



Let's ENGAGE with Math!

A graphic with a white background and a blue border. The text is arranged in four lines: 'Hey!' in blue, 'WELCOME' in large green letters, 'We're very' in yellow with 'very' in pink, and 'GLAD you're here!' in yellow. The word 'GLAD' is significantly larger than the other words in the bottom line.

Hey!
WELCOME
We're very
GLAD
you're here!

Session Objectives

- Engage in math activities and discussions about the math standards, including fluency and understanding math models used to solve mathematical problems.
- Understand how math is taught to students and why.

“How Many are Hiding?” Activity



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Progressing to Algebra



An important subset of the major work in grades K–8 is the progression that leads toward middle school algebra.

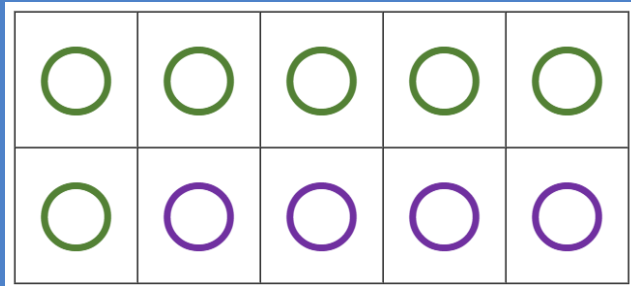
K	1	2	3	4	5	6	7	8
Know number names and the count sequence	Represent and solve problems involving addition and subtraction	Represent and solve problems involving addition and subtraction	Represent & solve problems involving multiplication and division	Use the four operations with whole numbers to solve problems	Understand the place-value system	Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers	Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers	Work with radical and integer exponents
Count to tell the number of objects	Understand and apply properties of operations and the relationship between addition and subtraction	Add and subtract within 20	Understand properties of multiplication and the relationship between multiplication and division	Generalize place value understanding for multi-digit whole numbers	Perform operations with multi-digit whole numbers and decimals to hundredths	Apply and extend previous understandings of multiplication and division to divide fractions by fractions	Understand the connections between proportional relationships, lines, and linear equations**	Understand the connections between proportional relationships, lines, and linear equations**
Compare numbers	Use place value understanding and properties of operations to add and subtract	Use place value understanding and properties of operations to add and subtract	Multiply & divide within 100	Use place value understanding and properties of operations to perform multi-digit arithmetic	Use equivalent fractions as a strategy to add and subtract fractions	Apply and extend previous understandings of multiplication and division to multiply and divide fractions	Analyze proportional relationships and use them to solve real-world and mathematical problems	Analyze and solve linear equations and pairs of simultaneous linear equations
Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from	Work with addition and subtraction equations	Measure and estimate lengths in standard units	Solve problems involving the four operations, and identify & explain patterns in arithmetic	Extend understanding of fraction equivalence and ordering	Apply and extend previous understandings of multiplication and division to multiply and divide fractions	Understand ratio concepts and use ratio reasoning to solve problems	Use properties of operations to generate equivalent expressions	Define, evaluate, and compare functions
Work with numbers 11–19 to gain foundations for place value	Extend the counting sequence	Relaes addition and subtraction to length	Develop understanding of fractions as numbers	Build fractions from unit fractions by applying and extending previous understandings of operations	Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition	Apply and extend previous understandings of arithmetic to algebraic expressions	Solve real-life and mathematical problems using numerical and algebraic expressions and equations	Use functions to model relationships between quantities
	Use place value understanding and properties of operations to add and subtract		Solve problems involving measurement and estimation of intervals of time, liquid volumes, & masses of objects	Understand decimal notation for fractions, and compare decimal fractions		Reason about and solve one-variable equations and inequalities		
	Measure lengths indirectly and by iterating length units		Geometric measurement: understand concepts of area and relate area to multiplication and to addition		Graph points in the coordinate plane to solve real-world and mathematical problems*	Represent and analyze quantitative relationships between dependent and independent variables		

* Indicates a cluster that is well thought of as a part of a student's progress to algebra, but that is currently not designated as major by the assessment consortia in their draft materials. Apart from the one asterisked exception, the clusters listed here are a subset of those designated as major in the assessment consortia's draft documents.

** Depends on similarity ideas from geometry to show that slope can be defined and then used to show that a linear equation has a graph which is a straight line and conversely.

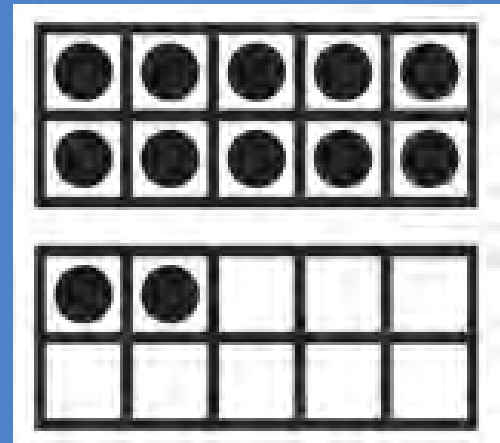
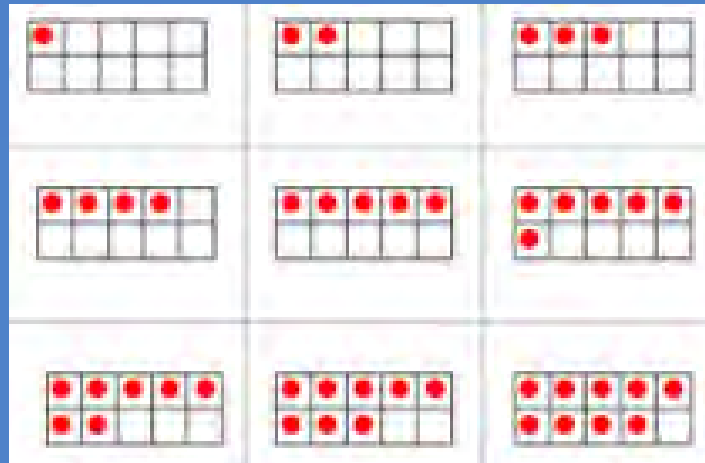
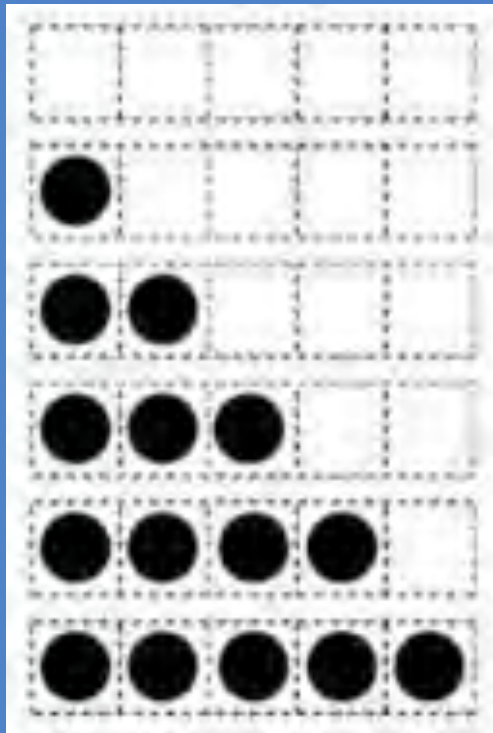


