

Key 9-3-11  
OK to print

1. When does  $P(A \text{ or } B) = P(A) + P(B)$   
When events A and B are mutually exclusive (disjoint).
2. When does  $P(A \cap B) = P(A) \cdot P(B)$ ?  
When A and B are independent events.
3. How do you prove that two events are independent? (three ways)  
 $P(A | B) = P(A)$  or  $P(B | A) = P(B)$   
Or  
 $P(A \cap B) = P(A) \cdot P(B)$   
OR  
Compare the "fractions" in the table to see if all the same

Questions 4-7 refer to the following study:

500 people used a home test for HIV, and then all underwent more conclusive hospital testing. The accuracy of the home test was evidenced in the following table:

	HIV	HEALTHY	Total
Positive home test	35	25	60
Negative home test	5	435	440
Total	40	460	500

A

4. What is the *predictive value* of the test? That is, what is the probability that a person has HIV and tests positive?  
A) 0.070  
B) 0.130  
C) 0.538  
D) 0.583  
E) 0.875

A

5. What is the *false-positive* rate? That is, what is the probability of testing positive given that the person does not have HIV?  
A) 0.054  
B) 0.050  
C) 0.130  
D) 0.417  
E) 0.875

E

6. What is the *sensitivity* of the test? That is, what is the probability of testing positive given that the person has HIV?  
A) 0.070  
B) 0.130  
C) 0.538  
D) 0.583  
E) 0.875

E

7. What is the *specificity* of the test? That is, what is the probability of testing negative given that the person does not have HIV?  
A) 0.125  
B) 0.583  
C) 0.870  
D) 0.950  
E) 0.946

8. When does the probability that event A or event B occurs equal the sum of their probabilities?

C

- A) When events A and B are both dependent.
- B) When events A and B are both independent.
- C) When events A and B are disjoint.
- D) When events A and B are both dependent and disjoint.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

↑  
zero  
then  
disjoint

9. When is the probability that both events A and B occur equal to the product of their probabilities?

B

- A) When events A and B are both dependent.
- B) When events A and B are both independent.
- C) When events A and B are disjoint.
- D) When events A and B are both independent and disjoint.

10. What is meant by saying two events are independent?

B

- A) The two events have no outcomes in common.
- B) Knowledge of the first event occurring has no effect on the outcome of the second event.
- C) The two events are mutually exclusive.
- D) It can be represented with  $P(A|B) = P(A \cap B) / P(B)$

this is conditional

#11-13. The probability that a car will come to a complete stop at a particular stop sign is 0.4. Assuming that the next four cars to arrive at the stop sign are independent of one another,

C

11. What is the probability that none of the four cars comes to a complete stop?

- A) 0
- B) 0.0256
- C) 0.1296
- D) 0.4
- E) 0.6

B

12. What is the probability that all four of the cars come to a complete stop?

- A) 0
- B) 0.0256
- C) 0.1296
- D) 0.4
- E) 0.6

C

13. What is the probability that at least one of the cars comes to a complete stop?

- A) 0.4
- B) 0.6
- C) 0.8704
- D) 0.9744
- E) 1.0

$$1 - 0.1296 =$$

14. At the McDonalds drivethru the probability a person orders fries is 0.78, a hamburger 0.70, and both 0.65. What is the probability a person....

a) just ordered fries?

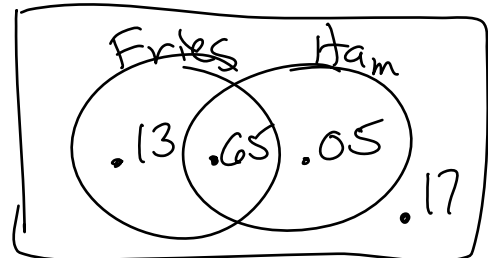
$$.13$$

b) ordered neither fries or hamburger?

$$.17$$

c) who is already know to have ordered fries also got a hamburger to go with it?

$$\frac{.65}{.78} = \boxed{.833}$$



15. The probability that a machine part is in working order is 0.8. Four machine parts are drawn at random from the day's production.

a. What is the probability that all 4 are broken?

$$P(\text{not working}) = .2 = P(w')$$

$$P(4 \text{ not working}) = P(w)' \cdot P(w)' \cdot P(w)' \cdot P(w)' = (.2)^4 = .0016$$

b. What is the probability that all 4 are in working order?

$$P(4 \text{ WORKING}) = (.8)^4$$

$$= .4096$$

c. What is the probability the first two work and the second two are broken?

$$(.8)(.8)(.2)(.2) = .0256$$

d. What is the probability that at least one is in working order?

$$P(\text{at least one}) = 1 - P(\text{none WORKING})$$

$$= 1 - P(4 \text{ not WORKING})$$

$$= 1 - .0016 = .9984 \text{ at least one WORKS}$$

16. A large computer software company received 500 applications for a single position. The applications are summarized in the table below.

