples of 10 from a given number up to 100.

1	addition and subtraction. Relate the strategy to a written method and explain the reasoning used.	

Alaska Mathematical Domains Grades K-8	Alaska Mathematical Content Standards (1st)	CES Interpretation of Standards (1st)
Measurement and Data (MD)	1.MD.1 - 1.MD.7	What knowledge and skills will students need in order to fully meet the standard?
Measure lengths indirectly and by iterating length units.	1.MD.1 - Measure and compare three objects using standard or nonstandard units.	1.MD.1 - The students will use standard or nonstandard units to measure and compare three objects by their length.
	1.MD.2 -Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps.	1.MD.2 - the students will use objects to measure items and focus on the attributes of the item by laying multiple copies of a shorter object of the same length end to end to equal a larger object.
Work with time and money.	1.MD.3. Tell and write time in half hours using both analog and digital clocks.	1.MD.3 - The students will be able to recognize and explain the difference between the two different hands on the clock. They will also be able to read and write time on both analog and digital clocks to the half hour.
	1.MD.4. Read a calendar distinguishing yesterday, today and tomorrow. Read and write a date.	1.MD.4 - The students will recognize the days of the week and their correct ordinal placement and be able to tell which day came before and which day came after the current day. They will also need to be able to read and write the corresponding date.
	1.MD.5. Recognize and read money symbols including $\$$ and $$\phi$$.	1.MD.5 - The students will correctly identify money symbols both $\$$ and $$\phi$$.

	1.MD.6. Identify values of coins (e.g., nickel = 5 cents, quarter = 25 cents). Identify equivalent values of coins up to \$1 (e.g., 5 pennies = 1 nickel, 5 nickels = 1 quarter).	identify values of coins (e.g., nickel = 5 cents, quarter = 25 cents) and be able to identify equivalent values of coins up to \$1 (e.g., 5 pennies = 1 nickel, 5 nickels = 1 quarter).
Represent and interpret data.	1.MD.7 - Organize, represent and interpret data with up to three categories. Ask and answer comparison and quantity questions about the data.	1.MD.7 - The students will collect and use data to answers a question. The data should be organized on a graph, chart or table. The student will interpret the data to answer the question.

Alaska Mathematical Domains Grades K-8	Alaska Mathematical Content Standards (1st)	CES Interpretation of Standards (1st)
Geometry (G)	1.G.1 - 1.G.3	What knowledge and skills will students need in order to fully meet the standard?
Reason with shapes and their attributes.	1.G.1. Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes. Identify shapes that have non-defining attributes (e.g., color, orientation, overall size). Build and draw shapes given specified attributes.	1.G.1 - the students will identify, name, build and draw shapes according to their defining and non-defining attributes.
	1.G.2. Compose (put together) two-dimensional or three-dimensional shapes to create a larger, composite shape, and compose new shapes from the composite shape.	1.G.2 - the students will create 2-D or 3-D shapes by using two or more geometric shapes to see how the shapes fit together to make different shapes.
	1.G.3. Partition circles and rectangles into two and four equal shares. Describe the shares using the words, Describe the whole as two of or four of the shares. Understand for these examples that decomposing (break apart) into more equal shares creates	1.G.3 - The students will partition regions into equal shares and combine them to form a larger share. They will use the following terms to describe the whole and it's equal parts, <i>halves, fourths</i> , and

	quarters and phrases half of, fourth of and quarter of.
	ana quarter oj.

The Alaska Mathematics Standards for Mathematical Practice are expected to be integrated into every mathematics lesson for all students Grades K-12. Below is a grade level analysis of these Practices as well as a few examples of how these Practice may be integrated into tasks that students accomplish.

ALASKA MATHEMATICAL PRACTICE GRADES K-12	CES ANALYSIS OF MATHEMATICAL PRACTICES AND EXAMPLES (2nd)
1. Make sense of problems and persevere in solving them.	1. Mathematically proficient students in Second grade examine problems and tasks, can make sense of the meaning of the task and find an entry point or a way to start the task. Second grade students also develop a foundation for problem solving strategies and become independently proficient on using those strategies to solve new tasks. In second grade, students' work continues to use concrete manipulatives and pictorial representations as wells as mental mathematics. Second grade students also are expected to persevere while solving tasks, that is, if students reach a point in which they are stuck, they can reexamine the task in a different way and continue to solve the task. Lastly, mathematically proficient students complete a task by asking themselves the question, "Does my answer make sense?"
2. Reason abstractly and quantitatively.	2. Mathematically proficient students in second grade make sense of quantities and relationships while solving tasks. This involves two processes- decontexualizing and contextualizing. In second grade, students represent situations by decontextualizing tasks into numbers and symbols. For example, in the task, "There are 25 children in the cafeteria and they are joined by 17 more children. How many children are in the cafeteria?" Second grade students translate that situation into an equation, such as: 25 + 17 = and then solve the problem. Students also contextualize situations during the problem solving process. For example, while solving the task above, students can refer to the context of the task to determine that they need to subtract 19 since 19 children leave. The processes of reasoning also other areas of mathematics such as determining the length of quantities when measuring with standard units.
3. Construct viable arguments and critique the reasoning of others.	3. Mathematically proficient students in second grade accurately use definitions and previously established solutions to construct viable arguments about mathematics. During discussions about problem solving strategies, students constructively critique the strategies and reasoning of their classmates. For example, while

*****	t commence of the commence of
	solving 74-18, students may use a variety of strategies, and after working on the task, can discuss and critique each others' reasoning and strategies, citing similarities and differences between strategies.
4. Model with mathematics.	4. mathematically proficient students in second grade model real-life mathematical situations with a number sentence or an equation, and check to make sure that their equation accurately matches the problem context. Second grade students use concrete manipulatives and pictorial representations to provide further explanations of the equation. Likewise, second grade students are able to create an appropriate problem situation from an equation. For example, students are expected to create a story problem for the equation 43+17= such as "There were 43 gumballs in the machine. Tom poured in 17 more gumballs. How many gumballs are now in the machine?"
5. Use appropriate tools strategically.	5. Mathematically proficient students in second grade have access to and use tools appropriately. These tools may include snap cubes, place value (base ten) blocks, hundred number boards, number lines, rulers and concrete geometric shapes. Students also have experiences with educational technologies, such as calculators and virtual manipulatives, which support conceptual understanding and higher-order thinking skills. During classroom instruction, students have access to various mathematical tools as well as paper, and determine which tools are the most appropriate to use. For example, while measuring the length of the hallway, students can explain why a yardstick is more appropriate to use than a ruler.
6. Attend to precision.	6. Mathematically proficient students in second grade are precise in their communication, calculations, and measurements. In all mathematical tasks, students in second grade communicate clearly, using grade level appropriate vocabulary accurately as well as giving precise explanations and reasoning regarding their process of finding solutions. For example, while measuring an object, care is taken to line up the tool correctly in order to get an accurate measurement. During tasks involving number sense, students consider if their answer is reasonable and check their work to ensure the accuracy of solutions.
7. Look for and make use of structure.	7. Mathematically proficient students in second grade carefully look for patterns and structures in the number system and other areas of mathematics. For example, students notice number patterns within the tens place as they connect skip count by 10s off the decade to the corresponding numbers on a 100s chart.

	While working in the numbers in Base ten domain, students work with the idea that 10 ones equals a ten, and 10 tens equals 1 hundred. In addition, second grade students also make use of structure when they work with subtraction as a missing addend problems, such as 50-33= can be written as 33+=50 and can be thought of as "How much more do I need to add to 33 to get 50?"
8. Look for and express regularity in repeated reasoning.	8. Mathematically proficient students in second grade begin to look for regularity in problem structures when solving mathematical tasks. For example, after solving two digit addition problems by decomposing numbers (33+25=30+20+3+5), students may begin to generalize and frequently apply that strategy independently on future tasks. Further, students begin to look for strategies to be more efficient in computations, including doubles strategies and making a ten. Lastly, while solving all tasks, second grade students accurately check for the reasonableness of their solutions during and after completing the task.

Second Grade Critical Areas - the critical areas are designed to bring focus to the standards at each grade by describing the big ideas that educators can use to build their curriculum and to guide instruction.

1. Extending understanding of base-ten notation

• students extend their understanding of the base-ten system. This includes ideas of counting in fives, tens, and multiples of hundreds, tens and ones, as well as number relationships involving these units, including comparing. Students understand multi-digit numbers (up to 1000) written in base-ten notation, recognizing that the digits in each place represent amounts of thousands, hundreds, tens, or ones.

2. Building fluency with addition and subtraction

• students use their understanding of addition to develop fluency with addition and subtraction within 100. They solve problems within 1000 by applying their understanding of models for addition and subtraction, and they develop, discuss, and use efficient, accurate, and generalizable methods to compute sums and differences of whole numbers in base-ten notation, using their understanding of place value and the properties of operations. They select and accurately apply methods that are appropriate for the context and the numbers involved to mentally calculate sums and differences for numbers with only tens or only hundreds.

3. Using standard units of measure

• students recognize the need for standard units of measure (centimeter and inch) and they use rulers and other measurement tools with the understanding that linear measure involves an iteration of units.

4. Describing and analyzing shapes

•

Alaska Mathematical Domains Grades K-8	Alaska Mathematical Content Standards (2nd)	CES Interpretation of Standards (2nd)
Operations and Algebraic Thinking (OA)	2.OA.1 - 2.OA.5	What knowledge and skills will students need in order to fully meet the standard?
Represent and solve problems involving addition and subtraction.	2.OA.1 -Use addition and subtraction strategies to estimate, then solve one-and two-step word problems (using numbers up to 100) involving situations of adding to, taking from, putting together, taking apart and comparing, with unknowns in all positions (e.g., by using objects, drawings and equations). Record and explain using equation symbols and a symbol for the unknown number to represent the problem.	2.OA.1 -
Add and subtract using numbers up to 20.	2.OA.2 -Fluently add and subtract using numbers up to 20 using mental strategies. Know from memory all sums of two one-digit numbers.	2.OA.2 -
Work with equal groups of objects to gain foundations for multiplication.	2.OA.3 -Determine whether a group of objects (up to 20) is odd or even (e.g., by pairing objects and comparing, counting by 2s). Model an even number as two equal groups of objects and then write an equation as a sum of two equal addends. 2.OA.4 -Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns. Write an equation to express the total as repeated addition (e.g., array of 4 by 5 would be	2.0A.3 - 2.0A.4 -
Identify and continue	5 + 5 + 5 + 5 = 20). 2.OA.5 -Identify, continue and label	2.OA.5 -

patterns.	number patterns (e.g., aabb, abab). Describe a rule that determines and continues a sequence or pattern.	
	_	

Alaska Mathematical Domains Grades K-8	Alaska Mathematical Content Standards (2nd)	CES Interpretation of Standards (2nd)
Number and Operations in Base Ten (NBT)	2.NBT.1 - 2.NBT.9	What knowledge and skills will students need in order to fully meet the standard?
Understand place value.	2.NBT.1 -Model and identify place value positions of three digit numbers. Include: a. 100 can be thought of as a bundle of ten tenscalled a "hundred". b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and o tens and o ones). 2.NBT.2 -Count up to 1000, skipcount by 5s, 10s and 100s. 2.NBT.3. Read, write, order up to 1000 using base-ten numerals, number names and expanded form. 2.NBT.4. Compare two three-digit numbers based on the meanings of the hundreds, tens and ones digits, using >, =, < symbols to record the results.	2.NBT.2 - 2.NBT.3 - 2.NBT.4 -
Use place value understanding and properties of operations to add and subtract.	 2. Fluently add and subtract using numbers up to 100. Use: strategies based on place value properties of operations and/or the relationship between addition and subtraction. 	2.NBT.5 - 2.NBT.6 - 2.NBT.7 - 2.NBT.8 -

2.NBT.6. Add up to four two-digit numbers using strategies based on place value and properties of operations.	2.NBT.9 -
2.NBT.7. Add and subtract using numbers up to 1000. Use:	
 concrete models or drawings and strategies based on place value properties of operations and/or relationship between addition and subtraction. 	
Relate the strategy to a written method and explain the reasoning used.	
Demonstrate in adding or subtracting three-digit numbers, hundreds and hundreds are added	
or subtracted, tens and tens are added or subtracted, ones and ones are added or subtracted and	ş.
sometimes it is necessary to compose a ten from ten ones or a hundred from ten tens.	
2.NBT.8. Mentally add 10 or 100 to a given number 100-900 and mentally subtract 10 or 100 from a given number.	
2.NBT.9 - Explain or illustrate the processes of addition or subtraction and their relationship using place value and the properties of operations.	

Alaska Mathematical Domains Grades K-8	Alaska Mathematical Content Standards (2nd)	CES Interpretation of Standards (2nd)
Measurement and Data (MD)	2.MD.1 - 2.MD.10	What knowledge and skills will students need in order to fully meet the standard?
Measure and estimate	2.MD.1 -Measure the length of an	2.MD.1 - Using both customary

lengths	in	standard
units.		

object by selecting and using standard tools such as rulers, yardsticks, meter sticks, and measuring tapes.

- 2.MD.2. Measure the length of an object twice using different length units for the two measurements. Describe how the two measurements relate to the size of the unit chosen.
- 2.MD.3. Estimate, measure and draw lengths using whole units of inches, feet, yards, centimeters and meters.
- 2.MD.4. Measure to compare lengths of two objects, expressing the difference in terms of a standard length unit.

and metric units, students will select an attribute to be measured, choose an appropriate unit of measurement and determine the number of units. Teachers provide rich tasks that ask students to perform real measurements, these foundational understandings are developed.

- Understand that larger units can be subdivided into equivalent units.
- Understand that same object or many objects of the same size such as paper clips can be repeatedly used to determine the length of an object.
- Understand the relationship between the size of a unit and the number of units needed.