Concrete → Representational → Abstract



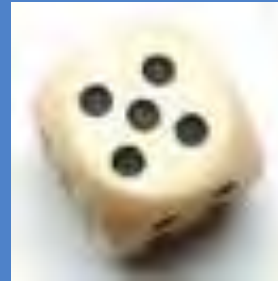
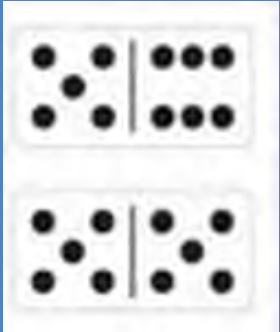
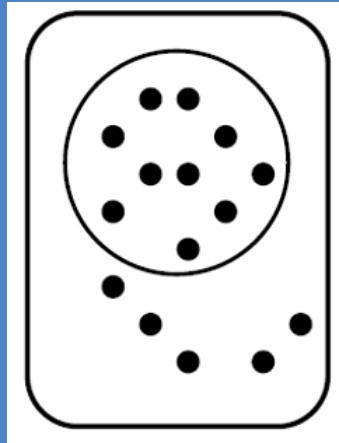
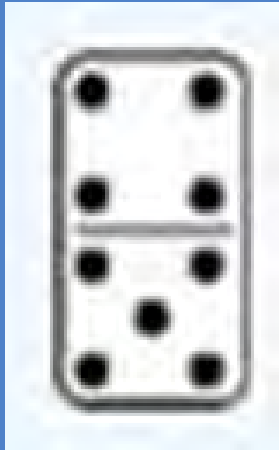
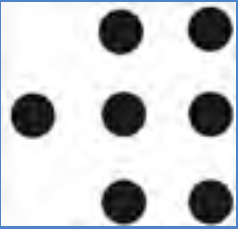
$$6 + 4 = 10$$

5- and 10-Frames, Double 10-Frames

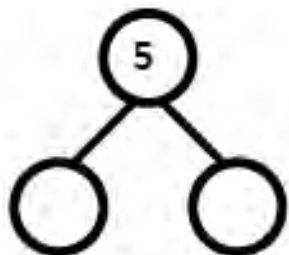


Subitizing

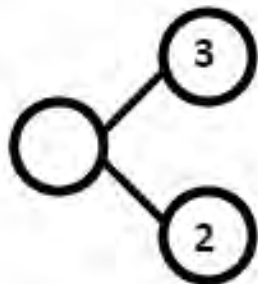
The ability to “see” a small amount of objects and know how many there are without counting each dot. For example, rolling dice – when “rolling” a 6-sided die, without counting the dots on the die, you “know” there are 5 dots when you “roll” number 5.



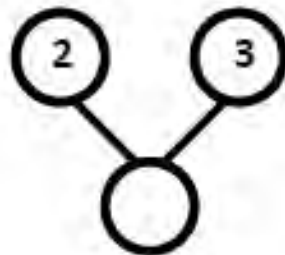
Number Bonds



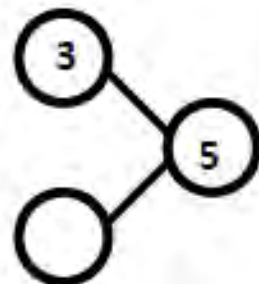
$$5 = \underline{\quad} + \underline{\quad}$$



$$3 + 2 = \underline{\quad}$$



$$2 + 3 = \underline{\quad}$$



$$5 - 3 = \underline{\quad}$$

A hand-drawn number bond diagram with a top circle containing the number 8 and two bottom circles containing the numbers 5 and 3, connected by lines.

• • • • •
• • •

$9 + 3 = \underline{12}$

1 2

$9 + 1 = 10$
 $10 + 2 = 12$

Number Bonds continued...

