

**Conditional Probability & Simpson's Paradox**

1. Suppose that the manager of the Chicago White Sox is faced with a difficult situation in the last inning of a baseball game later this evening and he asks for your input. Here is the situation. He has two right-handed batters, say Moe and Joe, available to pinch hit in the last inning to try to win the game. However, he is not sure which player to use. The performance of the players during this season is shown in the table below.

Player	Pitcher	Hits	At Bats
Joe	Right	40	100
	Left	80	400
Moe	Right	120	400
	Left	10	100

- Construct a two-way table of player (Joe or Moe) versus outcome (hit or no hit).
- Compute the overall batting average (hits divided by total number of times at bat) for each man.
- For each type of pitching, construct two-way tables for each player by outcome (hit or no hit).
- Use the tables in part (c) to compute the batting averages of the two players for each type of pitcher.
- Based on the calculations in (b) and (d), who would you recommend? Explain your recommendation in language that the manager will understand.

a)

	Hit	NoHit
Joe	120	380
Moe	130	370

b)

$$\text{Joe BA} = \frac{120}{500} = .240$$

$$\text{Moe BA} = \frac{130}{500} = .260$$

c)

Joe	Hit	NoHit	Avg
Right	40	60	.400
Left	80	320	.200

Moe	Hit	NoHit	Avg
Right	120	280	.300
Left	10	90	.100

d) In both cases I recommend Joe. He has higher average against both pitchers.

2. The influence of race on imposition of the death penalty for murder has been much studied and contested in the courts. The following three-way table classifies 326 cases in which the defendant was convicted of murder. The three variables are the defendant's race, the victim's race, and whether the defendant was sentenced to death.

	White Defendant	
	Death Penalty	No Death Penalty
White Victim	19	132
Black Victim	0	9

$$19/151 = .1258$$

$$0/9 = .000$$

	Black Defendant	
	Death Penalty	No Death Penalty
White Victim	11	52
Black Victim	6	97

$$11/63 = .1746$$

$$6/103 = .058$$

- Construct a two-way table of defendant's race by death penalty. Calculate the percentage of each race defendant sentenced to death.
- Using the two two-way tables above of victim race by death penalty, one table for white defendant and one table for black defendant; calculate the percentage of each victim where the defendant is sentenced to death.
- Show that Simpson's paradox holds: A higher percent of white defendants are sentenced to death overall, but for both black and white victims a higher percentage of black defendants are sentenced to death. Explain why the paradox holds in language that your grandmother would understand.

a)

	Death	No Death	%
White Def	19	141	$\frac{19}{160} = .11875$
Black Def	17	149	$\frac{17}{166} = .10241$

b) see above

c) Paradox true b/c sample sizes are NOT equal, Nor are marginal totals distributed similarly.

3. The University of California at Berkeley was charged with having discriminated against women in the graduate admissions process for the fall quarter of 1973. The table below identifies the number of acceptances and denials for both male and female applicants in the six largest graduate programs .

	Men Admitted	Men Denied	Women Admitted	Women Denied
Program A	511	314	89	19
Program B	352	208	17	8
Program C	120	205	202	391
Program D	137	270	132	243
Program E	53	138	95	298
Program F	22	351	24	317
Total	1195	1486	559	1276

(a) Start by ignoring the program distinction, collapsing the data into a two-way table of gender by admission status. To do this, find the total number of men accepted and denied and the total number of women accepted and denied. Fill in below.

	Admitted	Denied	Total
Men	1195	1486	2681
Women	559	1276	1835
Total	1754	2762	4516

(b) Consider for the moment just the men applicants. Of the men who applied to one of these programs, what proportion was admitted?

$$1195 / 2681 = .4457$$

(c) Now consider the women applicants; what proportion of them were admitted?

$$559 / 1835 = .30$$

(d) Do these proportions seem to support the claim that men were given preferential treatment in admissions decisions?

Yes, A little bit, but still close.

(e) Try to isolate the program(s) responsible for the mistreatment of women applicants. Calculate the **proportion** of men admitted out of men who tried, and the proportion of women admitted out of women who tried within each program. Record your results in the table below.

	Proportion of Men Admitted	Proportion of Women Admitted
Program A	.619	.824
Program B	.629	.680
Program C	.369	.341
Program D	.3366	.352
Program E	.277	.2417
Program F	.059	.070

(f) Which one program is responsible for the large discrepancy between men and women in the overall proportions admitted?

Program A has a large discrepancy.  
The others are within  $\pm 3\%$ , some with men higher, some w/ women.