See ex. page 24

- 2.MD.2 Second grade students measure an object using two different lengths. This experience helps students realize that the unit used is as important as the attribute being measured. This is a difficult concept for young children and will require numerous experiences for students to predict, measure, and discuss outcomes. See ex. page 25
- 2.MD.3 Students estimate the lengths of objects using inches, feet, CM, and meters prior to measuring. Estimation helps the students focus on the attribute being measured and the measuring process. As students estimate, the student has to

		T
		consider the size of the unit, helping them to become more familiar with the unit size. Estimation also creates a problem to be solved rather than a task to be completed. See ex. page 25 2.MD.4 -Students determine the difference in length between two objects by using the same tool and unit to measure both objects. Students choose two objects to measure, identify the appropriate tool and unit, measure and determine the difference in length. See ex. page 25
Relate addition and subtraction to length.	2.MD.5 -Solve addition and subtraction word problems using numbers up to 100 involving length that are given in the same units (e.g., by using drawings of rulers). Write an equation with a symbol for the unknown to represent the problem. 2.MD.6. Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1,2,, and represent whole-number sums and differences within 100 on a number line diagram.	2.MD.5 - Students apply the concept of length to solve addition and subtraction word problems with numbers within 100. Students should use the same unit of measurement in these problems. Equations may vary depending on students' interpretations of the task. See ex. on page 26 14+_= 23 23-14=_ 2.MD.6 -Building on their experiences with open number lines, students create number lines with evenly spaced points corresponding to the numbers to solve addition and subtraction problems to 100. They recognize the similarities between a number line and a ruler. See ex. on page 27.
Work with time and money.	2.MD.7 -Tell and write time to the nearest five minutes using a.m. and p.m. from analog and digital clocks.	2.MD.7 - In order to read an analog clock, students must be able to read a dial-type instrument. They must

	2.MD.8. Solve word problems involving dollar bills and coins using the \$ and ¢ symbols appropriately.	understand that the hour hand indicates the minutes between each hour. As students experience clocks with only hour hand looks different- but is still considered two. Discussing time as "about 2:00" "a little past 2:00" and "almost 3:00" helps build vocabulary to use when introducing time to the nearest five minutes.
		2.MD.8 - Student solve word problems involving either dollars or cents. Since students have not been introduced to decimals, problems focus on whole dollar amounts.
		Students will need numerous experiences with coin recognition and values of coins before using
Represent and interpret data.	2.MD.9 -Collect, record, interpret,	2.MD.9 -
data.	represent, and describe data in a table, graph or line plot.	2.MD.10 -
	2.MD.10. Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart and compare problems using information presented in a bar graph.	

Alaska Mathematical Domains Grades K-8	Alaska Mathematical Content Standards (2nd)	CES Interpretation of Standards (2nd)
Geometry (G)	2.G.1 - 2.G.3	What knowledge and skills will students need in order to fully meet the standard?
Reason with shapes and their attributes.	2.G.1 - Identify and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces compared visually, not by measuring. Identify	2.G.1 - 2.G.2 - 2.G.3 -

triangles, quadrilaterals, pentagons, hexagons and cubes.

2.G.2. Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.

2.G.3. Partition circles and rectangles into shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths.

Recognize that equal shares of identical wholes need not have the same shape.

The Alaska Mathematics Standards for Mathematical Practice are expected to be integrated into every mathematics lesson for all students Grades K-12. Below is a grade level analysis of these Practices as well as a few examples of how these Practice may be integrated into tasks that students accomplish.

ALASKA MATHEMATICAL PRACTICE GRADES K-12	CES ANALYSIS OF MATHEMATICAL PRACTICES AND EXAMPLES (3rd)
1. Make sense of problems and persevere in solving them.	1. Proficient students know that doing mathematics involves solving problems & discussing how they solved them. Students can explain the meaning of problems & look for ways to solve it by using concrete objects or pictures to conceptualize & solve problems. They should ask, "Does this make sense?", listen to other student's strategies & make connections between various methods for their problems.
2. Reason abstractly and quantitatively.	2. Proficient students should recognize that a number represents a specific quantity connecting to written symbols & create logical representations of the problem, consider appropriate units involved & the meaning of quantities.
3. Construct viable arguments and critique the reasoning of others.	3. Proficient students will construct arguments using concrete referents, such as objects, pictures, & drawings. They refine their mathematical communication skills as they participate in discussions that the teacher facilitates as students use explanations & verbal responses.
4. Model with mathematics.	4. Proficient students experiment with representing problem situations in multiple ways including numbers, words, drawing pictures, using objects, acting out, making a chart, list, or graph, creating equations, etc. Students require extensive opportunities to generate various math representations and to both equations & story problems, & explain connections between representations as well as between representations & equations. Students should be able to use all representations when needed & evaluate their results in the context of the problem & reflect whether the results make sense.
5. Use appropriate tools strategically.	5. Proficient students consider the available tools (including estimation) when solving a math problem & decide when certain tools might be helpful. Eg: graph paper, organized lists, tables,
6. Attend to precision.	6. Proficient students develop their math communication skills, try to use clear & precise language in their discussions with others & their own reasoning. They should specify units of measure &

The Craig City School District is dedicated to providing a meaningful, comprehensive, and engaging education to all students so they participate responsibly in the global society.

	state the meaning of the symbols they choose.
7. Look for and make use of structure.	7. Proficient students look closely to discover a pattern or structure. Students use properties of operations as strategies to multiply & divide (commutative & distributive properties)
8. Look for and express regularity in repeated reasoning.	8. Proficient students should notice repetitive actions in computation & look for more shortcut methods. Eg. use distributive property as a strategy for using products they know to solve products they don't know.

Third Grade Critical Areas - the critical areas are designed to bring focus to the standards at each grade by describing the big ideas that educators can use to build their curriculum and to guide instruction.

- 1. Developing understanding of multiplication and division and strategies for multiplication and division within 100
- 2. Developing understanding of fractions, especially unit fractions (fractions with numerator $\mathbf{1}$)
- 3. Developing understanding of the structure of rectangular arrays and area
- 4. Describing and analyzing two-dimensional shapes

Alaska Mathematical Domains Grades K-8	Alaska Mathematical Content Standards (3rd)	CES Interpretation of Standards (3rd)
Operations and Algebraic Thinking (OA)	3.OA.1 - 3.OA.9	What knowledge and skills will students need in order to fully meet the standard?
Represent and solve problems involving multiplication and division.	3.OA.1 - Interpret products of whole numbers (e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each). For example, show objects in rectangular arrays or describe a context in which a total number of objects can be expressed as 5 × 7. 3.OA.2 - Interpret whole-number quotients of whole numbers (e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56	3.OA.1 -what products of whole numbers are, express total number of objects in different ways (e.g., arrays, diagrams, manipulatives) 3.OA.2 - Use partition models & measurement (repeated subtraction) models.

	objects are partitioned into equal shares of 8 objects each). For example, deconstruct rectangular arrays or describe a context in which a number of shares or a number of groups can be expressed as $56 \div 8$. 3.OA.3 -Use multiplication and division numbers up to 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem). 3.OA.4 -Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations $8 \times ? = 48$, $5 = ? \div 3$, $6 \times 6 = ?$	3.OA.3 - Use a variety of representations for creating & solving one-step word problems involving multiplication & division in different problem structures. Student can reason through the problem mentally or verbally using pictures, numberline, Unknown variables introduced at this time. 3.OA.4 - Explore inverse operations of multiplication & division with comparing traditional notion of fact families.
Understand properties of multiplication and the relationship between multiplication and division.	3.OA.5 -Make, test, support, draw conclusions and justify conjectures about properties of operations as strategies to multiply and divide. (Students need not use formal terms for these properties.) Commutative property of multiplication: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. Associative property of multiplication: 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. Distributive property: Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 =	3.OA.5 -know & identify properties of multiplication & apply the rules in various situations.

	56. Inverse property (relationship) of multiplication and division. 3.OA.6 -Understand division as an unknown-factor problem. For example, find 32 ÷ 8 by finding the number that makes 32 when	3.OA.6 -Students are expected to solve problems using inverse operations & explain their processes.
Multiply and divide up to 100.	3.OA.7 -Fluently multiply and divide numbers up to 100, using strategies	3.OA.7 -By studying patterns and relationships in multiplication
	such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.	facts & relating multiplication & division, student build a foundation for fluency with multiplication & division facts. Students demonstrate fluency with multiplication facts through 10 & the related division facts. *Knowledge of procedures, when & how to use them appropriately, & skill in performing them flexibly, accurately, & efficiently.
Solve problems involving the four operations, and identify and explain patterns in arithmetic.	3.OA.8 -Solve and create two-step word problems using any of the four operations. Represent these problems using equations with a symbol (box, circle, question mark) standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.	3.OA.8 -Students begin to use formal algebraic language by using a letter for the unknown quantity in expressions or equations for 1 & 2-step problems. The 2-step problems should use the 4 operations. Adding & subtracting numbers should include numbers within 1,000 & multiplying & dividing numbers should include single-digit factors & products less than 100. *strategies: estimation,
	3.OA.9 -Identify arithmetic patterns (including patterns in the addition table or multiplication table) and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be	compatible numbers, rounding, discussion, checking for reasonableness. 3.OA.9 -Students will examine arithmetic patterns in addition &

multiplication.

will use various contexts, models,

notion of "same size & same

parts.

shape" (congruence) to explain

word problems & develop intuitive

decomposed into two equal addends.

	decomposed into two equal addenas.	multiplication.
Alaska Mathematical Domains Grades K-8	Alaska Mathematical Content Standards (3rd)	CES Interpretation of Standards (3rd)
Number and Operations in Base Ten (NBT)	3.NBT.1 - 3.NBT.3	What knowledge and skills will students need in order to fully meet the standard?
Use place value understanding and properties of operations to perform multi-digit arithmetic.	3.NBT.1 -Use place value understanding to round whole numbers to the nearest 10 or 100. 3.NBT.2 -Use strategies and/or algorithms to fluently add and subtract with numbers up to 1000, demonstrating understanding of place value, properties of operations, and/or the relationship between addition and subtraction. 3.NBT.3 -Multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., 9 x 80, 10 x 60) using strategies based on place value and properties of operations.	3.NBT.1 -Students will understand place value & number sense when rounding. 3.NBT.2 -Students will apply commutative & associative properties to explain their thinking & show their work by using strategies & steps & verify that answers are reasonable. 3.NBT.3 -Students will understand that multiplication of one-digit numbers with multiples of 10 represent groups of tens using the associative property.
Alaska Mathematical	Alaska Mathematical Content	CES Interpretation of Standards
Domains Grades K-8 Number and Operations Fractions (NF)	Standards (3rd) 3.NF.1 - 3.NF.3 (limited in this grade to fractions with denominators 2, 3, 4, 6, & 8)	(3rd) What knowledge and skills will students need in order to fully meet the standard?
Develop understanding of fractions as numbers.	3.NF.1 -Understand a fraction $1/b$ (e.g., $1/4$) as the quantity formed by 1 part when a whole is partitioned into b (e.g., 4) equal parts; understand a fraction a/b (e.g., $2/4$) as the quantity formed by a (e.g., $2/4$) parts of size $1/b$	3.NF.1 -Students will start with fractions with numerator of 1, which are formed by partitioning a whole into parts & reasoning about one part of the whole. They

formed by a (e.g., 2) parts of size 1/b.

3.NF.2 -Understand a fraction as a

represent fractions on a number

number on the number line;

(e.g., 1/4)

line diagram.

a. Represent a fraction 1/b (e.g., 1/4) on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b (e.g., 4) equal parts. Recognize that each part has size 1/b (e.g., 1/4) and that the endpoint of the part based at 0 locates the number 1/b(e.g., 1/4) on the number line. b. Represent a fraction a/b (e.g., 2/8) on a number line diagram or ruler by marking off a lengths 1/b (e.g., 1/8) from o. Recognize that the resulting interval has size a/b (e.g., 2/8) and that its endpoint locates the number a/b (e.g., 2/8) on the number line.

3.NF.2 -Students will use a number line diagram & other linear models to help reason & justify the location of fractions.

3.NF.3 -Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

a. Understand two fractions as equivalent if they are the same size (modeled) or the same point on a number line.

b. Recognize and generate simple equivalent fractions (e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent (e.g., by using a visual fraction model). c. Express and model whole numbers as fractions, and recognize and construct fractions that are equivalent to whole numbers. For example: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.

d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results

3.NF.3 -Students should be able to visualize fractions models & number lines to explore equivalent fractions. This includes writing whole numbers as fractions & relate it to division problems. This involves comparing fractions with or without visual fraction models including number lines. They should reason that comparisons are only valid if the wholes are identical with same denominators. They should also see that for unit fractions, the one with the larger denominator is smaller.

,	of comparisons with the symbols >, =, or <, and justify the conclusions (e.g., by using a visual fraction model).	
---	-------------------------------------------------------------------------------------------------------------------	--

Alaska Mathematical Domains Grades K-8	Alaska Mathematical Content Standards (3rd)	CES Interpretation of Standards (3rd)
Measurement and Data (MD)	3.MD.1 - 3.MD.10	What knowledge and skills will students need in order to fully meet the standard?
Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.	3.MD.1 -Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes or hours (e.g., by representing the problem on a number line diagram or clock).	3.MD.1 -Students will solve elapsed time problems using clock models or number lines.
	3.MD.2 -Estimate and measure liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). (Excludes compound units such as cm3 and finding the geometric volume of a container.) Add, subtract, multiply, or divide to solve and create one-step word problems involving masses or volumes that are given in the same units (e.g., by using drawings, such as a beaker with a measurement scale, to represent the problem). (Excludes multiplicative comparison problems [problems involving notions of "times as much."])	3.MD.2 -Students will reason about the units of mass & volume using units g, kg, & L. Multiple opportunities for weighing & filling containers are needed to develop understanding. Relationships between smaller units to larger units in the same system is emphasized as well as reasoning by estimating & using benchmarks to measure weight & capacity. (Weight & mass are not distinguished at this time.)
*	3.MD.3 -Select an appropriate unit of English, metric, or non-standard measurement to estimate the length, time, weight, or temperature (L)	3.MD.3 - Estimates of length, time, weight, or temperature should be used with appropriate units of measure.

Represent and interpret data.	3.MD.4 -Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. For example, draw a bar graph in which each square in the bar graph might represent 5 pets.	3.MD.4 - Students should read, solve problems & draw scaled graphs (picture, bar: horizontal & verticle) with different intervals. They will explore data concepts, pose a question, collect data, analyze & interpret data.
	3.MD.5 -Measure and record lengths using rulers marked with halves and fourths of an inch. Make a line plot with the data, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.	3.MD.5 -Students will work with fractions by measuring objects to a quarter of an inch and make a line plot graph with appropriate units & data recorded.
	3.MD.6 -Explain the classification of data from real-world problems shown in graphical representations. Use the terms minimum and maximum. (L)	3.MD.6 -Use real world graphical representations of classified data. Interpret data using terms: minimum & maximum.
Geometric measurement: understand concepts of area and relate area to multiplication and to addition.	3.MD.7 -Recognize area as an attribute of plane figures and understand concepts of area measurement. a. A square with side length 1 unit is said to have "one square unit" and can be used to measure area. b. Demonstrate that a plane figure which can be covered without gaps or overlaps by <i>n</i> (e.g., 6) unit squares is said to have an area of <i>n</i> (e.g., 6) square units.	3.MD.7 -Students understand the concept of covering a regions with "unit squares" then transition to pictorial representations on graph paper to experience area measurements.
	3.MD.8 -Measure areas by tiling with unit squares (square centimeters, square meters, square inches, square feet, and improvised units).	3.MD.8 -Students should be counting the square units to find the area in metric, customary, or non-standard square units.
	3.MD.9 -Relate area to the operations of multiplication and addition.	3.MD.9 -Students learn how to multiply length measurements to find the area of a rectangular

a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. For example, after tiling rectangles, develop a rule for finding the area of any rectangle. b. Multiply side lengths to find areas of rectangles with whole number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. c. Use area models (rectangular arrays) to represent the distributive property in mathematical reasoning. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths \bar{a} and b + c is the sum of a \times b and $a \times c$. d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into nonoverlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems. For example, the area of a 7 by 8 rectangle can be determined by decomposing it into a 7 by 3 rectangle and a 7 by 5 rectangle.

region using arrays. Students should understand & explain that one length tells how many unit squares in a row & the other how many rows there are.