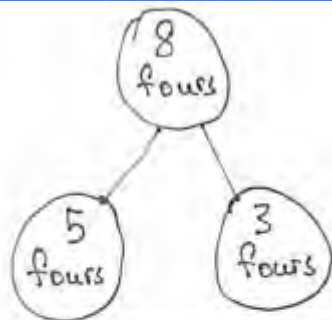
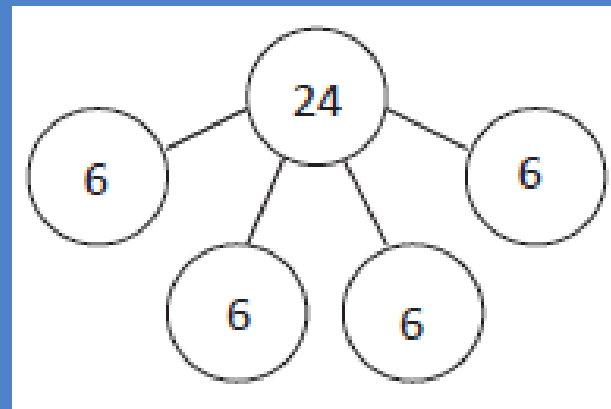
$$60 - 8 = 52$$

/\

50 10

$$10 - 8 = 2$$

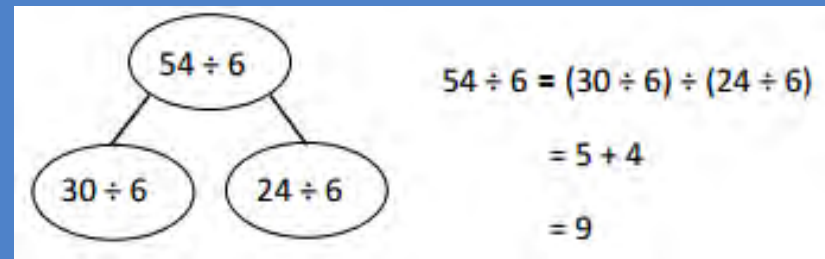
$$50 + 2 = 52$$



$$8 \text{ fours} = 5 \text{ fours} + 3 \text{ fours}$$

$$8 \times 4 = (5 \times 4) + (3 \times 4)$$

$$= (5 + 3) \times 4$$



$$54 \div 6 = (30 \div 6) + (24 \div 6)$$

$$= 5 + 4$$

$$= 9$$

Number Bonds continued...

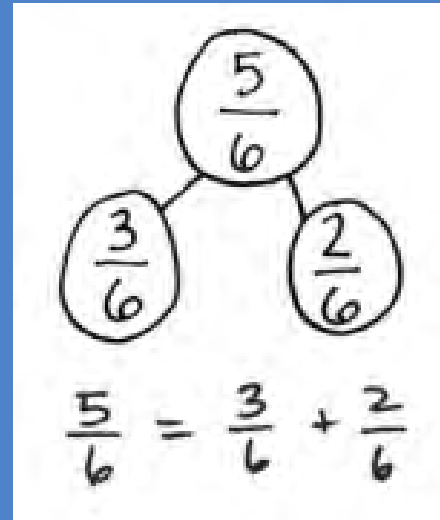
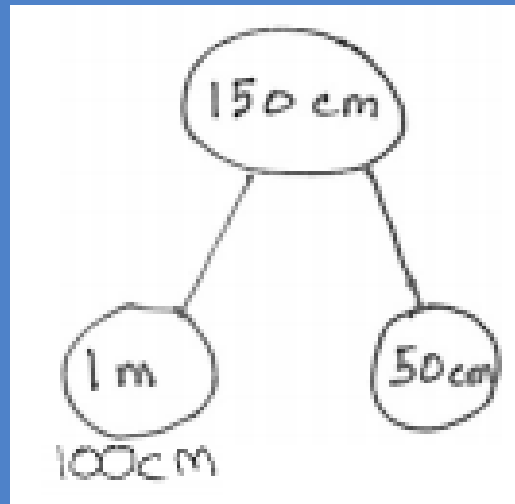
$$2 \text{ km } 608 \text{ m} + 3 \text{ km } 412 \text{ m}$$

$$\begin{aligned} 2 \text{ km} + 3 \text{ km} &= 5 \text{ km} \\ 608 \text{ m} + 412 \text{ m} &= 600 \text{ m} + 420 \text{ m} \\ &= 1,020 \text{ m} \end{aligned}$$

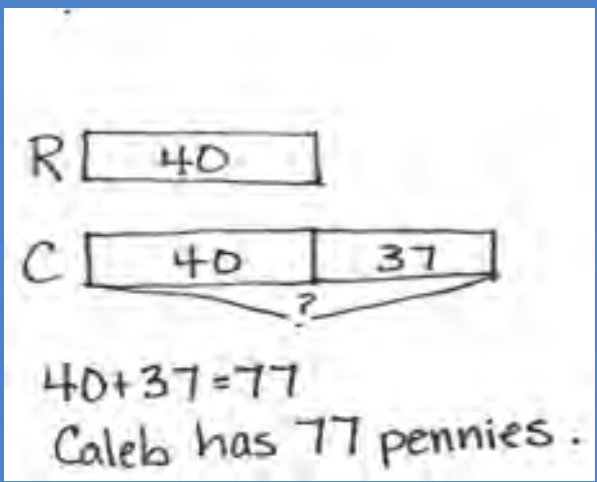
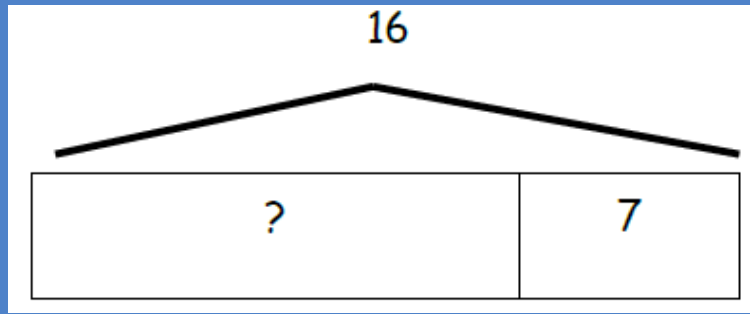
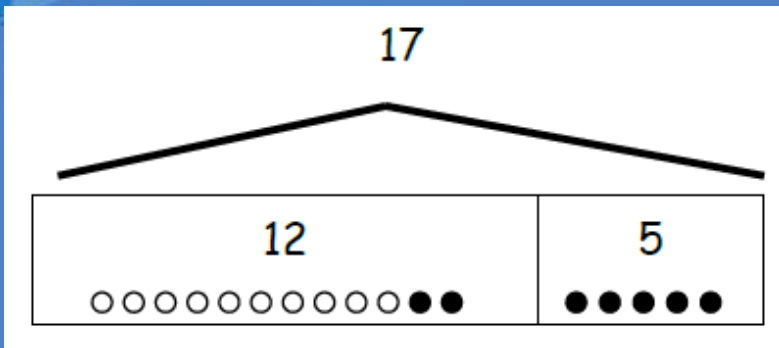
$$5 \text{ km} + 1 \text{ km } 20 \text{ m} = 6 \text{ km } 20 \text{ m}$$

