

## Intro to Sequences and Series

Sequence: a set of things (numbers) that are in order

### Finite Sequence

limited # of terms

ex. 2, 4, 6, 8

### Infinite Sequence

continues w/o stopping

ex. 2, 4, 6, 8, 10, ...

Writing the terms of a sequence:

Ex. Write the first <sup>four</sup> ~~two~~ terms of the sequence.

a.  $a_n = 2n + 1$

$$a_1 = 2(1) + 1 = 3$$

$$a_2 = 2(2) + 1 = 5$$

$$a_3 = 2(3) + 1 = 7$$

$$a_4 = 2(4) + 1 = 9$$

b.  $a_n = (-1)^n$

$$a_1 = (-1)^1 = -1$$

$$a_2 = (-1)^2 = 1$$

$$a_3 = (-1)^3 = -1$$

$$a_4 = (-1)^4 = 1$$

Series: when the terms of a sequence are added together

### Finite Series

ex.  $2 + 4 + 6$   
 $= 12$

### Infinite Series

ex.  $2 + 4 + 6 + 8 + \dots$

Find the Sum of a Series:

Ex. Find the sum of the series.

a.  $3 + 5 + 7 + 9$

24

b.  $3 + 4 + 5 + 6$

18

## 11.5 Arithmetic Sequences and Series

Arithmetic Sequence: A sequence in which the difference between consecutive terms is constant.

Common Difference: The constant difference is called the common difference and is denoted by the letter  $d$ .

Determine if each sequence is arithmetic. If so, find the common difference,  $d$ .

$$5, 12, 19, 26, 33, \dots \quad \text{Y/N} \quad d = \underline{7}$$

$\begin{array}{cccc} \checkmark & \checkmark & \checkmark & \\ +7 & +7 & +7 & \end{array}$

$$12, 8, 4, 0, -4, -8, \dots \quad \text{Y/N} \quad d = \underline{-4}$$

$\begin{array}{ccc} \checkmark & \checkmark & \checkmark \\ -4 & -4 & -4 \end{array}$

$$1, 3, 9, 27, 81, \dots \quad \text{Y/N} \quad d = \underline{\quad}$$

$\begin{array}{cc} \checkmark & \checkmark \\ +2 & +6 \end{array}$

$$\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \frac{9}{2}, \dots \quad \text{Y/N} \quad d = \underline{1}$$

$\begin{array}{ccc} \checkmark & \checkmark & \checkmark \\ +1 & +1 & +1 \end{array}$

### Rule for Arithmetic Sequence

The  $n^{\text{th}}$  term of an arithmetic sequence with first term  $a_1$  and common difference  $d$

ARITHMETIC TERM

$$a_n = a_1 + (n-1)d$$

1) Write a rule for the sequence 4, 9, 14, 19... Then find the 21<sup>st</sup> term.

$$a_1 = 4$$

$$d = 5$$

$$a_n = 4 + (n-1)5$$

$$= 4 + 5n - 5$$

$$\boxed{a_n = 5n - 1}$$

$$a_{21} = 5(21) - 1$$

$$\boxed{a_{21} = 104}$$

2) Write a rule for the  $n$ th term of the arithmetic sequence 59, 68, 77, 86, ...  
Then find  $a_9$ .

$$a_1 = 59$$

$$d = 9$$

$$a_n = 59 + 9(n-1)$$

$$= 59 + 9n - 9$$

$$\boxed{a_n = 9n + 50}$$

$$a_9 = 9(9) + 50$$

$$\boxed{a_9 = 131}$$

3) Write a rule for the  $n$ th term of the arithmetic sequence when  $a_1 = 7$  and the common difference  $d = -3$ .

$$a_n = 7 + (-3)(n-1)$$

$$= 7 - 3n + 3$$

$$\boxed{a_n = -3n + 10}$$

4) Write the rule for the  $n$ th term of the arithmetic sequence if  $a_7 = 18$  and the common difference is  $-2$ .