Figures could be decomposed to help find the area using rectilinear figure with all right angles & using distributive properties to solve. 5x7=, (5x5)+(5x2)

Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

3.MD.10 -Solve real world and mathematical problems involving perimeters of polygons, including:

- finding the perimeter given the side lengths,
- finding an unknown side length,
- exhibiting rectangles with the same perimeter and different areas,
- exhibiting rectangles with the

3.MD.10 -Students develop an understanding of perimeter through various experiences using addition & recognizing patterns that exist when finding the sum or the lengths & widths of rectangles. Tools such as: geoboards, tiles, graph paper, dot & graph paper should be utilized. They will justify & communicate their solutions using words, diagrams,

Till d Grade		
	same area and different perimeters.	pictures, numbersto find the missing length or width.
Alaska Mathematical Domains Grades K-8	Alaska Mathematical Content Standards (3rd)	CES Interpretation of Standards (3rd)
Geometry (G)	3.G.1 - 3.G.2	What knowledge and skills will students need in order to fully meet the standard?
Reason with shapes and their attributes.	3.G.1 -Categorize shapes by different attribute classifications and recognize that shared attributes can define a larger category. Generalize to create examples or non-examples. 3.G.2 -Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.	3.G.1 -Students categorize shapes by attributes & examine properties of geometric figures that belong to a category: quadrilaterals, squares, rectangles, rhombuses, parallelograms, trapezoids. 3.G.2 -Students develop the idea of fraction more formally by partitioning a whole into equal parts (circle, rectangle). Students should be responsible to partition parts to ½, ⅓, ¼, □, ⅓ in different ways.

The Alaska Mathematics Standards for Mathematical Practice are expected to be integrated into every mathematics lesson for all students Grades K-12. Below is a grade level analysis of these Practices as well as a few examples of how these Practice may be integrated into tasks that students accomplish.

ALASKA MATHEMATICAL PRACTICE GRADES K-12	CES ANALYSIS OF MATHEMATICAL PRACTICES AND EXAMPLES (4th)
Make sense of problems and persevere in solving them.	1. Proficient students know that doing mathematics involves solving problems and discussing how they solved them. Students can explain the meaning of a problem and look for ways to solve it. Students will use concrete objects, pictures, or models to help them conceptualize and solve the project. Students will check their thinking by asking, "Does this make sense?" They listen to strategies of others and will try different approaches. They often will use another method to check their answers.
2. Reason abstractly and quantitatively.	2. Proficient students will recognize that a number represents a specific quantity. They connect the quantity to written symbols and create a logical representation of the problem, considering both the appropriate units involved and the meaning of quantities. They extend this understanding from whole numbers to their work with fractions and decimals. Students write simple expressions, record calculations with numbers, and represent or round numbers using place value concepts.
3. Construct viable arguments and critique the reasoning of others.	3. Proficient students will construct arguments using concrete referents, such as objects, pictures, and drawings. They explain their thinking and make connections between models and equations. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?" They explain their thinking to others and respond to others' thinking.
4. Model with mathematics.	4. Proficients students will experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, or raph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They will use all of these representations as needed. STudents will evaluate their results in the context of the situation and reflect on whether the results make sense.

5. Use appropriate tools strategically.	5. Proficient students will consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they might use other measurement tools to understand the relative size of units within a system and express measurements given in larger units in terms of smaller units.
6. Attend to precision.	6. Proficient students will develop their mathematical communication skills, they will try to use clear and precise language in their discussions with others in their own reasoning. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, they use appropriate labels when creating a line plot.
7. Look for and make use of structure.	7. Proficient students look closely to discover a pattern or structure. For instance, students use properties of operations to explain calculations (partial products model). They relate representations of counting problems such as tree diagrams and arrays to the multiplication principle of counting. They generate number or shape patterns that follow a given rule.
8. Look for and express regularity in repeated reasoning.	8. Proficient students will notice repetitive actions in computations to make generalizations. Students use models to explain calculations and understand how algorithms work. They also use models to examine patterns and generate their own algorithms. For example, students use visual fraction models to write equivalent fractions.

Fourth Grade Critical Areas - the critical areas are designed to bring focus to the standards at each grade by describing the big ideas that educators can use to build their curriculum and to guide instruction.

1. Developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends

*Students generalize their understanding of place value to 1,000,000, understanding the relative sizes of numbers in each place. They apply their understanding of models for multiplication (equal-sized groups, arrays, area models), place value, and properties of operations, in particular the distributive property, as they develop, discuss, and use efficient, accurate, and generalizable methods for multiplying whole numbers; understand and explain why the procedures work based on place value and properties of operations; and use them to solve problems. Students apply their understanding of models for division, place value, properties of operations, and the relationship of division to multiplication as they develop, discuss and use efficient, accurate, and generalizable procedures to find quotients involving multi-digit dividends. They select and accurately apply appropriate methods to estimate and mentally calculate quotients, and interpret remainders based upon the context.

- 2. Developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers *Students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g. 15/9=5/3), and they develop methods for generating and recognizing equivalent fractions. Students extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number.
- 3. Understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry

*Students describe, analyze, compare, and classify two-dimensional shapes. Through building, drawing, and analyzing two dimensional shapes, students deepen their understanding of properties of two-dimensional objects and the use of them to solve problems involving symmetry.

Alaska Mathematical Domains Grades K-8 Alaska Mathematical Content Standards (4th) CES Interpretation of Standards (4th)

Tour di Grade		
Operations and Algebraic Thinking (OA)	4.0A.1 - 4.0A.6 Use the four operations with whole numbers to solve problems.	What knowledge and skills will students need in order to fully meet the standard?
Use the four operations with whole numbers to solve problems.	4.0A.1 -Interpret a multiplication equation as a comparison (e.g., interpret 35 = 5 x7 as a statement that 35 is 5 groups of 7 and 7 groups of 5). (Commutative property) Represent verbal statements of multiplicative comparisons as multiplication equations.	4.0A.1 -A multiplicative comparison is a situation in which one quantity is multiplied by a specified number to get another quantity (e.g., "a is n times as much as b"). Students should be able to identify and verbalize which quantity is being multiplied and which number tells how many times.
		Students should be given opportunities to write and identify equations and statements for multiplicative comparisons. Example: $5 \times 8 = 40$ Sally is five years old. Her mom is eight times older. How old is Sally's mom? $5 \times 5 = 25$ Sally has five times as many pencils as Mary. If Sally has 5 pencils, how many does Mary have?
	4.OA.2 - Multiply or divide to solve word problems involving multiplicative comparison (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem or missing numbers in an array). Distinguish multiplicative comparison from additive comparison.	4.OA.2 - This standard calls for students to translate comparative situations into equations with an unknown and solve. Students need many opportunities to solve contextual problems. *Refer to examples on Math Standards p. 5

	4.OA.3 -Solve multistep word problems posed with whole numbers and having wholenumber answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.	4.OA.3 -The focus in this standard is to have student use and discuss various strategies. It refers to estimation strategies, including using compatible numbers (numbers that sum to 10 or 100) or rounding. Problems should be structured so that all acceptable estimation strategies will arrive at a reasonable answer. Students need many opportunities solving multistep story problems using all four operations. *Refer to examples on Math Standards p. 7-9
Gain familiarity with factors and multiples.	* Find all factor pairs for a whole number in the range 1.–100. * Explain the correlation/differences between multiples and factors. * Determine whether a given whole number in the range 1.–100 is a multiple of a given one-digit number. * Determine whether a given whole number in the range 1–100 is prime or composite.	4.OA.4 -This standard requires students to demonstrate understanding of factors and multiples of whole numbers. This standard also refers to prime and composite numbers. Prime numbers have exactly two factors, the number one and their own number. For example, the number 17 has the factors of 1 and 17. Composite numbers have more than two factors. For example, 8 has the factors 1, 2, 4, and 8. *Refer to examples on Math Standards p. 9-10.
Generate and analyze patterns.	4.OA.5 -Generate a number or shape pattern, table, t-chart, input/output function that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself.	4.OA.5 -Patterns involving numbers or symbols either repeat or grow. Students need multiple opportunities creating and extending number and shape patterns. Numerical patterns

Be able to express the pattern in algebraic terms. For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.	allow students to reinforce facts and develop fluency with operations. Patterns and rules are related. A pattern is a sequence that repeats the same process over and over. A rule dictates what that process will look like. Student investigate different patterns to find rules, identify features in the patterns, and justify the reasons for those features. *Refer to examples on Math Standards p. 11-12.
4.0A.6 -Extend patterns that use addition, subtraction, multiplication, division or symbols, up to 10 terms, represented by models (function machines), tables, sequences, or in problem situations (L)	4.OA.6 -

Alaska Mathematical Domains Grades K-8	Alaska Mathematical Content Standards (4th)	CES Interpretation of Standards (4th)
Number and Operations in Base Ten (NBT)	4.NBT.1 - 4.NBT.6	What knowledge and skills will students need in order to fully meet the standard?
Generalize place value understanding for multi- digit whole numbers.	4.NBT.1 -Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that	4.NBT.1 -This standard calls for students to extend their understanding of place value related to multiplying and dividing by multiples of 10. In this standard, students should reason about the magnitude of digits in a

 $700 \div 70 = 10$ by applying concepts of place value and division.

number. Students should be given opportunities to reason and analyze the relationships of numbers that they are working with.

In the base-ten system, the value of each place is 10 times the value of the place to the immediate right. Because of this, multiplying by 10 yields a product in which each digit of the multiplicand is shifted one place to the left.

*Refer to examples on Math Standards p. 13.

4.NBT.2 -Read and write multidigit whole numbers using base-ten numerals, number names, and expanded form.

Compare two multi-digit numbers based on the value of the digits in each place, using >, =, and < symbols to record the results of comparisons.

4.NBT.2 -This standard refers to various ways to write numbers. Students should have flexibility with the different number forms. Traditional expanded form is 285 = 200 + 80 + 5. Written form or number name is two hundred eighty-five. However, students should have opportunities to explore the idea that 285 could be 28 tens plus 5 ones or 1 hundred, 18 tens, and 5 ones.

To read numerals between 1,000 and 1,000,000, students need to understand the role of commas. Each sequence of three digits made by commas is read as hundreds, tens, and ones, followed by the name of the appropriate base-thousand unit (thousand, million, billion, trillion, etc.). Thus, 457,000 is read "four hundred fifty seven thousand." The same methods students used for comparing and rounding

	4.NBT.3 -Use place value understanding to round multi-digit whole numbers to any place using a variety of estimation methods; be able to describe, compare, and contrast solutions.	numbers in previous grades apply to these numbers, because of the uniformity of the base-ten system. Students should also be able to compare two multi-digit whole numbers using appropriate symbols. 4.NBT.3 -This standard refers to place value understanding, which extends beyond an algorithm or procedure for rounding. The expectation is that students have a deep understanding of place value and number sense and can explain and reason about the answers they get when they round. Students should have numerous experiences using a number line and a hundreds chart as tools to support their work with rounding. *Refer to examples on Math Standards p. 14-15.
Use place value understanding and properties of operations to perform multi-digit arithmetic.	4.NBT.4 -Fluently add and subtract multi-digit whole numbers using any algorithm. Verify the reasonableness of the results.	4.NBT.4 -Students build on their understanding of addition and subtraction, their use of place value and their flexibility with multiple strategies to make sense of the standard algorithm. They continue to use place value in describing and justifying the processes they use to add and subtract. This standard refers to fluency, which means accuracy, efficiency (using a reasonable amount of steps and time), and flexibility (using a variety of strategies such as the distributive property). This is the first grade level in which

proficient at using the standard algorithms add and subtract.
However, other previously learned strategies are still appropriate for students to use.

Computation algorithm - A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. Computation strategy - Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another.

In mathematics, an algorithm is defined by its steps and not by the way those steps are recorded in writing. With this in mind, minor variations in methods of recording standard algorithms are acceptable. As with addition and subtraction, students should use methods they understand and can explain. Visual representations such as area and array diagrams that students draw and connect to equations and other written numerical work are useful for this purpose. By reasoning repeatedly about the connection between math drawings and written numerical work, students can come to see multiplication and division algorithms as abbreviations or summaries of their reasoning about quantities. Students can invent and use fast special strategies while also working towards understanding

4.NBT.5 -Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

general methods and the standard algorithm.

*Refer to examples on Math Standards p. 17-18.

4.NBT.5 -Students who develop flexibility in breaking numbers apart have a better understanding of the importance of place value and the distributive property in multi-digit multiplication. Students use base ten blocks, are models, partitioning. compensation strategies, etc. when multiplying whole numbers and use words and diagrams to explain their thinking. They use the terms factor and product when communicating their reasoning. Multiple strategies enable students to develop fluency with multiplication and transfer that understanding to division. Use of the standard algorithm for multiplication is an expectation in the 5th grade.

Another part of understanding general base-ten methods for multi-digit multiplication is understanding the role played by the distributive property. This allows numbers to be decomposed into base-ten units, products of the units to be computed, and then combined. By decomposing the factors into like base-ten units and applying the distributive property, multiplication computations are reduced to single-digit multiplications and products of numbers with

	multiples of 10, 100, and 1000. Students can connect diagrams of areas or arrays to numerical work to develop understanding of general base-ten multiplication methods. Computing products of two two-digit numbers requires using the distributive property several times when the factors are decomposed into base-ten units. *Refer to examples on Math Standards p. 19-21.
4.NBT.6 -Find whole-number quotients and remainders with one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	4.NBT.6 - In fourth grade, students build on their third grade work with division within 100. Students need opportunities to develop their understandings by using problems in and out of context. General methods for computing quotients of multi-digit numbers and one-digit numbers rely on the same understandings as for multiplication, but cast in terms of division. One component is quotients of multiples of 10, 100, or 1000. and one-digit numbers. *Refer to examples on Math Standards p. 21-25.

g

Alaska Mathematical Domains Grades K-8	Alaska Mathematical Content Standards (4th)	CES Interpretation of Standards (4th)
Number and Operations Fractions (NF)	4.NF.1 - 4.NF.7 (limited in this grade to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, & 100)	What knowledge and skills will students need in order to fully meet the standard?
Extend understanding of fraction equivalence and	4.NF.1 -Explain why a fraction a/b is equivalent to a fraction	4.NF.1 - This standard refers to visual fraction models. This

 $The \ Craig \ City \ School \ District \ is \ dedicated \ to \ providing \ a \ meaningful, \ comprehensive, \ and \ engaging \ education \ to \ all \ students \ so \ they \ participate \ responsibly \ in \ the \ global \ society.$

Fourth Grade		
ordering.	$(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.	includes area models, number lines or it could be a collection/set model. This standard extends the work in third grade by using additional denominators. (5, 10, 12, and 100) This standard addresses equivalent fractions by examining the idea that equivalent fractions can be created by multiplying both the numerator and denominator by the same number or by dividing a shaded region into various parts. **Refer to examples on Math Standards p. 26-27.
	4.NF.2 -Compare two fractions with different numerators and different denominators (e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2). Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions (e.g., by using a visual fraction model).	4.NF.2 -This standard calls students to compare fractions by creating visual fraction models or finding common denominators or numerators. Students' experiences should focus on visual fraction models rather than algorithms. When tested, models may or may not be included. Students should learn to draw fraction models to help them compare. Students must also recognize that they must consider the size of the whole when comparing fractions (ie, ½ and ⅓ of two medium pizzas is very different from ½ of one medium and ⅓ or one large).