OR

$$\begin{array}{r} 2,608 \text{ m} + 3,412 \text{ m} \\ \begin{array}{ccc} \swarrow & \downarrow & \searrow \\ 2,000 & 600 & 8 \end{array} \\ \begin{array}{ccc} \swarrow & \downarrow & \searrow \\ 3,000 & 400 & 12 \end{array} \\ \hline 5,000 \text{ m} + 1,000 \text{ m} + 20 \text{ m} = 6,020 \text{ m} \end{array}$$



Tape Diagrams



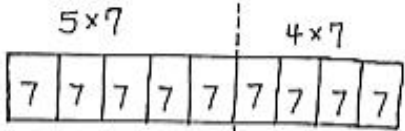
Twenty-four people line up to use the canoes at the park. Three people are assigned to each canoe. How many canoes are used?

3	?
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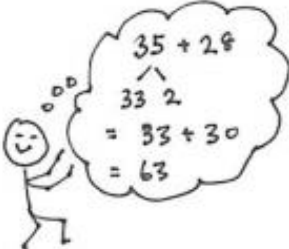
24 people
? canoes

$? \times 3 = 24$
 $? = 8$
The people use 8 canoes.

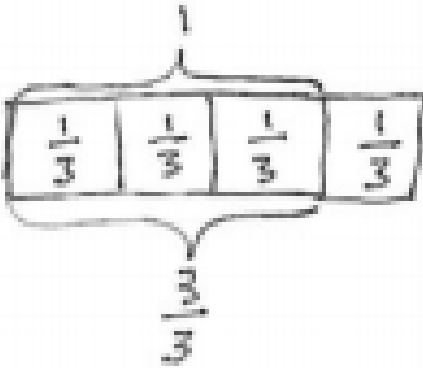
Tape Diagrams continued...

5×7 4×7


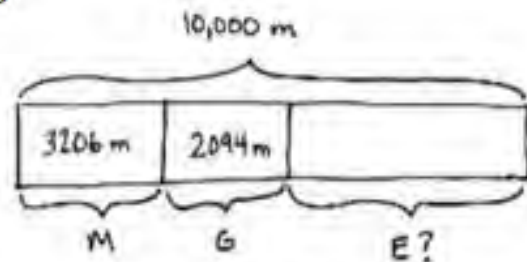
$9 \times 7 =$
 $(5 \times 7) + (4 \times 7) =$
 $35 + 28 = 63$
 $9 \times 7 = 63$



$\frac{4}{3} = \frac{3}{3} + \frac{1}{3}$
 $= 1 + \frac{1}{3}$



Martha, George, and Elizabeth sprint a combined distance of 10,000 meters. Martha sprints 3,206 meters. George sprints 2,094 meters. How far does Elizabeth sprint? Solve using an algorithm or a simplifying strategy.



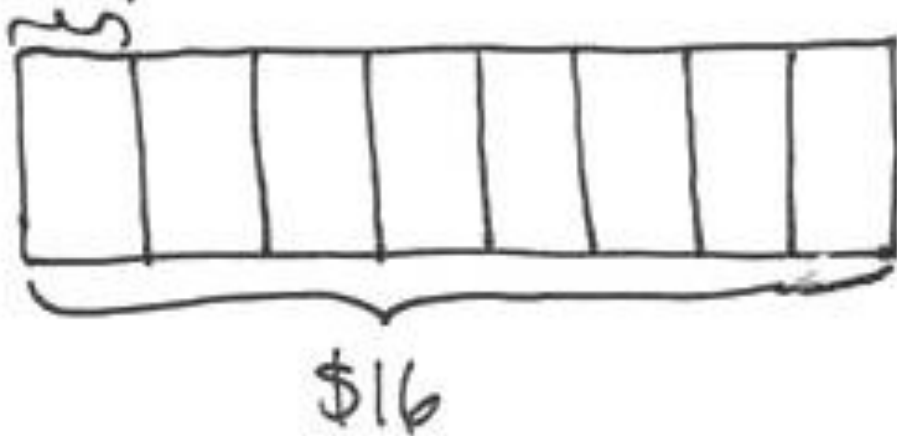
Tape Diagram Activity

Olga saves \$2 a week to buy a toy car.
The car costs \$16. How many weeks will
it take her to save enough to buy the toy?

Tape Diagrams continued...

Olga saves \$2 a week to buy a toy car. The car costs \$16. How many weeks will it take her to save enough to buy the toy?

1 week, \$2



$$\$16 \div \$2 = 8$$

It will take Olga 8 weeks to save enough money.

Arrays

Diagram illustrating different array arrangements of 12 'x' characters:

- 1 row of 12
- 2 rows of 6
- 3 rows of 4
- 4 rows of 3
- 6 rows of 2
- 12 rows of 1

Diagram illustrating a 16x3 array of 'o' characters, grouped into four 4x3 sub-arrays:

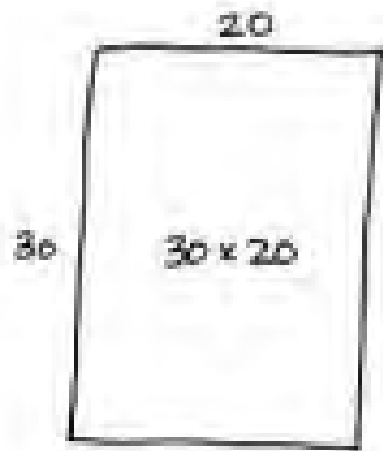
$$16 \times 3 = 4 \times (4 \times 3)$$

Diagram illustrating a multiplication problem using arrays of 'L' characters:

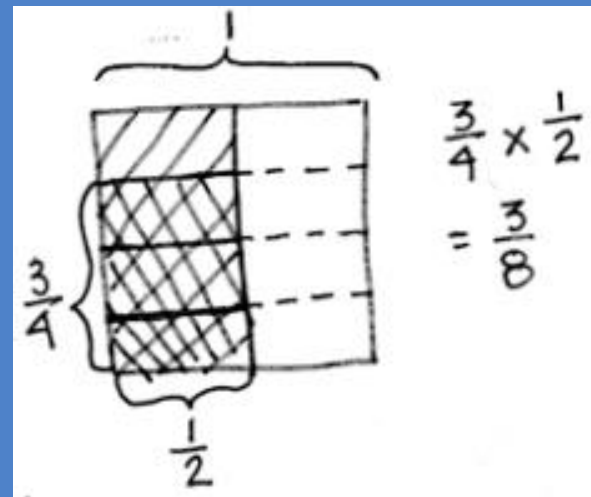
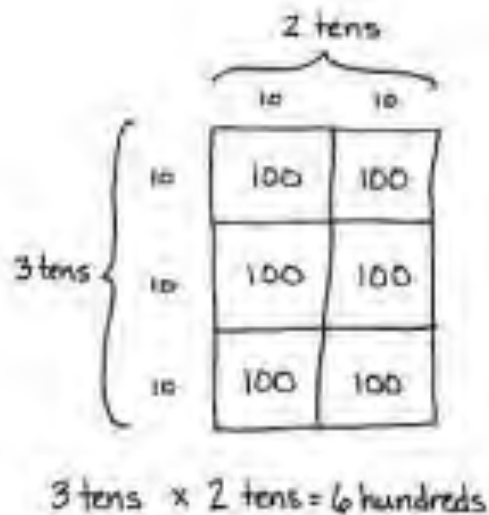
L L L L	4000
L L L L	x 3
L L L L	12,000

4 thousands \times 3 = 12 thousands

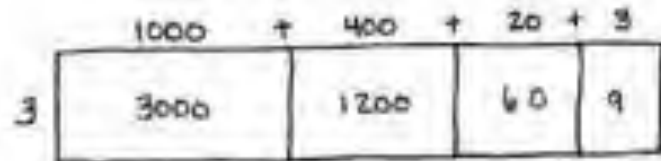
Area Models



$$30 \times 20 = 600$$



$$\begin{array}{r} 1423 \\ \times \quad 3 \\ \hline 4269 \end{array}$$



Area Models in Algebra 1

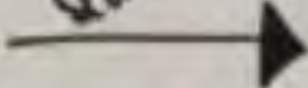
$$24 \times 35$$

	20	+4
30	600	120
+5	100	20

$$600 + 100 + 120 + 20$$
$$= 840$$

This model builds conceptual understanding even though it might not be most efficient.

Elementary
to
Secondary



$$(2x+3)(x+7)$$

	2x	+3
x	2x ²	3x
+7	14x	21

$$2x^2 + 14x + 3x + 21$$
$$= 2x^2 + 17x + 21$$

This is the SAME model with more challenging content. Check out that awesome vertical alignment!

The Progression of Multiplication

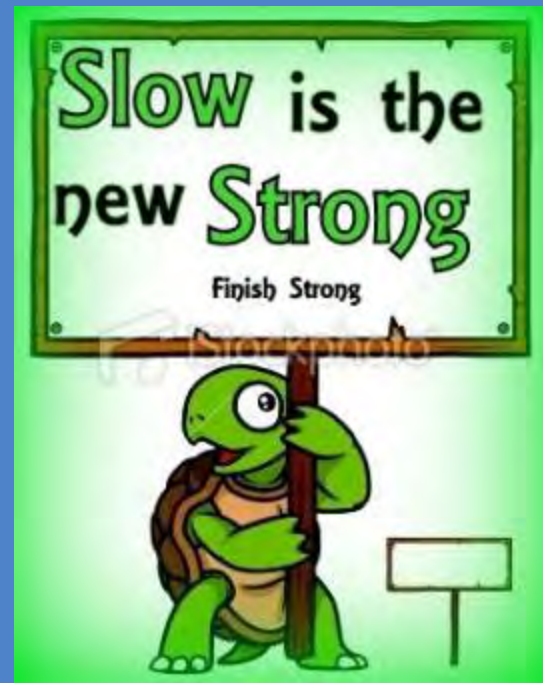
Video by Graham Fletcher

<https://vimeo.com/149428217>

The diagram illustrates the progression of multiplication through several stages:

- Concrete Stage:** Shows 3 packs of 100 items each. A note says "3 packs each pack had". Below it, a drawing of a pack of 100 is shown with the number 3 written next to it. A note says "use and STRUCTURE!".
- Intermediate Stage:** Shows a drawing of 100 items (represented as 10 tens) with a note "100 + 20 = 120". Below it, a drawing of 12 tens (120) is shown with a note "120".
- Abstract Stage:** Shows a drawing of 12 tens (120) with a note "120". Below it, a drawing of 30 tens (300) is shown with a note "30 30 30 30". A note says "30 x 4 = 120 = 3 x 40" and "Connection".
- Final Stage:** Shows a drawing of 400 items (represented as 4 hundreds) with a note "400". Below it, a drawing of 100 items (represented as 10 tens) is shown with a note "100". Below that, a drawing of 6 items (represented as 6 ones) is shown with a note "6". A note says "4 x 100 = 400", "10 x 10 = 100", "6 x 1 = 6" and "506".