Sex/Degree	Computer Science	Computer Engineering	Business	
Male	120	100	80	300
Female	60	20	120	200
	180	120	200	500

a. What is the probability that a randomly selected applicant had a degree in Computer Science or Computer Engineering?

$$P(\text{CS OR CE}) = P(\text{CS}) + P(\text{CE}) - P(\text{CS} \cap \text{CE})$$

*← mutually exclusive can't have both degrees*

$$= \frac{180}{500} + \frac{120}{500} - 0 = .60$$

b. What is the probability that a randomly selected applicant was a female and had a Business degree?

$$P(\text{F and Bus}) = \frac{120}{500} = .24$$

c. What is the maximum number of applicants that may be selected before we are concerned with "replacement"?

$$n < 5\% \quad 5\%(500) = 25 \quad n < 25$$

d. Assuming that two applicants are selected independently of each other, what is the probability that both were female applicants with Computer Engineering degrees?

$$P(\text{F and CE}) = \frac{20}{500} \cdot \frac{20}{500} = .0016$$

*assume independent so can multiply probabilities.*

17. A college math professor has surveyed his records and found the following frequency distribution of the grades of 1187 students who have taken his calculus classes.

Grade	A	B	C	D	F
Frequency	125	352	461	187	62

- a. What is the probability that a randomly selected student received an A?

$$P(A) = \frac{125}{1187} = \boxed{.1053}$$

- b. What is the probability that a randomly selected student received an A or a B?

$$P(A \text{ or } B) = \frac{125 + 352}{1187} = \boxed{.4019}$$

- c. What is the probability that two independently selected students both received an A?

Assume independent (and large sample) so can multiply.

$$P(2 \text{ A's}) = P(A) \cdot P(A) = .1053 \times .1053 = \boxed{.01109}$$

- d. What is the probability that two independently selected students both failed Calculus?

Assume independent so can multiply.

$$P(2 \text{ F's}) = P(F) \cdot P(F) = \frac{62}{1187} \cdot \frac{62}{1187} = \boxed{.0027}$$

18. A woman lamented that she had just given birth to her eighth child, and all were girls! Her doctor had assured her that the chance of the eighth child being a girl was only 1 in 100.

- a. What was the real probability that the eighth child would be a girl?

- b. Before the birth of the first child, what was the probability that the woman would give birth to eight girls in a row?

A. .5, .0039

B. .0039, .0039

C. .5, .5

D. .0039, .4

E. .5, .01

every birth is independent  
so the 8<sup>th</sup> baby had 50% girl  
 $P(G)^8 = (.5)^8 = .0039$

19. Suppose  $P(A) = 0.35$ ,  $P(B) = 0.50$ , and  $P(A \text{ and } B) = 0.45$ .

Are A and B independent events? Show work to justify your answer.

If independent,  $P(A \text{ and } B) = P(A \cap B) = P(A) \cdot P(B)$   
 $.45 \neq .35 \cdot .50$   
 So NOT independent

20. 35% of cars on the road are over 10 years old, while 40% of cars on the road have over 100,000 miles. I am driving a 1999 Chrysler Sebring convertible with 109,100 miles. Am I correct to assume my car is one of the 14% of old, long-driven cars on the road? Explain why or why not.

$P(\text{over 10 yrs}) * P(> 100k \text{ miles}) =$   
 is only true if these two variables are independent  
 But they are NOT independent, so cannot  
 multiply and cannot assume.

Questions 21 – 25 refer to the following study: One thousand students at a city high school were classified both according to GPA and whether or not they consistently skipped classes.

