11.6 Geometric Sequences and Series

multiplication

Geometric Sequence: A sequence in which the ratio of any term to the previous term is constant.

$$\frac{a_2}{a_1} = r \qquad \frac{a_3}{a_2} = r \qquad \frac{a_4}{a_3} = r \ \ldots$$

Common Ratio: denoted by the letter r.

Determine if each sequence is geometric. If so, find the common ratio, r.

32, 16, 8, 4, 2 .... (Y) / N r= 1/2

·1/2 ·1/2 ·1/2

12, 8, 4, 0, -4, -8,..... Y / (N) r= ____

-4 -4

1, 3, 9, 27, 81, ... (Y) / N r= 3

·3 ·3

8, 4, 2, 1, ½,... (Y) / N r= 1/2

×1/2 ·1/2

Writing a Rule for a Geometric Sequence

The n th term of a geometric sequence with first term a_1 and common ratio r.

Rule for a Geometric Sequence

$$a_n = a_1 r^{n-1}$$

·1/2

1) Write a rule for the geometric sequence: 32, 16, 8, 4, 2... Then find the 8th term.

·1/2

$a_1 = 32$

$r = \frac{1}{2}$

$$a_n = 32\left(\frac{1}{2}\right)^{n-1}$$

$a_8 = 32\left(\frac{1}{2}\right)^{8-1}$

$= .25$ or $\frac{1}{4}$

$$a_n = a_1 (r)^{n-1}$$

2) Write a rule for the nth term of the geometric sequence when $a_4 = 54$ and the common ratio is $r = 3$.

① Find a_1

$a_1 =$

$r = 3$

$54 = a_1 (3)^{4-1}$

$\frac{54}{27} = \frac{a_1 (27)}{27}$

$a_1 = 2$

② $\boxed{a_n = 2(3)^{n-1}}$

3) Write the rule for the nth term of a geometric sequence: 5, 20, 80, 320... (×4)

$a_1 = 5$

$r = 4$

$\boxed{a_n = 5(4)^{n-1}}$

Sum of Finite Geometric Series: The sum of the first n terms of a geometric series is denoted by S_n.

GEOMETRIC SUM $\quad S_n = a_1 \left(\frac{1 - r^n}{1 - r} \right)$

4) Evaluate. $\sum_{n=1}^{20} 5(2)^{n-1}$

$a_1 = 5$

$n = 20$

$r = 2$

$S_{20} = 5 \left(\frac{1 - 2^{20}}{1 - 2} \right)$

$= \boxed{5,242,875}$

5) Find the sum of the first 10 terms of the geometric series 3 + 9 + 27 + 81 + ... (×3)

$a_1 = 3$

$n = 10$

$r = 3$

$S_{10} = 3 \left(\frac{1 - 3^{10}}{1 - 3} \right)$

$= 88572$