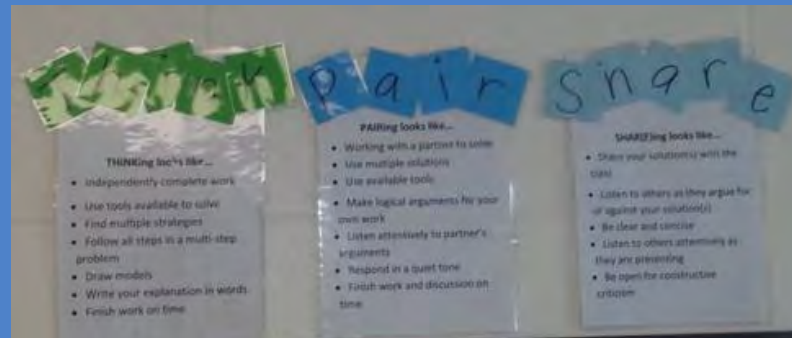
Arrows indicate the flow from the concrete stage to the intermediate stage, then to the abstract stage, and finally to the final stage.



Number Talk (mini version): Activity

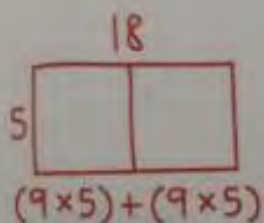
Solve 18×5 using models.

“Think, Pair, Share” - Work on your own, then work with a partner to continue solving, then share your strategy with the group.



Possible Activity Solutions

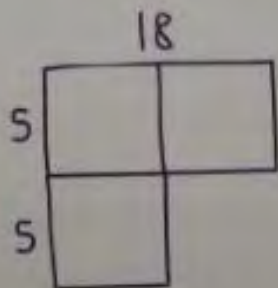
Neil



$$(9 \times 5) + (9 \times 5)$$

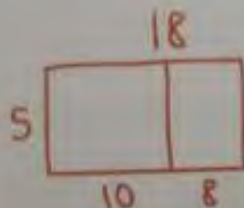
$$45 + 45 = 90$$

Ricardo



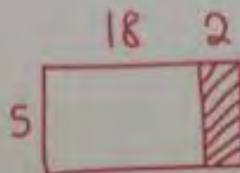
$$18 \times 5 = 9 \times 10$$

Sammi



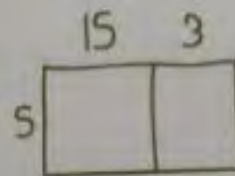
$$(10 \times 5) + (8 \times 5)$$
$$50 + 40 = 90$$

Jaime



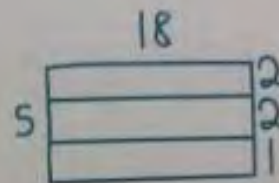
$$20 \times 5 = 100$$
$$2 \times 5 = 10$$
$$100 - 10 = 90$$

Ariane



$$15 \times 5 = 75$$
$$3 \times 5 = 15$$
$$75 + 15 = 90$$

Bryan



$$(18 \times 2) + (18 \times 2) + 18$$
$$36 + 36 + 18 = 90$$

Mathematical Mindset by Jo Boaler

Number Talk: 2015

23,
28,

It's a year.
It will not be 2015 again.
It will be 2016 in
10 days!

2015

2,000 + 15 = 2,015

2,015 + 1 = 2,016

2,000 is made out of
20 100's.

1,015 + 1,000 = 2,015

2015

1,000 + 1,000 + 10 + 5 = 2,015

2,000 + 10 + 5 = 2,015

The image shows a whiteboard with a large circle drawn around it. Inside the circle, there are several handwritten notes and diagrams. At the top left, there is a note: "It's a year. It will not be 2015 again. It will be 2016 in 10 days!". In the center, the number "2015" is written and circled. To the right of the center, there are two diagrams: one showing "2015" branching into "2,000" and "15", with the equation "2,000 + 15 = 2,015" below it; and another showing "2,015" branching into "2,000", "10", and "5", with the equation "2,000 + 10 + 5 = 2,015" below it. To the left of the center, there is a diagram showing "2015" branching into "1,015" and "1,000", with the equation "1,015 + 1,000 = 2,015" above it. Below the center, there is a diagram showing "2015" branching into "1,000", "1,000", "10", and "5", with the equation "1,000 + 1,000 + 10 + 5 = 2,015" above it. To the right of this diagram, there is another diagram showing "2,015" branching into "2,000", "10", and "5", with the equation "2,000 + 10 + 5 = 2,015" above it. In the top right corner of the whiteboard, the numbers "23," and "28," are written.

The Progression of Division


Video by Graham Fletcher

<https://vimeo.com/153668928>

4th grade 4 digit dividends ÷ 1 digit divisors

models representation ↔ written expression

$144 \div 9$



so on....

Area model for partial quotients

$3672 \div 9$

200	200	8
9	1800	1800
		72

$3672 \div 9$

3672
-9
3663
-9
3654
900
2754
-1800
954
900
54
54
0

5th grade 4 digit dividend ÷ 2-digit divisors

Whole # quotients ONLY

$1257 \div 32$

10	10	10	5	4
320	320	320	160	128

my products of 32

- $1 \times 32 = 32$
- $2 \times 32 = 64$
- $3 \times 32 = 96$
- $10 \times 32 = 320$

$1257 \div 32$

1257
-320
937
-320
617
-320
297
-160
137
-128
9

9 remainder

Decimals?

$124 \div 0.04 = 31$

10	10	10	1
.4	.4	.4	0.04

$1.24 \div 0.04$

1.24
.40
.84
.40
.44
.40
0.04
0.04
0

out $25 + 6$

100	.24
-----	-----



Math Language That Needs to Expire

Instead of saying...	Say...
<i>borrowing and carrying</i>	<i>trading and regrouping</i>
<i>___ out of ___</i>	<i>fraction and attribute</i>
<i>reducing fractions</i>	<i>simplifying fractions</i>
How are shapes <i>similar</i> ?	How are shapes the <i>same</i> or <i>different</i> ?
Reading “=” symbol as “ <i>makes</i> ”	<i>equals</i> or <i>is the same as</i>
<i>plugging</i> a number into	<i>substitute values</i>
<i>top number/bottom number</i>	<i>numerator/denominator</i>

How to Help Kids Solve Word Problems

Ask your child:

1. What do you already know?
2. What do you need to find out?
3. How can you retell the story problem in your own words?
4. How can you use math tools to solve the problem?
5. How could you draw a picture or model of the problem?
6. Does your solution make sense? Why or why not?