GPA	< 2.0	2.0 – 3.0	> 3.0	Total
Many skipped classes	80	25	5	110
Few skipped classes	175	450	265	890
Total	255	475	270	1000

21. What is the probability that a student has a GPA between 2.0 and 3.0?

- A. 0.025
- B. 0.227
- C. 0.450
- D. 0.475**
- E. 0.506

$$P(2 < \text{GPA} < 3) = \frac{475}{1000}$$

22. What is the probability that a student has a GPA under 2.0 and has skipped many classes?

- A. 0.080**
- B. 0.281
- C. 0.285
- D. 0.314
- E. 0.727

$$P(\text{GPA} < 2 \cap \text{skipped many}) = \frac{80}{1000}$$

23. What is the probability that a student has a GPA under 2.0 or has skipped many classes?

- A. 0.080
- B. 0.281
- C. 0.285**
- D. 0.314
- E. 0.727

$$\begin{aligned} P(\text{GPA} < 2 \cup \text{skipped many}) &= P(\text{GPA} < 2) + P(\text{skip many}) \\ &\quad - P(\text{GPA and skip many}) \\ &= .255 + .110 - .080 \\ &= .285 \end{aligned}$$

24. What is the probability that a student has a GPA under 2.0 given that he has skipped many classes?

- A. 0.080
- B. 0.281
- C. 0.285
- D. 0.314
- E. 0.727**

$$P(\text{GPA} < 2 \mid \text{skip many}) = \frac{80}{110}$$

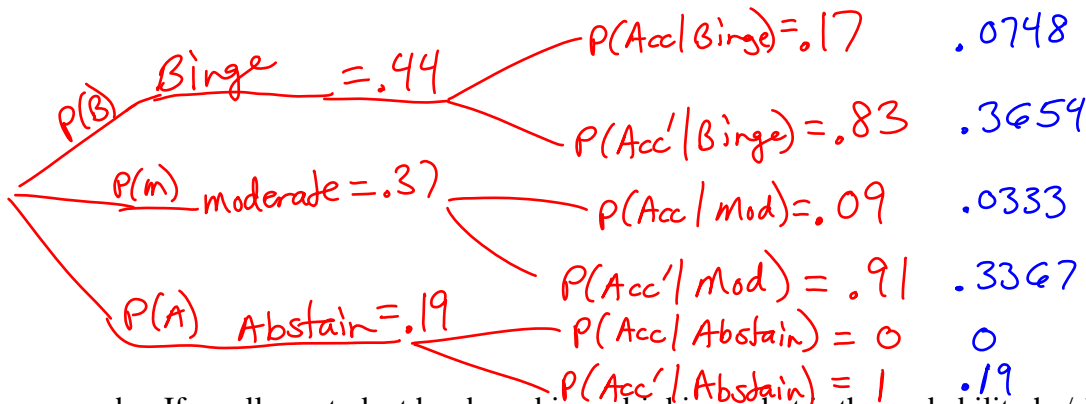
25. What is the probability that a student has a GPA below 3.0, if they have skipped many classes?

- A. 0.144
- B. 0.227
- C. 0.830
- D. 0.955**
- E. Cannot be determined because NOT independent

$$P(\text{GPA} < 3.0 \mid \text{skipped many}) = \frac{105}{110}$$

26. For men, binge drinking is defined as having five or more drinks in a row, and for women as having four or more drinks in a row. 44% of college students engage in binge drinking, 37% drink moderately, and 19% abstain entirely. Another study, published in the *American Journal of Health Behavior*, finds that among binge drinkers 17% have been involved in an alcohol-related automobile accident, while among moderate drinkers, only 9% have been involved in such accidents.

a. Draw a tree diagram for this problem and label the probabilities of each branch.



b. If a college student has been binge drinking, what is the probability he/she has an automobile accident?

$$P(\text{Acc} \mid \text{Binge}) = 0.17$$

c. What is the probability that a college student is a binge drinker and has an accident?

$$P(\text{Binge and Acc}) = P(\text{Binge}) \cdot P(\text{Acc}) = 0.44 \times 0.17 = 0.0748$$

d. If a student drove and was NOT involved in an accident, what is the probability that student was a drunk driver who lucked out?

$$P(\text{Binge} \cup \text{moderate drink} \mid \text{Accident}) = \frac{0.3654 + 0.3367}{0.3654 + 0.3367 + 0.19} = \boxed{0.787}$$