Build fractions from unit fractions by applying and extending previous understanding of operations on whole numbers.

4.NF.3 -Understand a fraction a/bwith a > 1 as a sum of fractions 1/b. a. Understand addition and subtraction of fractions as joining and separating parts referring to

4.NF.3 -A fraction with a numerator of one is called a unit fraction. When students investigate fractions other than unit fractions, such as 2/3, they should be able to join (compose) or separate (decompose) the fractions of

*Refer to examples on Math

Standards p. 28-30.

the same whole.

Example: $\frac{2}{3} = \frac{1}{3} + \frac{1}{3}$ Being able to visualize this decomposition into unit fractions helps students when adding or subtracting fractions. Students need multiple opportunities to work with mixed numbers and be able to decompose them in more than one way. Students may use visual models to help develop this understanding. *Refer to examples on Math Standards p. 31-.

b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions (e.g., by using a visual fraction model). Examples: 3/8 = 1/8 + 1/8 + 1/83/8 = 1/8 + 2/8; 21/8 = 1+1+1/8 = 8/8 + 8/8 + 1/8.

Students should justify their breaking apart (decomposing) of fractions using visual fraction models. The concept of turning mixed numbers into improper fractions needs to be emphasized using visual fraction models. *Refer to examples on Math Standards p. 32.

c. Add and subtract mixed numbers with like denominators (e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction).

A separate algorithm for mixed numbers in addition and subtraction is not necessary. Students will tend to add or subtract the whole numbers first and then work with the fractions using the same strategies they have applied to problems that contained only fractions. *Refer to examples on Math Standards p. 33-35.

d. Solve word problems involving addition and subtraction of fractions referring to the same

A cake recipe calls for you to use 3/4 cup of milk, 1/4 cup of oil, and 2/4 cup of water. How much

whole and having like denominators (e.g., by using visual fraction models and equations to represent the problem).

liquid was needed to make the cake?

*Refer to examples on Math Standards p. 35.

4.NF.4 - Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

a. Understand a fraction a/b as a multiple of 1/b. For example, use a visual fraction model to represent 5/4 as the product

5 × (1/4), recording the conclusion by the equation

5/4 = 5 × (1/4).

4.NF.4 - This standard builds on students' work of adding fractions and extending that work into multiplication.

*Refer to examples on Math Standards p. 35-36.

b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express 3 ×(2/5) as 6 × (1/5), recognizing this product as 6/5.

(In general, n ×(a/b) = (n ×a)/b.)

This standard extended the idea of multiplication as repeated addition. For example, $3 \times (2/5) = \Box + \Box + \Box = 6/5 = 6 \times (1/5)$. Students are expected to use and create visual fraction models to multiply a whole number by a fraction. *Refer to examples on Math

c. Solve word problems involving multiplication of a fraction by a whole number (e.g., by using visual fraction models and equations to represent the problem). Check for the reasonableness of the answer. For example, if each person at a party

When introducing this standard make sure student use visual fraction models to solve word problems related to multiplying a whole number by a fraction. *Refer to examples on Math Standards p. 37-39.

Standards p. 36.

	will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?	
Understand decimal notation for fractions, and compare decimal fractions.	4.NF.5 -Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.	4.NF.5 - This standard continues the work of equivalent fractions by having students change fractions with a 10 in the denominator into equivalent fractions that have a 100 in the denominator. In order to prepare for work with decimals (4.NF.6 and 4.NF.7), experiences that allow students to shade decimal grids (10x10 grids) can support this work. Student experiences should focus on working with grids rather than algorithms. Students can also use base ten blocks and other place value models to explore the relationship between fractions with denominators of 10 and 100. Students in fourth grade work with fractions having denominators of 10 and 100. Students in fourth grade work with fractions having denominators 10 and 100. Because it involves partitioning into 10 equal parts and treating the parts as numbers called one tenth and one hundredth, work with these fractions can be used as preparation to extend the base ten system to non-whole numbers. *Refer to examples on Math Standards p. 39-40.

4.NF.6 -Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.

4.NF.6 -Decimals are introduced for the first time. Students should have ample opportunities to explore and reason about the idea that a number can be represented as both a fraction and a decimal.

Students make connections between fractions with denominators of 10 and 100 and the place value chart. By reading fraction names, students say 32/100 as thirty-two hundredths and rewrite this as 0.32 or represent it on a place value model.

*Refer to examples on Math Standards p. 41.

4.NF.7 -Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions (e.g., by using a visual model).

4.NF.7 - Students should reason that comparisons are only valid when they refer to the same whole. Visual models include area models, decimal grids decimal circles, number lines, and meter sticks.

The decimal point is used to signify the location of the ones place, but its location may suggest there should be a "oneths" place to its right in order to create symmetry with respect to the decimal point. However, because one is the basic unit from which the other base ten units are derived, the symmetry occurs instead with respect to the ones place.

*Refer to examples on Math Standards p. 41-42.

Alaska Mathematical	Alaska Mathematical Content	CES Interpretation of
Domains Graes K-8	Standards (4th)	Standards (4th)
Measurement and Data (MD)	4.MD.1 - 4.MD.9	What knowledge and skills will students need in order to fully meet the standard?

Solve problems involving measurement and conversion of measurement from a larger unit to a smaller unit, and involving time.

4.MD.1 -Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two column table. For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36).

4.MD.2 -Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

4.MD.3 -Apply the area and perimeter formulas for rectangles in real world and mathematical problems. For example, find the width of a rectangular room given

4.MD.1 -

4.MD.2 -

4.MD.3 -

4.MD.4 -

the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.	
4.MD.4 -Solve real-world problems involving elapsed time between U.S. time zones (including Alaska Standard time) (L)	

Represent and interpret data.	4.MD.5 -Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.	4.MD.5 - 4.MD.6 -
	4.MD.6 -Explain the classification of data from real-world problems shown in graphical representations including the use of terms mean, range, median and mode with a given set of data. (L)	
Geometric measurement: understand concepts of angle and measure angles.	4.MD.7 -Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand the following concepts of angle measurement: a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the	4.MD.7 - 4.MD.8 - 4.MD.9 -
,	fraction of the circular arc between the points where the two rays intersect the circle. An angle that	

	4.MD.8 -Measure and draw angles in whole-number	
	degrees using a protractor.	5
P 4, 1	Estimate and sketch angles of	296
	specified measure.	7. 12
	4.MD.9 - Recognize angle measure	
	as additive. When an angle is	
	divided into non-overlapping	
	parts, the angle measure of the	
- 1	whole is the sum of the angle	
a	measures of the parts. Solve	
	addition and subtraction problems	70
5 1	to find unknown angles on a	
	diagram in real world and	
	mathematical problems (e.g., by	1
	using an equation with a symbol	
	for the unknown angle measure).	

Alaska Mathematical Domains Grades K-8	Alaska Mathematical Content Standards (4th)	CES Interpretation of Standards (4th)
Geometry (G)	4.G.1 - 4.G.3	What knowledge and skills will students need in order to fully meet the standard?
Draw and identify lines and angles, and classify shapes by properties of their lines and angles.	 4.G.1 -Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular, parallel, and intersecting line segments. Identify these in plane figures. 4.G.2 -Classify plane figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles. 	4.G.1 - 4.G.2 - 4.G.3 -

4.G.3 -Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

The Alaska Mathematics Standards for Mathematical Practice are expected to be integrated into every mathematics lesson for all students Grades K-12. Below is a grade level analysis of these Practices as well as a few examples of how these Practice may be integrated into tasks that students accomplish.

ALASKA MATHEMATICAL PRACTICE GRADES K-12	CES ANALYSIS OF MATHEMATICAL PRACTICES AND EXAMPLES (5th)
1. Make sense of problems and persevere in solving them.	1. 5th grade students will demonstrate their understanding of operations with whole numbers, decimals, and fractions including mixed numbers. These students need to have an understanding of a problem being logical or not.
2. Reason abstractly and quantitatively.	2. Students in grade 5 should be able to understand the relationships between whole numbers, and fractions and decimals. They need to show this understanding by writing expressions that demonstrate their understanding.
3. Construct viable arguments and critique the reasoning of others.	3. 5th grade students should be able to come up with reasonable arguments and critique for others, using questioning and supporting mathematical facts.
4. Model with mathematics.	4. 5th grade students need to be able to create visual models such as graphs, charts, lists, and pictures, to show what they have learned.
5. Use appropriate tools strategically.	5. Students will consider the available tools when solving problems and decide if they might be helpful to solve the problem.
6. Attend to precision.	6. Students will use clear and concise language in their discussions and reasoning. They will use appropriate terms.
7. Look for and make use of structure.	7. Students will look for and identify patterns.
8. Look for and express regularity in repeated reasoning.	8. Students use repeated reasoning to understand algorithms and make generalizations about patterns. They make connections between previously learned material.

Fifth Grade Critical Areas - the critical areas are designed to bring focus to the standards at each grade by describing the big ideas that educators can use to build their curriculum and to guide instruction.

- 1. Developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions)
- 2. Extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations
- 3. Developing understanding of volume

Alaska Mathematical Domains Grades K-5	Common Core Standard (5th)	CES Interpretation of Standards (5th)
Operations and Algebraic Thinking (OA)	5.OA.1 - 5.OA.3	What knowledge and skills will students need in order to fully meet the standard?
Write and interpret numerical expressions.	Operations and Algebraic Thinking 5.OA.1 Use parentheses to construct numerical expressions, and evaluate numerical expressions with these symbols. 5.OA.2. Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them.	5.OA.1 - (PEMDAS) 5.OA.2 - Students will be able write out and solve expressions that include multiple steps Ex: "Double five and then add 26."
Analyze patterns and relationships.	5.OA.3. Generate two numerical patterns using two given rules.	5.OA.3 - Students in 5th grade will be able to create a double line

Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.	graph using ordered pairs. They will also have an understanding of independent and dependent variables related to the graph.
----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------	------------------------------------------------------------------------------------------------------------------------------

Alaska Mathematical Domains Grades K-8	Alaska Mathematical Content Standards (5th)	CES Interpretation of Standards (5th)
Number and Operations in Base Ten (NBT)	5.NBT.1 - 5.NBT.7	What knowledge and skills will students need in order to fully meet the standard?
Understand the place value system.	5.NBT.1. Recognize that in a multi- digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.	5.NBT.1 - Students will be able to recognize number relationships in place value. For example, they will know that a number in the tens place is 10 times bigger than a number in the ones place.
	5.NBT.2. Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain and extend the patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10. 5.NBT.3. Read, write, and compare decimals to thousandths.	5.NBT.2 - Students will understand how to multiply and divide quicker using zeros and the power of 10s. 5.NBT.3 - 5th grade students will continue to write numbers with and without decimals, using expanded form.

	a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form [e.g., 347.392 = 3 x 100 + 4 x 10 + 7 x 1 + 3 (1/10) + 9 (1/100) + 2 (1/1000)]. b. Compare two decimals to thousandths place based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. 5.NBT.4. Use place values understanding to round decimals to any place.	5.NBT.4 - Students should be able to round decimals to the thousandths place, using a variety of methods such as a number line or base ten model.
Perform operations with multi-digit whole numbers and with decimals to hundredths.	5.NBT.5. Fluently multiply multidigit whole numbers using a standard algorithm 5.NBT.6. Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, number lines, real life situations, and/or area models.	5.NBT.5 - 5th graders will be able to solve multiplication and division problems using the standard algorithm. 5.NBT.6 - 5th graders will be able to solve real-life, division and multiplication, story problems. These students will solve these problems using models, arrays, or written out equations
	5.NBT.7. Add, subtract, multiply, and divide decimals to hundredths,	5.NBT.7 - Students will be able to add, subtract, multiply, and divide decimals to the hundredths using

	using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between the operations. Relate the strategy to a written method and explain their reasoning in getting their answers.	a variety of strategies that relate to place value, properties of operations. They will also need to write out their explanations to how they solved the problem.
--	------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------	-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------

Alaska Mathematical Domains Grades K-8	Alaska Mathematical Content Standards (5th)	CES Interpretation of Standards (5th)
Number and Operations Fractions (NF)	5.NF.1 - 5.NF.7	What knowledge and skills will students need in order to fully meet the standard?
Use equivalent fractions as a strategy to add and subtract fractions.	5.NF.1. Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.) 5.NF.2. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators (e.g., by using visual fraction models or equations to represent the problem). Use benchmark fractions and number sense of fractions to estimate mentally and check the reasonableness of answers. For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.	5.NF.1 - Students will be able to add and subtract fractions with unlike denominators including mixed numbers. They will also need to be able to add and subtract them using models while explaining their reasoning. 5.NF.2 - 5th graders will be able to solve word problems that ask the student to add or subtract fractions with unlike denominators. They will need to show how to simplify each problem by rounding the fractions, finding the least common denominator, and using models.
Apply and extend	5.NF.3. Interpret a fraction as	

previous understandings of multiplication and division to multiply and divide fractions.

division of the numerator by the denominator $(a/b = a \div b)$. Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers (e.g., by using visual fraction models or equations to represent the problem). For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?

5.NF.4 -Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

a. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. For example, use a visual fraction model to show $(2/3) \times 4 = 8/3$, and create a story context for this equation. Do the same with $(2/3) \times (4/5) = 8/15$. (In general, $(a/b) \times (c/d) = ac/bd$.)

b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as

5.NF.5 -Interpret multiplication as

5.NF.3 - 5th grade students should understand the relationship between fractions and division. They need to have an understanding that division is equal sharing.

5.NF.4 - Students need to understand that the multiplication of a fraction of a whole number could be represented as repeated addition of a unit.

5.NF.5 - This standard calls for students to examine the magnitude of products in terms of relationships between the two types of problems.