Encourage children to approach word problems with careful thinking, confidence, and a persevering attitude.

Encourage kids to learn multiple approaches to solving math problems, so that they can choose the approach that works best for them, and so that they develop a full understanding of the concepts before they move on to more challenging levels.

Parents often ask...

“What is the point of my child explaining their work if he/she can get the right answer?”

Jo Boaler answers, “Explaining your work is what, in mathematics, we call reasoning, and reasoning is central to the discipline of mathematics. Students need to produce arguments that convince others by carefully reasoning their way from one idea to another, using logical connections.”

Jo Boaler, Author of *Mathematical Mindset*

Math Messages



Jo Boaler & her Students

<https://www.youcubed.org/students/>

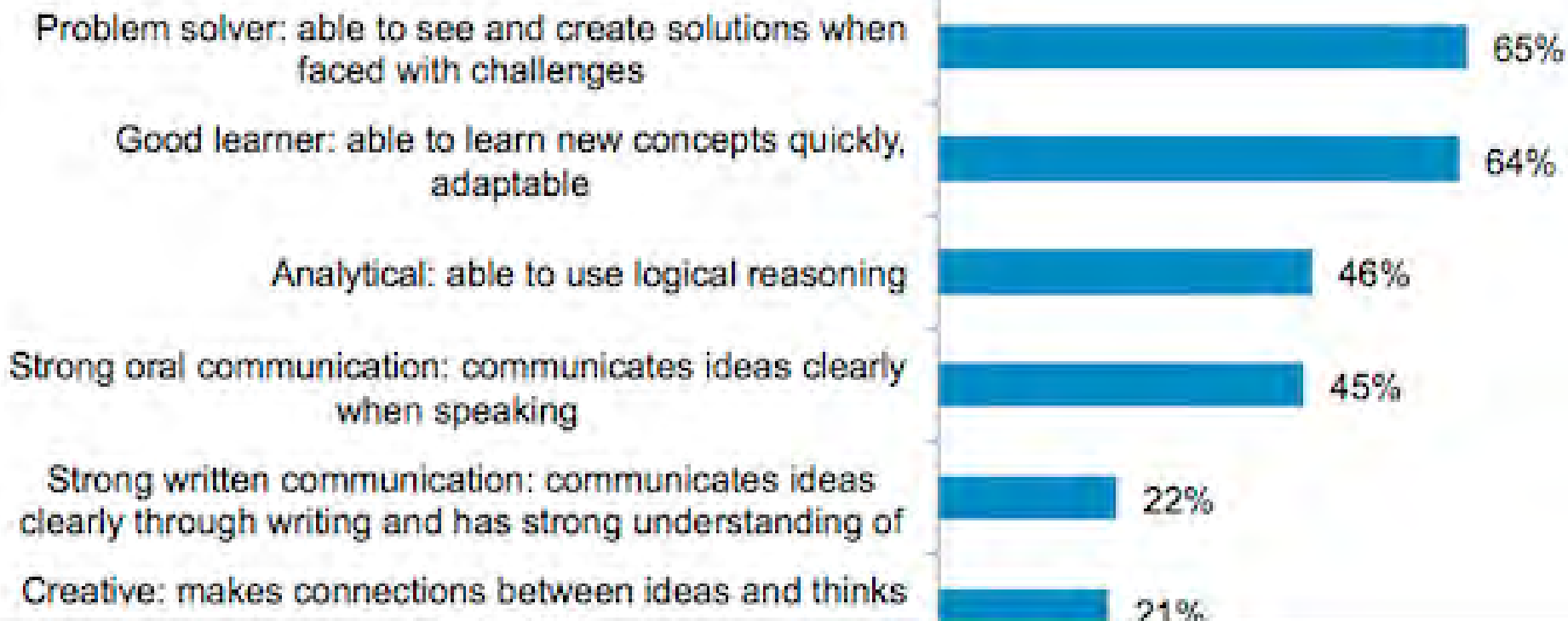
Math I Can

<https://www.amazon.com/gp/withmathican>

Fortune 500 Most Valued Skills

	1970	1999
1	Writing	Teamwork
2	Computational Skills	Problem Solving
3	Reading Skills	Interpersonal Skills
4	Oral Communications	Oral Communications
5	Listening Skills	Listening Skills
6	Personal Career Development	Personal Career Development
7	Creative Thinking	Creative Thinking
8	Leadership	Leadership
9	Goal Setting / Motivation	Goal Setting / Motivation
10	Teamwork	Writing

Linked in: Most Important Skills



Most Important Personality Traits



Key Areas of Focus in Mathematics

Grade	Focus Areas
K-2	Addition and subtraction – concepts, skills, problem solving and place value
3-5	Multiplication and division of whole numbers and fractions – concepts, skills, and problem solving
6	Ratios and proportional relationships; early expressions and equations
7	Ratios and proportional relationships; arithmetic of rational numbers
8	Linear algebra and linear functions