\* must find  $a_1$  first!

$$a_1 = 30$$

$$d = -2$$

$$18 = a_1 + (-2)(7-1)$$

$$18 = a_1 + (-2)(6)$$

$$18 = a_1 - 12$$

$$+12 \quad +12$$

$$30 = a_1$$

$$a_n = 30 + (-2)(n-1)$$

$$= 30 - 2n + 2$$

$$\boxed{a_n = -2n + 32}$$

5) Write the rule for the  $n$ th term of the arithmetic sequence if  $a_9 = 11$ ,  $a_{18} = 56$ .

\* find common difference first

\* then find  $a_1$  →

$$a_1 = -29$$

$$d = 5$$

$$\frac{56 - 11}{18 - 9} = \frac{45}{9} = 5$$

$$11 = a_1 + 5(9-1)$$

$$11 = a_1 + 5(8)$$

$$11 = a_1 + 40$$

$$-40 \quad -40$$

$$-29 = a_1$$

$$a_n = -29 + 5(n-1)$$

$$\boxed{a_n = 5n - 34}$$

6) The first row of a concert hall has 25 seats, and each row after the first has one more seat than the row before it. There are 32 rows of seats.

a) Write a rule for the number of seats in the  $n^{\text{th}}$  row.

$$a_1 = 25$$

$$d = 1$$

$$a_n = 25 + 1(n-1)$$

$$= 25 + n - 1$$

$$\boxed{a_n = n + 24}$$

b) Thirty five students from a class want to sit in the same row. How close to the front can they sit?

$n = \text{The row \#}$

$$35 = n + 24$$

$$\begin{array}{r} -24 \quad -24 \\ \hline \end{array}$$

$$11 = n$$

$$\boxed{\text{The } 11^{\text{th}} \text{ row}}$$

7) Fred is getting better at math. On his first quiz he scored 57 points, 61 points on his second quiz and then 65 on his third quiz. If his scores continued to increase at the same rate, what will be his score on his 9<sup>th</sup> quiz? Show all work.

$$a_1 = 57$$

$$\begin{array}{r} 57, 61, 65 \\ \quad \vee \quad \vee \\ \quad +4 \quad +4 \end{array}$$

$$d = 4$$

$$a_9 = 57 + 4(9-1)$$

$$a_9 = 57 + 4(8)$$

$$a_9 = 89$$

$$\boxed{89 \text{ points}}$$

Sum of Finite Arithmetic Series: The sum of the first  $n$  terms of an arithmetic series is denoted by  $S_n$ .

ARITHMETIC SUM

$$S_n = n \left( \frac{a_1 + a_n}{2} \right)$$

8) Evaluate.  $\sum_{n=1}^{25} (3n+1)$

$n=25$

$a_1 = 3(1)+1 = 4$

$a_{25} = 3(25)+1 = 76$

$S_{25} = 25 \left( \frac{4+76}{2} \right)$   
 $= \boxed{1000}$

9) Evaluate.  $\sum_{n=3}^{20} (n+5)$

change problem

$n = 20 - 3 = 17$

$a_3 = 3 + 5 = 8$

$a_{20} = 20 + 5 = 25$

$S_{17} = 17 \left( \frac{8+25}{2} \right)$

$= \boxed{280.5}$

10) Find the sum of the first 18 terms of the arithmetic series  $100 + 110 + 120 + 130 + \dots$

$n=18$

$a_{18} = 100 + 10(18-1)$

$+10 \quad +10$

$a_1 = 100$

$= 100 + 10(17)$

$S_{18} = 18 \left( \frac{100+270}{2} \right)$

$a_{18} = ?$

$a_{18} = 270$

$= \boxed{3330}$

11) Find the sum of the first 10 terms of the arithmetic series  $2 + 5 + 8 + 11 + 14 + 17 + \dots$

$n=10$

$a_{10} = 2 + 3(10-1)$

$+3 \quad +3$

$a_1 = 2$

$= 2 + 3(9)$

$S_{10} = 10 \left( \frac{2+29}{2} \right)$

$a_{10} = ?$

$= 29$

$= \boxed{155}$