5.NF.6 -This standard builds on all work done in this cluster. Students should be given ample opportunities to use various strategies to solve word problems involving the multiplication of a fraction by a mixed number.

5.NF.7 - Is the first time that students are dividing with fractions.

Division of a fraction by a

scaling (resizing), by:

- a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.
- b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1. (Division of a fraction by a fraction is not a requirement at this grade.)
- 5.NF.6. Solve real world problems involving multiplication of fractions and mixed numbers (e.g., by using visual fraction models or equations to represent the problem).
- 5.NF.6. Solve real world problems involving multiplication of fractions and mixed numbers (e.g., by using visual fraction models or equations to represent the problem).
- 5.NF.7 -Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.

 a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for (1/3) ÷ 4, and use a visual

fraction is not a requirement at this grade level.

fraction model to show the quotient. Use the relationship between multiplication and division to explain that $(1/3) \div 4 = 1/12$ because $(1/12) \times 4 = 1/3$.

b. Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $4 \div (1/5) = 20$ because $20 \times (1/5) = 4$. c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions (e.g., by using visual fraction models and equations to represent the problem). For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?

Alaska Mathematical Domains Grades K-8

Alaska Mathematical Content Standards (5th) CES Interpretation of Standards (5th)

Measurement and Data (MD)	5.MD.1 - 5.MD.7	What knowledge and skills will students need in order to fully meet the standard?
Convert like measurement units within a given measurement system and solve problems involving time.	5.MD.1 -Identify, estimate measure, and convert equivalent measures within systems English length (inches, feet, yards, miles) weight (ounces, pounds, tons) volume (fluid ounces, cups, pints, quarts, gallons) temperature (Fahrenheit) Metric length (millimeters, centimeters, meters, kilometers) volume (milliliters, liters), temperature (Celsius), (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems using appropriate tools.	5.MD.1 - Calls for students to convert measurements within the same system of measurement in the context of multi-step, realword problems. Both customary and standard measurements systems are included. Students build on work done in previous years. They should explore how the base 10 system supports conversions in the metric system.
Represent and interpret data.	5.MD.2 -Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving information presented in line plots. For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.	5.MD.2 -Students work with fractions by measuring objects to ½ of a unit. This includes length, mass, and liquid volume. Students ame a line plot of data and add/subtract fractions.
Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.	5.MD.3 -Recognize volume as an attribute of solid figures and understand concepts of volume measurement. a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. b. A solid figure that can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units. (L) 5.MD.4 -Estimate and measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and non-standard units.	5.MD.3,.4,.5- Students explore the concept of volume.

- 5.MD.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.
- a. Estimate and find the volume of a right rectangular prism with wholenumber side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Demonstrate the associative property of multiplication by using the product of three whole numbers to find volumes (length x width x height).
- b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole number edge lengths in the context of solving real world and mathematical problems. c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.
- 5.MD.6. Estimate and measure volumes by counting unit cubes, using cubic, cm, cubic in, cubic ft, and non-standard units.
- 5.MD.7. Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.
- a. Estimate and find the volume of a right rectangular prism with wholenumber side lengths by packing it with unit cubes, and show that the volume is

	the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Demonstrate the associative property of multiplication by using the product of three whole numbers to find volumes (length x width x height); b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole number edge lengths in the context of solving real world and mathematical problems. c. Recognize volume as additive. Find volumes of solid figures composed of	
	c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular	
U	prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.	

Alaska Mathematical Domains Grades K-8	Alaska Mathematical Content Standards (5th)	CES Interpretation of Standards (5th)
Geometry (G)	5.G.1 - 5.G.4	What knowledge and skills will students need in order to fully meet the standard?
Graph points on the coordinate plane to solve real-world and mathematical problems.	5.G.1 - Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the	5.G.1 & 5.G.2 - These standards deal with the first quadrant in the coordinate plane and how to understand they can look at a point from two different ways.

	coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate). Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation. 5.G.2 -Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret	
Classify two- dimensional (plane) figures into categories based on their properties.	coordinate values of points in the context of the situation. 5.G.3 - Understand that attributes belonging to a category of two-dimensional (plane) figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.	5.G.3 -This standard calls for students to reason and discuss attributes of shapes. 5.G.4 -This builds on what is covered in 4th grade.
	5.G.4 - Classify two-dimensional (plane) figures in a hierarchy based on attributes and properties.	

MIDDLE SCHOOL MATHEMATICS CURRICULUM

Grades 6-8

SECTION CONTENTS

SIXTH GRADE	38
SEVENTH GRADE	38
SEVENTH/EIGHTH GRADE	40
EIGHTH GRADE	42
EIGHTH GRADE/ALGEBRA I	40

Sixth Grade

Established Goals	Transfer Goals		
	Awaitin	waiting Teacher Submission	ambinission
Alaska Standards	Units of Instruction	Students will understand	Students will be able to

Seventh Grade

Established Goals	Transfer Goals		
	Awanthim	waiting Teacher Submission	sulbimissiom
Alaska Standards	Units of Instruction	Students will understand	Students will be able to

Seventh/Eighth Grade

Established Goals	Transfer Goals		
	Awaittim	waitting Teacher Submission	wide maissing and the same of
Alaska Standards	Units of Instruction	Students will understand	Students will be able to
		,	

Eighth Grade

Established Goals	Transfer Goals		
	Awaittim	waitting Teacher Submission	molssion
Alaska Standards	Units of Instruction	Students will understand	Students will be able to
	y		

Eighth Grade/Algebra I

Established Goals	Transfer Goals		
	Awaittim	waiting Teacher Submission	albmissiom
Alaska Standards	Units of Instruction	Students will understand	Students will be able to
		•	

HIGH SCHOOL MATHEMATICS CURRICULUM

Grades 9-12

SECTION CONTENTS

PRE-ALGEBRA
ALGEBRA I
GEOMETRY
ALGEBRA II
PRE-CALCULUS
ADVANCED PLACEMENT CALCULUS
FINANCIAL ALGEBRA
TRADES MATH

PRE-ALGEBRA			
Length: Prerequisites: None Grade Level: 9	Awaittimg	Awaithing Teacher Submission	16mnîssîom
Established Goals	Transfer Goals		
Alaska Standards	Units of Instruction	Students will understand	Students will be able to

Algebra 1 - Unit 1 - Functions		
Established Goals	Transfer	
Craig City School District is dedicated to providing a meaningful,	Students will be able to independently use the Describe a variety of functions using appropriate the students will be able to independently use the students will be able to independent with the students will be able to independent will be able to independent with the students will be able to independent will be able to independent with the students will be able to indep	•
comprehensive, and engaging education to	Meaning	
all students so they responsibly participate in the global society. Craig graduates will use mathematics innovatively to problem solve, think critically, and understand the world around them. MP.4 Model with mathematics F-IF.4 F-IF.5 F-IF.7B F-IF.9 F-LE.5	UNDERSTANDINGS Students will understand that Some relations are functions and others are not	ESSENTIAL QUESTIONS Students will keep considering Can I identify important quantities in situations and describe their relationships using graphs?
	Acquisition	
	Students will know The definitions of domain and range How to use and interpret function notation	Students will be skilled at Finding inputs and outputs of functions Describing linear graphs using slope and intercept Describing parabolas using vertex, line of symmetry and orientation Describing square root, cube root, and absolute value functions

	Algebra 1 - Unit 2 - Linear Relation	nships
Established Goals	Trans	fer
Craig City School District is dedicated to providing a meaningful, comprehensive, and	Students will be able to independently use the Use multiple representations of linear functionand descriptions.	-
engaging education to all students so they	Meaning	
responsibly participate in the global society. Craig graduates will use mathematics innovatively to problem solve, think critically, and understand the world	UNDERSTANDINGS Students will understand that Slope is a rate of change There are connections between a growth pattern with an initial value and y=mx+b	ESSENTIAL QUESTIONS Students will keep considering Can I create a representation of a problem, consider the units involved, and understand the meaning of the quantities using tables, graphs, and equations?
around them.	Acquis	ition
MP.2 Reason abstractly and quantitatively A-CED.2 A-SSE.1a A-SSE.1b F-IF.4 F-IF.7b F-IF.7e F-IF.8a F-IF.9	Students will know The slopes of horizontal and vertical lines Speed is a rate	Students will be skilled at Using slope triangles to compare relative steepness of lines Using slope-intercept form of a linear equation (y=mx+b) Writing the equation of a line given the graph of a motion problem Finding the equation of a line algebraically given the slope

and a point

Finding the equation of a line algebraically given two points

F-IF.9 F-BF.3

Algebra 1 - Unit 3 - Simplifying and Solving		
Established Goals	Transi	fer
Craig City School District is dedicated to providing a meaningful, comprehensive, and engaging education to	Students will be able to independently use their learning to Simplify expressions, including those using exponents and distributive property, using a variety of tools Solve linear equations	
all students so they responsibly	Meaning	
participate in the global society. Craig graduates will use mathematics innovatively to problem solve, think critically, and	UNDERSTANDINGS Students will understand that Area models and algebra tiles can aid in understanding multiplication of expressions Real-life situations can be represented algebraically	ESSENTIAL QUESTIONS Students will keep considering How can algebra tiles and area models hip me better understand multiplication?
understand the world around them.	Acquisition	
MP.5 Use appropriate tools strategically A-APR.1 A-APR.4 A-APR.7+ A-SSE.1a A-SSE.1b A-SSE.2	Students will know The Laws of Exponents How to represent multiplication of expressions with algebra tiles and with generic rectangles	Students will be skilled at Connecting situations to equations Using the distributive property with polynomial expressions Solving linear equations that involve multiplication and the distributive property Solving linear equations involving absolute value Solving multivariable equations for one of the variables

	Algebra 1 - Unit 4 - Systems of Equ	ations
Established Goals	Trans	fer
Craig City School District is dedicated to providing a meaningful,	Students will be able to independently use the Solve situational problems by writing and sol	•
comprehensive, and engaging education to	Meaning	
all students so they responsibly participate in the global society. Craig graduates will use mathematics innovatively to problem solve, think critically, and understand the world around them.	UNDERSTANDINGS Students will understand that The intersection of linear graphs is the same as the solution achieved algebraically How the graphs of systems that have no solutions relate to the algebraic solution How the graphs of systems with infinite solutions relate to the algebraic solution	ESSENTIAL QUESTIONS Students will keep considering Am I using correct vocabulary and clear explanations in discussions with my team while paying attention to small details?
MP.6 Attend to	Acquisition	
precision N-Q.1 N-Q.2 A-SSE.1b A-CED.1 A-CED.2 A-REI.5 A-REI.6 A-REI.10 F-LE.1b	How to solve a system of equations using substitution How to solve a system of equations using elimination	Students will be skilled at Writing and solving one and two variable equations from word problems Recognizing when one method is more efficient than another

Algebra 1 - Unit 5 - Sequences		
Established Goals	Transfer	
Craig City School District is dedicated to providing a meaningful, comprehensive, and	Students will be able to independently use th Use algebra skills in new situations	eir learning to
engaging education to all students so they	Meaning	
responsibly participate in the global society. Craig graduates will use mathematics innovatively to problem solve, think critically, and	UNDERSTANDINGS Students will understand that There are connections between arithmetic sequences and linear equations There are connections between geometric sequences and exponential equations	ESSENTIAL QUESTIONS Students will keep considering When patterns are repeated, can I find shortcuts that lead to equations?
understand the world around them.	Acquisition	
MP.8 Look for and express regularity in repeated reasoning N-Q.2 F-LE.1c F-LE.2 F-IF.3 F-IF.7e F-BF.2	Students will know The specialized vocabulary associated with sequences How to recognize an arithmetic sequence How to recognize a geometric sequence	Students will be skilled at Writing equations for the nth term of arithmetic sequences Writing equations for the nth term of geometric sequences Writing recursive equations for sequences Converting between explicit and recursive equations

about:blank Page 1 of 1

Algebra 1 - Unit 7 - Exponential Functions

Established Goals

Craig City School
District is dedicated to
providing a
meaningful,
comprehensive, and
engaging education to
all students so they
responsibly
participate in the
global society.

Craig graduates will use mathematics innovatively to problem solve, think critically, and understand the world around them.

MP.1 Make sense of problems and persevere in solving them

F-IF.4

F-IF.5

F-IF.6

F-IF.7b

T TT

F-IF.7e

F-IF.8b

F-IF.9

A-CED.1

A-CED.2

F-LE.1a

F-LE.1C

F-LE.2

F-LE.5

A-SSE.1b

A-SSE.3c

N-Q.1

N-Q.2

N-RN.1

N-RN.2

F-BF.1a

A-REI.10

Transfer

Students will be able to independently use their learning to . . .

Build more advanced algebra skills, such as solving for an indicated variable or rewriting exponential expressions

Meaning

UNDERSTANDINGS

Students will understand that . . .

 Exponential functions have multiple representations including tables, graphs, equations, and applications **ESSENTIAL QUESTIONS**

Students will keep considering . . .

Am I making connections between the multiple representations and making sense of the situations?

Acquisition

Students will know . . .

- How to distinguish between growth in linear situations and exponential situations
- How to interpret expressions using negative exponents
- How to interpret expressions using fractional exponents

Students will be skilled at . . .