Required Fluencies K-6

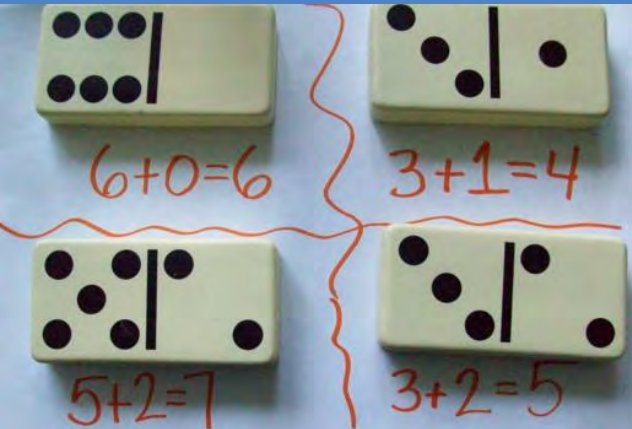
Grade	Standard	Expected Fluency
K	K.OA.A.5	Add/Subtract within 5
1	1.OA.C.6	Add/Subtract within 10
2	2.OA.B.2 2.NBT.B.5	Add/Subtract within 20 (Know single digit sums from memory) Add/Subtract within 100
3	3.OA.C.7 3.NBT.A.2	Multiply/Divide within 100 (Know single digit products from memory) Add/Subtract within 1000
4	4.NBT.B.4	Add/Subtract within 1,000,000
5	5.NBT.B.5	Multi-digit multiplication
6	6.NS.B.2 6.NS.B.3	Multi-digit division Multi-digit decimal operations
7	7.NS.A.1,2 7.EE.B.3 7.EE.B.4	Fluency with rational number arithmetic Solve multistep problems with positive and negative rational numbers in any form Solve one-variable equations of the form $px + q = r$ and $p(x + q) = r$ fluently
8	8.EE.C.7 8.G.C.9	Solve one-variable linear equations, including cases with infinitely many solutions or no solutions Solve problems involving volumes of cones, cylinders, and spheres together with previous geometry work, proportional reasoning and multi-step problem solving in grade 7



Ways to Help with Math Fluency at Home



More Ways to Help at Home



**GAMES ARE
A GREAT
MOTIVATOR
FOR KIDS
TO LEARN
MATH.**

More Ways to Help at Home



More Ways to Help at Home

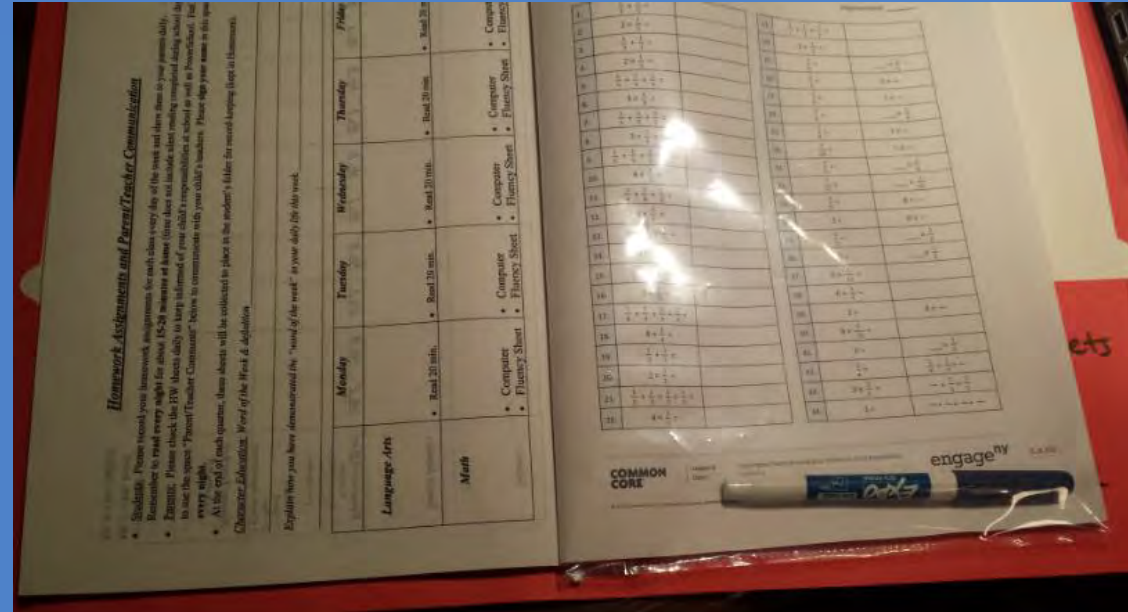
Printed HW Assignments Grade 8 (prints)

Home Activity

A

Student: _____ # Correct: _____

1	$20 + 2 =$		23	$58 + 2 =$	
2	$4 + 2 =$		24	$96 + 3 =$	
3	$24 + 2 =$		25	$86 + 2 =$	
4	$30 + 3 =$		26	$93 + 3 =$	
5	$6 + 3 =$		27	$88 + 4 =$	
6	$36 + 3 =$		28	$99 + 3 =$	
7	$40 + 4 =$		29	$86 + 3 =$	
8	$8 + 4 =$		30	$56 + 2 =$	
9	$48 + 4 =$		31	$40 + 4 =$	
10	$2 + 2 =$		32	$80 + 4 =$	
11	$40 + 2 =$		33	$60 + 4 =$	
12	$42 + 2 =$		34	$68 + 4 =$	
13	$2 + 3 =$		35	$20 + 2 =$	
14	$60 + 3 =$		36	$40 + 2 =$	
15	$63 + 3 =$		37	$30 + 2 =$	
16	$4 + 4 =$		38	$36 + 2 =$	
17	$80 + 4 =$		39	$30 + 3 =$	
18	$84 + 4 =$		40	$39 + 3 =$	
19	$40 + 5 =$		41	$45 + 3 =$	
20	$50 + 5 =$		42	$60 + 3 =$	
21	$80 + 5 =$		43	$57 + 2 =$	
22	$70 + 5 =$		44	$51 + 3 =$	



Adults should...

- Encourage children and focus on the reasoning in the math rather than getting a correct answer.
- Help children realize math is not about speed.
- Help children recognize that struggling moments are opportunities for growth. Making mistakes is okay.
- Say, “Let’s work and learn together to figure the problem out.” when working on math with your child, rather than sharing with your child the idea that you were not good/struggled at math in school or you disliked math or not a “math person.”
- Encourage number sense.
- Encourage a growth mindset...the idea that ability and smartness change as you work and learn more.

Q & A

You have

Questions

We have

Answers

Thank you!



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