- Modeling situations using step functions
- Using properties of exponents to transform expressions for exponential functions
- Rewriting expressions using negative and fractional exponents
- Graphing exponential functions
- Using exponential functions to model real-life situations and solve problems
- Finding exponential functions when given two points

Algebra 1 - Unit 8 - Quadratic Functions		
Established Goals	Transf	fer
Craig City School District is dedicated to	Students will be able to independently use the	eir learning to
providing a meaningful,	Find connections between different represen	ntations of a quadratic function
comprehensive, and engaging education to	Meaning	
all students so they responsibly participate in the global society. Craig graduates will use mathematics	UNDERSTANDINGS Students will understand that • There are multiple connections between different representations of quadratics: an equation, a table, a situation, and a graph	ESSENTIAL QUESTIONS Students will keep considering Can I explain my understanding of mathematics accurately to others?
problem solve, think	Acquisition	
innovatively to	That the solutions to quadratic equations are the x-intercepts of the corresponding parabola The corresponding parabola	Students will be skilled at Changing a quadratic written as a sum into its factored form Applying shortcuts for factoring some quadratics Factoring simple cubic equations Using the Zero Product Property to solve quadratic equations Using completing the square to convert quadratics functions in standard form to vertex (graphing) form Using square roots to solve a problem

Algebra 1 - Unit 9 - Solving Quadratics and Inequalities		
Established Goals	Transfer	
Craig City School District is dedicated to providing a	Students will be able to independently use th	eir learning to
meaningful, comprehensive, and engaging education to	Use multiple methods to solve quadratic equations, deciding which method of solvi is most efficient in a particular case Use their knowledge of solving equations to solve inequalities	
all students so they responsibly participate in the		
global society.	Mean	ing
Craig graduates will use mathematics innovatively to problem solve, think critically, and understand the world	UNDERSTANDINGS Students will understand that There are multiple methods for solving quadratics, but some are more efficient than others in a particular situation	ESSENTIAL QUESTIONS Students will keep considering Can I look closely to see a pattern or structure in these functions?
around them.	Acquisi	tion
MP.7 Look for and make use of structure A-SSE.3b A-REI.3 A-REI.4a A-REI.4b A-REI.10 A-REI.12 A-CED.1 A-CED.2 A-CED.3 F-IF.8a F-LE.6 N-Q.2	Students will know How to derive the quadratic formula using algebraic methods Algebraic equations can have multiple solutions	Students will be skilled at Solving quadratic equations by using completing the square Solving quadratic equations by using the quadratic formula Simplifying and solving linear inequalities in one variable Sketching the region defined by a linear inequality Solve systems of two linear inequalities in two variables and sketch the solution sets Representing a given situation as a system of inequalities

about:blank Page 1 of 1

Algebra 1 - Unit 11 - Functions and Data		
Established Goals	Transfer	
Craig City School District is dedicated to providing a	Students will be able to independently use their learning to	
meaningful, comprehensive, and engaging education to	Compare multiple methods of displaying data Solve complicated culminating problems usin	
all students so they responsibly	Meani	ing
participate in the global society. Craig graduates will use mathematics innovatively to problem solve, think critically, and understand the world	UNDERSTANDINGS Students will understand that Center, shape, spread, and outliers are used to compare and analyze data Statistics are used to make predictions	ESSENTIAL QUESTIONS Students will keep considering Am I taking advantage of everything I have learned this year to really engage with the mathematics and understand the problem I am solving?
	Acquisition	
around them. MP.1 Make sense of problems and persevere in solving them F-IF.1 F-IF.4 F-IF.5 F-IF.7a F-IF.8b F-BF.3 F-BF.4a S-ID.1 S-ID.2 S-ID.3 S-ID.6abc S-ID.7 S-ID.8 N-Q.2 N-Q.3 A-SSE.3a A-CED.1 A-CED.3 F-LE.1ac F-LE.2 F-LE.5	Students will know The definition of standard deviation and how it is used The differences between histograms and boxplots The vocabulary of relations including domain, range, function, intercepts, and symmetry	Students will be skilled at Constructing histograms and boxplots by hand Find inverse functions Finding the standard deviation of a data set Collecting and analyzing data using least squares regressions, residuals, and associations

····	*****	the state of the s
Geometry - Unit 1- Shapes and Transformations		
Established Goals	Transi	er
Craig City School District is dedicated to providing a meaningful, comprehensive, and	Students will be able to independently use their learning to This is an introductory unit, intended to familiarize students with shapes, attributes of shapes and transformations. Independent use of these ideas is not expected yet.	
engaging education to all students so they	Meaning	
responsibly participate in the global society. Craig graduates will use mathematics innovatively to problem solve, think critically, and understand the world around them.	UNDERSTANDINGS Students will understand that • there are multiple ways to classify and describe shapes • precise description with appropriate vocabulary is needed for clear communication • investigations are a way to explore new mathematical ideas	ESSENTIAL QUESTIONS Students will keep considering How can I communicate my ideas precisely to others?
MP.6 Attend to	Acquisition	
g-CO.2 G-CO.3 G-CO.4 G-CO.5 G-CO.6 G-GMD.4 G-GPE.5	Students will know How to identify and describe rotations, reflections, and translations of geometry figures. How to characterize shapes using descriptors such as rotational symmetry or parallel lines	Students will be skilled at Performing rotations, reflections, and translations of figures on a grid

about:blank

Geometry - Unit 3 - Justification and Similarity		
Established Goals	Trans	fer
Craig City School District is dedicated to providing a meaningful, comprehensive, and	Students will be able to independently use their learning to support a mathematical statement in multiple ways.	
engaging education to all students so they	Meaning	
responsibly participate in the global society. Craig graduates will use mathematics	UNDERSTANDINGS Students will understand The special relationships between shapes that are similar or congruent	ESSENTIAL QUESTIONS Students will keep considering Can I apply the mathematics I know to problems in everyday life?
innovatively to problem solve, think	Acquisition	
problem solve, think critically, and understand the world around them. MP.4 Model with mathematics G-CO.2 G-CO.3 G-CO.12 G-SRT.1a,b G-SRT.2 G-SRT.5 G-C.1	Students will know How to solve problems using similar figures and proportions Congruence is a special case of similarity The conditions for triangle similarity	Students will be skilled at Using flowcharts Creating if-then conditional statements Constructing dilations Identifying corresponding sides and angles

	Geometry - Unit 4 -Trigonometry and F	Probability
Established Goals	Trans	fer
Craig City School District is dedicated to providing a meaningful, comprehensive, and engaging education to	Students will be able to independently use their learning to Use tangent ratios to solve problems and use different models to represent and calculate probabilities	
all students so they responsibly	Meaning	
participate in the global society. Craig graduates will use mathematics innovatively to problem solve, think critically, and understand the world	UNDERSTANDINGS Students will understand There are multiple ways to represent probabilities Real-world situations can be modeled using tangent ratios	ESSENTIAL QUESTIONS Students will keep considering Can I use the available tools to solve problems and decide which tool might be the most helpful?
around them.	Acquisition	
MP.5 Use appropriate tools strategically G-SRT.6 G-SRT.8 S-CP.1 S-CP.7 S-MD.6+	Students will know That the tangent ratio is the slope of the line How to find expected value in games of chance	Students will be skilled at Finding missing measurements in right triangles using tangent ratios Using tree diagrams and area models to represent probability problems Formalizing probabilities of unions, intersections and complements of events

	Geometry - Unit 5 - Completing the Trial	ngle Toolkit
Established Goals	Transfer	
Craig City School District is dedicated to providing a meaningful, comprehensive, and	Students will be able to independently use the Find missing side lengths and angles of any to	-
engaging education to all students so they	Meaning	
responsibly participate in the global society. Craig graduates will use mathematics innovatively to	UNDERSTANDINGS Students will understand Sufficient information must be given to solve a triangle	ESSENTIAL QUESTIONS Students will keep considering Which tool should I use to find missing parts of triangles?
problem solve, think critically, and	Acquisition	
critically, and understand the world around them. MP.5 Use appropriate tools strategically G-SRT.4 G-SRT.6 G-SRT.7 G-SRT.8 G-SRT.9+ G-SRT.10+ G-SRT.11+ G-CO.10	Students will know Side length ratios for 45-45-90 triangles and for 30-60-90 triangles Trigonometric ratios of sine and cosine, as well as the inverses of these functions Pythagorean triples	Students will be skilled at Using Law of Sines and Law of Cosines to solve non-right triangles Applying all trigonometric ratios to find missing measurements in right triangles

about:blank Page 1 of 1

	Geometry - Unit 7 - Proof and Quadr	ilaterals	
Established Goals	Transfer		
Craig City School District is dedicated to providing a meaningful, comprehensive, and engaging education to	Students will be able to independently use their learning to Write a convincing argument in a variety of formats (flowchart and two-column proofs)		
all students so they responsibly	Meaning		
participate in the global society. Craig graduates will use mathematics innovatively to problem solve, think critically, and understand the world around them.	UNDERSTANDINGS Students will understand The relationships of sides, angles and diagonals of special quadrilaterals (parallelograms, rhombi, trapezoids, kites, rectangles, and squares) The concept of "proof"	ESSENTIAL QUESTIONS Students will keep considering How can I use the given information to construct arguments? How can I justify my conclusions? How can I respond to the arguments of others?	
MP.3 Construct viable arguments and critique the reasoning of others G-CO.1 G-CO.10 G-CO.11 G-MG.3 G-SRT.4 G-SRT.5 G-GPE.4 G-GPE.6	Acquisition		
	Students will know How to find the midpoint of a line segment The properties of special quadrilaterals	Students will be skilled at Using algebraic tools to explore quadrilaterals on a coordinate plane Applying concepts of rigid transformations and congruence in writing proofs	

G-GPE.7

Geometry - Unit 8 - Polygons and Circles				
Established Goals Craig City School District is dedicated to providing a meaningful, comprehensive, and engaging education to all students so they responsibly participate in the global society. Craig graduates will use mathematics innovatively to problem solve, think critically, and understand the world around them. MP.8 Look for and express regularity in repeated reasoning G-GMD. G-SRT.5 G-C.5 G-MG.1 G-MG.3	Transfer			
	Students will be able to independently use their learning to Apply knowledge of circles and polygons to solve problems in a variety of contexts			
	Meaning			
	UNDERSTANDINGS Students will understand How the area and perimeter of of similar figures are related How angles (interior/exterior) are related to the number of sides of a regular polygon	ESSENTIAL QUESTIONS Students will keep considering Can I find the shortcuts and generalize the rules for finding the perimeters and areas of regular polygons?		
	Acquisition			
	Students will know Properties of special polygons such a regular and non-convex	Students will be skilled at Finding the area and circumference of circles and parts of circles Finding the measure of interior and exterior angles of a regular polygon		

	Geometry - Unit 9 - Solids and Consti	ructions	
Established Goals Craig City School District is dedicated to providing a meaningful, comprehensive, and engaging education to all students so they responsibly participate in the global society. Craig graduates will use mathematics innovatively to problem solve, think critically, and understand the world around them.	Students will be able to independently use their learning to Solve problems involving three-dimensional objects Select and use appropriate tools to perform geometric constructions		
			Meaning
	UNDERSTANDINGS Students will understand There are multiple ways to represent 3 dimensional objects such as a mat plan, a net, and side and top views How volume changes when and three dimensional shape in enlarge proportionally	ESSENTIAL QUESTIONS Students will keep considering How can I represent it? What tools can i use? How can I construct it?	
	MP.5 Use appropriate tools strategically G-GMD.1 G-GMD.3 G-MG.1 G-MG.2 G-CO.9 G-CO.10 G-CO.12 G-CO.13 G-C.3	Acquisition	
Students will know • How to use a compass and straightedge to perform a variety of constructions including segments, angles, bisectors, parallel and perpendicular lines		Students will be skilled at Finding the volume and surface area of three-dimensional solids such as prisms and cylinders Constructing familiar geometric shapes (such as a rhombus or a regular hexagon) using construction tools such as tracing paper, compass and straightedge, or a technology tool	

Geometry - Unit 10 - Circles and Conditional Probability				
Established Goals	Transfer			
Craig City School District is dedicated to providing a meaningful, comprehensive, and engaging education to all students so they responsibly participate in the global society. Craig graduates will use mathematics innovatively to problem solve, think critically, and understand the world around them.	Students will be able to independently use their learning to Solve a variety of problems using tangents, chords, angles, and arc of circles Solve probability problems using multiple strategies			
	Meaning			
	UNDERSTANDINGS Students will understand The difference between arc measure and arc length The concept of conditional probability The definition of independence	ESSENTIAL QUESTIONS Students will keep considering What tools do I have available to help me solve this problem?		
	Acquisition			
MP.5 Use appropriate tools strategically G-MG.1 G-C.2 G-C.3 G.C-5 S-CP.2 S-CP.3 S-CP.4 S-CP.5 S-CP.6 S-CP.7 S-CP.7+ S-CP.7+ S-CP.9+ S-MD.6+ S-MD.7+	Students will know That the perpendicular bisector of a chord passes through the center of the circle Circle vocabulary including major/minor arcs, chords, inscribed angles, and tangents The relationship between inscribed angles and intercepted arcs That the tangent line is perpendicular to the radius of the circle The Multiplication Rule and the Fundamental Principle of Counting The difference between permutations and combinations and when to use each Formulas for permutations and for computations	Students will be skilled at Finding the relationship between lengths created by intersecting chords Calculating conditional probability from relative frequencies in two-way tables Determining if categorical variables are associated Displaying probabilities in multiple ways Using the Fundamental Principle of counting to count permutations and other outcomes where they are too many to list		

	Geometry - Unit 11 - Solids and Ci	rcles	
Established Goals	Trans	fer	
Craig City School District is dedicated to providing a meaningful, comprehensive, and	Students will be able to independently use the Solve a variety of problems involving pyramic	_	
engaging education to all students so they	Mean	ing	
responsibly participate in the global society. Craig graduates will use mathematics innovatively to	UNDERSTANDINGS Students will understand The relationships between lengths of segments created when tangents and secants intersect outside the circle	ESSENTIAL QUESTIONS Students will keep considering What information do I need, what do I already know, and how can I use this information to solve the problem?	
problem solve, think critically, and	Acquisition		
understand the world around them. MP.1 Make sense of problems and persevere in solving them G-GMD.1 G-GMD.3 G-GMD.4 G-MG.1 G-C.2 G-C.4+ G-C.5	Students will know The properties of special polyhedra, called Platonic solids Definitions of tangent lines and secant lines	Students will be skilled at Finding the volume and surface area of pyramids, cones, and spheres Finding the measures of angles and arcs that are formed by tangents and secants Finding the cross-section of a solid	

about:blank

	Geometry - Unit 12 - Conics and Cl	osure
Established Goals Craig City School	Trans	fer
District is dedicated to providing a meaningful,	Students will be able to independently use the Make connections between geometry and al	-
comprehensive, and engaging education to	Mean	ing
all students so they responsibly participate in the global society. Craig graduates will use mathematics innovatively to problem solve, think	UNDERSTANDINGS Students will understand How circles and parabolas can be defined using both geometry and algebra There are many patterns and relationships between mathematical ideas	ESSENTIAL QUESTIONS Students will keep considering How can I connect these ideas to previous topics, and can I make it simpler or make a generalization?
critically, and understand the world	Acquisi	tion
around them. MP.7 Look for and make use of structure G-GPE.1 G-GPE.2 G-GPE.4 G-MG.3 G-GMD.4 S-MD.7+	Students will know How to write the equation of a circle How to identify conic sections	Students will be skilled at Completing the square to write the equation of a circle Using the geometric definition of a parabola to derive the algebraic equation of a parabola

ALGEBRA II			
Length: Prerequisites: None Grade Level: 9-12	Awanthim	Awaitting Teacher Submission	ulbmissiom
Established Goals	Transfer Goals		
Alaska Standards	Units of Instruction	Students will understand	Students will be able to

PRE-CALCULUS			
	Awaitting	Awaitting Teacher Submission	bmission
Length: Prerequisites: None Grade Level: 9-12			
Established Goals	Transfer Goals		
Alaska Standards	Units of Instruction	Students will understand	Students will be able to

Awaiting Teacher Submission Students will be able to.... Students will understand.... Units of Instruction Transfer Goals ADVANCED PLACEMENT CALCULUS Length: Prerequisites: None Grade Level: 9-12 Alaska Standards **Established Goals**

ALGEBRA ALGEBRA Length: Prerequisites: None Grade Level: 9-12	Transfer Goals	Units of Instruction Students will understand Students will be able to	
ALGEBRA Length: Prerequisites: None Grade Level: 9-12		Alaska Standards Un	

TRADES MATH Length: Prerequisites: None Grade Level: 9-12	Awaitting	Awaiting Teacher Submission	1bmissiom
Established Goals	Transfer Goals		
Alaska Standards	Units of Instruction	Students will understand	Students will be able to

MATHEMATICS CURRICULUM APPENDEX

Standards for Mathematical Practice

Specific expectations for grade bands K-2, 3-5, 6-8 and 9-12 can be found starting on page 97 of the Alaska English/Language Arts and Mathematics Standards document.

		Mathematically proficient students will:		
		explain the meaning of the problem to themselves.		
		look for a way to start and note the strategies that will help		
		solve the problem.		
		identify and analyze givens, constraints, relationships and		
1.	Make sense of	goals.		
	problems and	make inferences about the form and meaning of the solution.		
	persevere in solving	design a plan to solve the problem.		
	them.	• use effective problem solving strategies.		
		• evaluate the progress and change the strategy if necessary.		
		solve the problem using a different methods and compare		
		solutions.		
		ask, "Does this make sense?"		
		make sense of quantities and their relationships in problem		
		solutions.		
		use two complementary abilities when solving problems		
		involving number relationships.		
		 Decontextualize- be able to reason abstractly and 		
		represent a situation symbolically and manipulate the		
2.	Reason abstractly and	symbols		
	quantitatively.	Contextualize- make meaning of the symbols in the		
		problem		
		• understand the meaning of quantities and are flexible in the use		
		of operations and their properties.		
		• create a logical representation of the problem.		
		attends to the meaning of quantities, not just how to compute		
		them.		
		analyze problems and use stated mathematical assumptions, definitions and actablished results in agreements.		
,	C	definitions, and established results in construction arguments.		
٤.	Construct viable	• justify conclusions with mathematical ideas.		
	arguments and critique the reasoning	• listen to arguments of others and ask useful question to		
	of others.	determine if an argument makes sense.		
	of others.	• ask clarifying questions or suggest ideas to improve/revise the argument.		
		 compare two arguments and determine correct or flawed logic. 		
		- compare two arguments and determine correct of flawed togic.		



		Y
4.	Model with mathematics.	 understand this is a way to reason quantitatively and abstractly (able to decontextualize and contextualize). apply the math they know to solve problems in everyday life. are able to simplify a complex problem and identify important quantities to look at relationships. represent mathematics to describe a situation either with an equation or a diagram and interpret the results of a mathematical situation. reflect on whether the results make sense possibly improving/revising the model.
		ask, "How can I represent this mathematically?"
5.	Use appropriate tools strategically.	 use available tools recognizing the strengths and limitations of each. use estimation and other mathematical knowledge to detect possible errors. identify relevant external mathematical resources to pose and solve problems. use technological tools to deepen their understanding of mathematics.
6. Attend to precision.		 communicate precisely with others and try to use clear mathematical language when discussing their reasoning. understand meanings of symbols used in mathematics and can label quantities appropriately. express numerical answers with a degree of precision appropriate for the problem context. calculate efficiently and accurately.
7.	Look for and make use of structure.	 apply general mathematical rules to specific situations. look for overall structure and patterns in mathematics. see complicated things as a single object or as being composed of several objects. be able to look at problems from a different perspective.
8.	Look for and express regularity in repeated reasoning	 see repeated calculations and look for generalizations and shortcuts. see the overall process of the problem and still attend to details. understand the broader application of patterns and see the structure in similar situations. continually evaluate the reasonableness of their intermediate results.





Shifts for Alaska Mathematics Standards

Focus: In each grade or course, focus deeply on 2 - 4 topics.

Focus: The Standards call for a greater focus in mathematics. Rather than racing to cover topics in today's mile-wide, inch-deep curriculum, teachers use the power of the eraser and significantly narrow and deepen the way time and energy is spent in the math classroom. They focus deeply on the major work* of each grade so that students can gain strong foundations: solid conceptual understanding, a high degree of procedural skill and fluency, and the ability to apply the math they know to solve problems inside and outside the math classroom.

2. **Coherence: think** across grades, and **link** to major topics within grades.

Thinking across grades: The Standards are designed around coherent progressions from grade to grade. Principals and teachers carefully connect the learning across grades so that students can build new understanding onto foundations built in previous years. Teachers can begin to count on deep conceptual understanding of core content and build on it. Each standard is not a new event, but an extension of previous learning.

Linking to major topics: Instead of allowing additional or supporting topics to detract from the focus of the grade, these topics can serve the grade level focus. For example, instead of data displays as an end in themselves, they support grade-level word problems.

- 3. **Rigor:** in major topics* pursue:
 - conceptual understanding,
 - procedural skill and fluency, and
 - application with equal intensity.

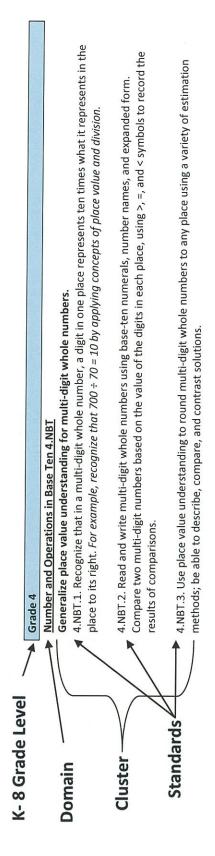
Conceptual understanding: The Standards call for conceptual understanding of key concepts, such as place value and ratios. Teachers support students' ability to access concepts from a number of perspectives so that students are able to see math as more than a set of mnemonics or discrete procedures.

Procedural skill and fluency: The Standards call for speed and accuracy in calculation. Teachers structure class time and/or homework time for students to practice core functions such as single-digit multiplication so that students have access to more complex concepts and procedures

Application: The Standards call for students to use math flexibly for applications. Teachers provide opportunities for students to apply math in context. Teachers in content areas outside of math, particularly science, ensure that students are using math to make meaning of and access content.

*Priorities in Support of Conceptual Understanding and Fluency K-2 Addition and subtraction – concepts, skills, and problem solving 3-6 Multiplication and division of whole numbers and fractions – concepts, skills and problem solving Ratios and proportional relationships; early expressions and equations Ratios and proportional relationships; arithmetic of rational numbers Linear algebra

Guide to Reading the Mathematical Content Standards



Domains are intended to convey coherent groupings of content. All domains are underlined. Clusters are groups of related standards. Cluster headings are bolded.

Standards define what students should understand and be able to do. Standards are numbered. G-GMD-9 Explain volume formulas and use them to solve problems Geometric Measurement and Dimension **◆**Geometry Standards

> High School Conceptual

G-GMD.2. (+) Give an informal argument using Cavalieri's principle for the formulas for G-GMD.3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve he volume of a sphere and other solid figures. Standards 4

G-GMD.1. Explain how to find the formulas for the circumference of a circle, area of a

Domain

Category

circle, volume of a cylinder, pyramid, and cone.

(+) Additional standards for advanced courses (*) Standards with connection to modeling

Page 2 of 6

Mathematical Content Standards Overview

Kindergarten	Canal 1	
Wilder Bar Cell	Grade 1	Grade 2
Counting and Cardinality	Counting and Cardinality	
 Know number names and the count sequence. 	 Know ordinal names and counting flexibility. 	
 Count to tell the number of objects. 	 Count to tell the number of objects. 	
Compare numbers.	Compare numbers.	
Operations and Algebraic Thinking	Operations and Algebraic Thinking	Operations and Algebraic Thinking
Understand addition as putting together and	Represent and solve problems involving and dittion	9 Doming and a second of
adding to and understand subtraction as	and cubtraction	• Represent and solve problems involving
taling onest and taling from	and subtraction.	addition and subtraction.
dantify and continue actions	 Understand and apply properties of operations 	 Add and subtract up to 20.
deficitly and continue patterns.	and the relationship between addition and	 Work with equal groups of objects to gain
	subtraction.	foundations for multiplication.
	 Add and subtract up to 20. 	 Identify and continue patterns.
	 Work with addition and subtraction equations. 	
	 Identify and continue patterns. 	
Number and Operations in Base Ten	Number and Operations in Base Ten	Number and Operations in Base Ten
 Work with numbers 11–19 to gain foundations 	Extend the counting sequence.	Understand place value.
for place value.	 Understand place value. 	 Use place value understanding and
	 Use place value understanding and properties of 	properties of operations to add and subtract
	operations to add and subtract.	
Measurement and Data	Measurement and Data	Measurement and Data
 Describe and compare measurable attributes. 	Measure lengths indirectly and by iterating	Measure and estimate lengths in standard
 Classify objects and count the number of 	length units.	units.
objects in categories.	 Work with time and money. 	Relate addition and subtraction to length
 Work with time and money. 	 Represent and interpret data. 	Work with time and money.
		 Represent and interpret data.
Geometry	Geometry	Geometry
 Identify and describe shapes. 	Reason with shapes and their attributes.	 Reason with shapes and their attributes.
 Analyze, compare, create, and compose 		
shapes.		

Grado 3	A chan's	
	Grade 4	Grade 5
Operations and Algebraic Thinking	Operations and Algebraic Thinking	Operations and Algebraic Thinking
 Represent and solve problems involving multiplication and division. 	Use the four operations with whole numbers to solve problems	Write and interpret numerical expressions.
 Understand properties of multiplication and the relationship between multiplication and division. 	 Gain familiarity with factors and multiples. Generate and analyze patterns. 	Analyze patterns and relationships.
Multiply and divide up to 100.	Number and Operations in Base Ten	Number and Operations in Base Ten
 Solve problems involving the four operations, and identify and explain patterns in arithmetic. 	 Generalize place value understanding for multi-digit whole numbers. 	 Understand the place value system. Perform operations with multi-digit whole
Number and Operations in Base Ten	 Use place value understanding and properties of operations to nerform multi-digit 	numbers and with decimals to hundredths.
Use place value understanding and properties of operations to perform multi-digit arithmetic.	arithmetic.	
	Number and Operations—Fractions	Number and Operations—Fractions
Number and Operations—Fractions	Extend understanding of fraction equivalence	Use equivalent fractions as a strategy to add
Develop understanding of fractions as	and ordering.	and subtract fractions.
numbers.	Build fractions from unit fractions by applying and extending pravious understandings of	Apply and extend previous understandings of multiplication and division to multiplication.
Measurement and Data	operations on whole numbers.	divide fractions.
Solve problems involving measurement and	 Understand decimal notation for fractions, and 	
estimation of intervals of time, liquid volumes, and masses of objects.	compare decimal fractions.	
Represent and interpret data.	Measurement and Data	Measurement and Data
Geometric measurement: understand concepts	Solve problems involving measurement and	Convert like measurement units within a given
of area and relate area to multiplication and to	conversion of measurements from a larger unit	measurement system and solve problems
Geometric measurement: recognize perimeter	 Represent and interpret data. 	Represent and interpret data.
as an attribute of plane figures and distinguish	 Geometric measurement: understand 	Geometric measurement: understand
between linear and area measures.	concepts of angle and measure angles.	concepts of volume and relate volume to multiplication and to addition.
Geometry	Geometry	Geometry
 Reason with shapes and their attributes. 	Draw and identify lines and angles, and classify shapes by properties of their lines and angles.	 Graph points on the coordinate plane to solve real-world and mathematical problems. Classify two-dimensional figures into categories based on their properties.

Grade b	Grade 7	Grade 8
Ratios and Proportional Relationships	Ratios and Proportional Relationships	Functions
Understand ratio concents and use ratio	Analyze proportional relationships	in the state of th
reasoning to solve problems.	them to solve real-world and mathematical problems.	Use functions to model relationships between quantities.
The Number System	The Number System	The Number System
 Apply and extend previous understandings of multiplication and division to divide fractions by fractions. Compute fluently with multi-digit numbers and 	 Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers. 	Know that there are numbers that are not rational, and approximate them by rational numbers.
find common factors and multiples.	Expressions and Equations	Expressions and Equations
 Apply and extend previous understandings of numbers to the system of rational numbers. 	Use properties of operations to generate equivalent expressions. Solve real-life and mathematical problems.	 Work with radicals and integer exponents. Understand the connections between
Expressions and Equations	using numerical and algebraic expressions and	proportional relationships, mies, and mear
 Apply and extend previous understandings of arithmetic to algebraic expressions. Reason about and solve one-variable equations 	equations.	Analyze and solve linear equations and pairs of simultaneous linear equations.
and inequalities.	Geometry	Geometry
 Represent and analyze quantitative relationships between dependent and independent variables. 	Draw, construct and describe geometrical figures and describe the relationships between them.	Understand congruence and similarity using physical models, transparencies, or geometry software.
Geometry	involving angle measure, area, surface area.	Theorem.
 Solve real-world and mathematical problems involving area, surface area, and volume. 	and volume.	 Solve real-world and mathematical problems involving volume of cylinders, cones and spheres.
Statistics and Probability	Statistics and Probability	Statistics and Probability
Develop understanding of statistical variability.Summarize and describe distributions.	 Use random sampling to draw inferences about a population. 	Investigate patterns of association in bivariate data.
	 Draw informal comparative inferences about two populations. 	
	 Investigate chance processes and develop, use, and evaluate probability models. 	

HIGH SCHOOL CONTENT STANDARDS BY CONCEPTUAL CATEGORIES

Algebra	Seeing Structure in Expressions Interpret the structure of expressions. Write expressions in equivalent forms to solve problems. Arithmetic with Polynomials and Rational Expressions Perform arithmetic operations on polynomials. Understand the relationship between zeros and factors of polynomials.	 Use polynomial identities to solve problems. Rewrite rational expressions. 	Creating Equations and Inequalities* Create equations and inequalities that describe numbers or relationships. Reasoning with Equations and Inequalities Understand solving equations as a process of	 reasoning and explain the reasoning. Solve equations and inequalities in one variable. Solve systems of equations. Represent and solve equations and inequalities graphically
Number and Quantity	The Real Number System Extend the properties of exponents to rational exponents. Use properties of rational and irrational numbers. Quantities* Reason quantitatively and use units to solve problems.	Perform arithmetic operations with complex numbers.	 Kepresent complex numbers and their operations on the complex plane. + Use complex numbers in polynomial identities and equations. Vector and Matrix Quantities	 Represent and model with vector quantities. + Perform operations on vectors. + Perform operations on matrices and use matrices in applications. +
Modeling	Modeling links classroom mathematics and statistics to everyday life, work, and decisionmaking. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Specific modeling standards appear	throughout the high school standards indicated by an asterisk (*). If the asterisk appears on the heading for a group	of standards, it should be understood to apply to all standards in that group. There are other individual standards under clusters, domains and conceptual categories that have connections to modeling.	Additionally, model with mathematics is a Standard for Mathematical Practice. This practice will be started in kindergarten.

^{*}Standards with connections to modeling. If asterisk appears on the category, domain, or cluster for a group of standards, it should be understood to apply to all standards in that group. There may be individual standards within clusters with connections to modeling.

⁺ Standards include additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics. There may be individual standards within clusters which include additional mathematics.

HIGH SCHOOL CONTENT STANDARDS BY CONCEPTUAL CATEGORIES

Functions	Geometry	Statistics and Probability*
Interpreting Functions	Congruence	Interpreting Categorical and Quantitative Data
 Understand the concept of a function and 	 Experiment with transformations in the plane. 	Summarize, represent, and interpret data on
use function notation.	 Understand congruence in terms of rigid motions. 	a single count or measurement variable.
 Interpret functions that arise in 	 Prove geometric theorems. 	 Summarize, represent, and interpret data on
applications in terms of the context.	 Make geometric constructions. 	two categorical and quantitative variables.
 Analyze functions using different 		 Interpret linear models.
representations.	Similarity, Right Triangles, and Trigonometry	
	 Understand similarity in terms of similarity 	Making Inferences and Justifying Conclusions
Building Functions	transformations.	 Understand and evaluate random processes
 Build a function that models a 	 Prove theorems involving similarity. 	underlying statistical experiments.
relationship between two quantities.*	 Define trigonometric ratios and solve problems 	 Make inferences and justify conclusions
 Build new functions from existing 	involving right triangles.	from sample surveys, experiments, and
functions.	 Apply trigonometry to general triangles. + 	observational studies.
Linear, Quadratic, and Exponential Models*	Circles	Conditional Probability and the Rules of
 Construct and compare linear, quadratic, 	 Understand and apply theorems about circles. 	Probability
and exponential models and solve	 Find arc lengths and areas of sectors of circles. 	 Understand independence and conditional
problems.		probability and use them to interpret data.
 Interpret expressions for functions in 	Expressing Geometric Properties with Equations	 Use the rules of probability to compute
terms of the situation they model.	 Translate between the geometric description and the 	probabilities of compound events in a
	equation for a conic section.	uniform probability model.
Trigonometric Functions	 Use coordinates to prove simple geometric theorems 	
 Extend the domain of trigonometric 	algebraically.	Using Probability to Make Decisions
functions using the unit circle.		 Calculate expected values and use them to
 Model periodic phenomena with 	Geometric Measurement and Dimension	solve problems. +
trigonometric functions.	 Explain volume formulas and use them to solve 	 Use probability to evaluate outcomes of
 Prove and apply trigonometric identities. 	problems.	decisions. +
	Visualize relationships between two-dimensional and	
	tnree-dimensional objects.	
	Modeling with Geometry	
	Apply geometric concepts in modeling situations.*	

Glossary for Alaska Mathematics Standards

addition and subtraction within 5, 10, 20, 100, or 1000

Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0-5, 0-10, 0-20, or 0-100, respectively. Example: 8 + 2 = 10 is an addition within 10, 14 - 5 = 9 is a subtraction within 20, and 55 - 18 = 37 is a subtraction within 100.

additive inverses

Two numbers whose sum is 0 are additive inverses of one another. Example: 3/4 and -3/4 are additive inverses of one another because 3/4 + (-3/4) = (-3/4) + 3/4 = 0.

associative property of addition

See Table 3 in this Glossary.

associative property of multiplication

See Table 3 in this Glossary.

bivariate data

Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team. Box plot. A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.¹

cardinality

Cardinal numbers, known as the "counting numbers," indicate quantity.

commutative property

See Table 3 in this Glossary.

complex fraction

A fraction A/B where A and/or B are fractions (B nonzero).

computation algorithm

A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. See also: computation strategy.

computation strategy

Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. See also: computation algorithm.

congruent

Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

¹ Adapted from Wisconsin Department of Public Instruction, http://dpi.wi.gov/standards/mathglos.html, accessed Mar 2, 2010.

counting on

A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the stack all over again. One can find the total by counting on—pointing to the top book and saying "eight," following this with "nine, ten, eleven. There are eleven books now."

dot plot

See: line plot

dilation

A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor.

expanded form

A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten. For example, 643 = 600 + 40 + 3.

expected value

For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

first quartile

For a data set with median M, the first quartile is the median of the data values less than M. Example: For the data set $\{1, 3, 6, 7, 10, 12, 14, 15, 22, 120\}$, the first quartile is $6.^2$ See also: median, third quartile, interquartile range.

fraction

A number expressible in the form a/b where a is a whole number and b is a positive whole number. (The word fraction in these standards always refers to a non-negative number.) See also: rational number.

identity property of 0

See Table 3 in this Glossary.

independently combined probability models

Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.

integer

A number expressible in the form a or -a for some whole number a.

interquartile range

A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set $\{1, 3, 6, 7, 10, 12, 14, 15, 22, 120\}$, the interquartile range is 15 - 6 = 9. See also: first quartile, third quartile.

² Many different methods for computing quartiles are in use. The method defined here is sometimes called the More and McCabe method. See Langford, E., "Quartiles in Elementary Statistics," *Journal of Statistics Education*, Volume 14, number 3 (2006).

line plot

A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line. Also known as a dot plot.³

mean

A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list.⁴ Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.

mean absolute deviation

A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.

median

A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.

midline

In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values. Multiplication and division within 100. Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0-100. Example: $72 \div 8 = 9$.

multiplicative inverses

Two numbers whose product is 1 are multiplicative inverses of one another. Example: 3/4 and 4/3 are multiplicative inverses of one another because $3/4 \times 4/3 = 4/3 \times 3/4 = 1$.

number line diagram.

A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

ordinality

Ordinal numbers indicate the order or rank of things in a set (e.g., sixth in line; fourth place).

percent rate of change

A rate of change expressed as a percent. Example: if a population grows from 50 to 55 in a year, it grows by 5/50 = 10% per year.

probability distribution

The set of possible values of a random variable with a probability assigned to each.

probability

A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin, selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

³ Adapted from Wisconsin Department of Public Instruction, op. cit.

⁴ To be more precise, this defines the *arithmetic mean*.

probability model

A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. See also: uniform probability model.

properties of equality

See Table 4 in this Glossary.

properties of inequality

See Table 5 in this Glossary.

properties of operation

See Table 3 in this Glossary.

random variable

An assignment of a numerical value to each outcome in a sample space. Rational expression. A quotient of two polynomials with a non-zero denominator.

rational expression

A quotient of two polynomials with a non-zero denominator.

rational number

A number expressible in the form a/b or -a/b for some fraction a/b. The rational numbers include the integers.

rectilinear figure

A polygon all angles of which are right angles.

rigid motion

A transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

repeating decimal

The decimal form of a rational number. See also: terminating decimal.

sample space

In a probability model for a random process, a list of the individual outcomes that are to be considered.

scatter plot

A graph in the coordinate plane representing a set of bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot.⁵

similarity transformation

A rigid motion followed by a dilation.

⁵ Adapted from Wisconsin Department of Public Instruction, op. cit.

tape diagram

A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar model, fraction strip, or length model.

terminating decimal

A decimal is called terminating if its repeating digit is 0.

third quartile

For a data set with median M, the third quartile is the median of the data values greater than M. For example: For the data set $\{2, 3, 6, 7, 10, 12, 14, 15, 22, 120\}$, the third quartile is 15. See also: median, first quartile, interquartile range.

transitivity principle for indirect measurement

If the length of object A is greater than the length of object B, and the length of object B is greater than the length of object C, then the length of object A is greater than the length of object C. This principle applies to measurement of other quantities as well.

uniform probability model

A probability model which assigns equal probability to all outcomes. See also: probability model.

vector

A quantity with magnitude and direction in the plane or in space, defined by an ordered pair or triple of real numbers.

visual fraction model

A tape diagram, number line diagram, or area model.

whole numbers

The numbers 0, 1, 2, 3,...

Table 1: Common addition and subtraction situations¹

	Result Unknown	Change Unknown	Start Unknown
Add To	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5-2=?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 - ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? - 2 = 3
	Total Unknown	Addend Unknown	Both Addends Unknown ²
Put Together/ Take Apart ³	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3+?=5,5-3=?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, $5 = 5 + 05 = 1 + 4$, $5 = 4 + 15 = 2 + 3$, $5 = 3 + 2$
	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare ⁴	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy?	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have?	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have?
	("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 - 2 = ?	(Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5-3=?,?+3=5

¹Adapted from Box 2-4 of the National Research Council (2009, op. cit., pp. 32, 33).

²These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.

³Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

⁴For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

Table 2: Common multiplication and division situations⁵

	Unknown Product	Group Size Unknown ("How many in each group?" Division)	Number of Groups Unknown ("How many groups?" Division)
	3 x 6 = ?	3 x ? = 18, and 18 ÷ 3 = ?	? x 6 = 18, and 18 ÷ 6 = ?
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag?	If 18 plums are to be packed 6 to a bag, then how many bags are needed?
	Measurement example. You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	Measurement example. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	Measurement example. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Array ⁶ , Area ⁷	There are 3 rows of apples with 6 apples in each row. How many apples are there?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be?
	Area example. What is the area of a 3 cm by 6 cm rectangle?	Area example. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	Area example. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost?	A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost?	A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat?
	Measurement example. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	Measurement example. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	Measurement example. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	a x b = ?	$a \times ? = p, and p \div a = ?$? $x b = p$, and $p \div b = ?$

⁵The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

⁶The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

Table 3. The properties of operations.

Here a, b and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

Associative property of addition	(a + b) + c = a + (b + c)
Commutative property of addition	a+b=b+a
Additive identity property of 0	a + 0 = 0 + a = a
Existence of additive inverses	For every a there exists $-a$ so that $a + (-a) = (-a) + a = 0$.
Associative property of multiplication	$(a \times b) \times c = a \times (b \times c)$
Commutative property of multiplication	$a \times b = b \times a$
Multiplicative identity property of 1	$a \times 1 = 1 \times a = a$
Existence of multiplicative inverses	For every $a \neq 0$ there exists $1/a$ so that $a \times 1/a = 1/a \times a = 1$.
Distributive property of multiplication over addition	$a \times (b + c) = a \times b + a \times c$

Table 4. The properties of equality.

Here a, b and c stand for arbitrary numbers in the rational, real, or complex number systems.

Reflexive property of equality	a = a
Symmetric property of equality	If $a = b$, then $b = a$.
Transitive property of equality	If $a = b$ and $b = c$, then $a = c$.
Addition property of equality	If $a = b$, then $a + c = b + c$.
Subtraction property of equality	If $a = b$, then $a - c = b - c$.
Multiplication property of equality	If $a = b$, then $a \times c = b \times c$.
Division property of equality	If $a = b$, then b may be substituted for a in any
	expression containing a.
Substitution property of equality	If $a = b$ and $c \neq 0$, then $a \div c = b \div c$.

Table 5. The properties of inequality.

Here a, b and c stand for arbitrary numbers in the rational or real number systems.

	to a solution of the factorial of the fa
9"	Exactly one of the following is true: $a < b$, $a = b$, $a > b$.
	If $a > b$ and $b > c$ then $a > c$.
	If $a > b$, then $b < a$.
	If $a > b$, then $-a < -b$.
	If $a > b$, then $a \pm c > b \pm c$.
	If $a > b$ and $c > 0$, then $a \times c > b \times c$.
V	If $a > b$ and $c < 0$, then $a \times c < b \times c$.
	If $a > b$ and $c > 0$, then $a \div c > b \div c$.
	If $a > b$ and $c < 0$, then $a \div c < b \div c$.

CULTURAL STANDARDS



Culturally-knowledgeable students are well grounded in the cultural heritage and traditions of their community.

Students who meet this cultural standard are able to:

- 1) assume responsibilities for their role in relation to the well-being of the cultural community and their lifelong obligations as a community member;
- 2) recount their own genealogy and family history;
- 3) acquire and pass on the traditions of their community through oral and written history;
- 4) practice their traditional responsibilities to the surrounding environment;
- 5) reflect through their own actions the critical role that the local heritage language plays in fostering a sense of who they are and how they understand the world around them;
- 6) live a life in accordance with the cultural values and traditions of the local community and integrate them into their everyday behavior; and
- 7) determine the place of their cultural community in the regional, state, national, and international political and economic systems.



Culturally-knowledgeable students are able to build on the knowledge and skills of the local cultural community as a foundation from which to achieve personal and academic success throughout life.

Students who meet this cultural standard are able to:

- 1) acquire insights from other cultures without diminishing the integrity of their own;
- 2) make effective use of the knowledge, skills, and ways of knowing from their own cultural traditions to learn about the larger world in which they live;
- 3) make appropriate choices regarding the long-term consequences of their actions; and
- 4) identify appropriate forms of technology and anticipate the consequences of their use for improving the quality of life in the community.



Culturallyknowledgeable students are able to actively participate in various cultural Students who meet this cultural standard are able to:

- 1) perform subsistence activities in ways that are appropriate to local cultural traditions;
- 2) make constructive contributions to the governance of their community and the well-being of their family;



(continued)

- 3) attain a healthy lifestyle through which they are able to maintain their social, emotional, physical, intellectual, and spiritual well-being; and
- 4) enter into and function effectively in a variety of cultural settings.



Culturallyknowledgeable students are able to engage effectively in learning activities that are based on traditional ways of knowing and learning. Students who meet this cultural standard are able to:

- 1) acquire in-depth cultural knowledge through active participation and meaningful interaction with Elders;
- 2) participate in and make constructive contributions to the learning activities associated with a traditional camp environment;
- 3) interact with Elders in a loving and respectful way that demonstrates an appreciation of their role as culture-bearers and educators in the community;
- 4) gather oral and written history information from the local community and provide an appropriate interpretation of its cultural meaning and significance;
- 5) identify and utilize appropriate sources of cultural knowledge to find solutions to everyday problems; and
- engage in a realistic self-assessment to identify strengths and needs and make appropriate decisions to enhance life skills.



Culturally-knowledgeable students demonstrate an awareness and appreciation of the relationships and processes of interaction of all elements in the world around them.

Students who meet this cultural standard are able to:

- 1) recognize and build upon the interrelationships that exist among the spiritual, natural, and human realms in the world around them, as reflected in their own cultural traditions and beliefs as well as those of others;
- 2) understand the ecology and geography of the bioregion they inhabit;
- 3) demonstrate an understanding of the relationship between world view and the way knowledge is formed and used;
- 4) determine how ideas and concepts from one knowledge system relate to those derived from other knowledge systems;
- 5) recognize how and why cultures change over time;
- 6) anticipate the changes that occur when different cultural systems come in contact with one another;
- determine how cultural values and beliefs influence the interaction of people from different cultural backgrounds; and
- 8) identify and appreciate who they are and their place in the world.