11.7 Infinite Geometric Series

The fifth term of a geometric series is $\frac{1}{2}$ and the common ratio is $\frac{1}{2}$. Write the rule for the n^{th} term.

$a_5 = \frac{1}{2}$

$r = \frac{1}{2}$

$$\frac{1}{2} = a_1 \left(\frac{1}{2}\right)^{5-1}$$

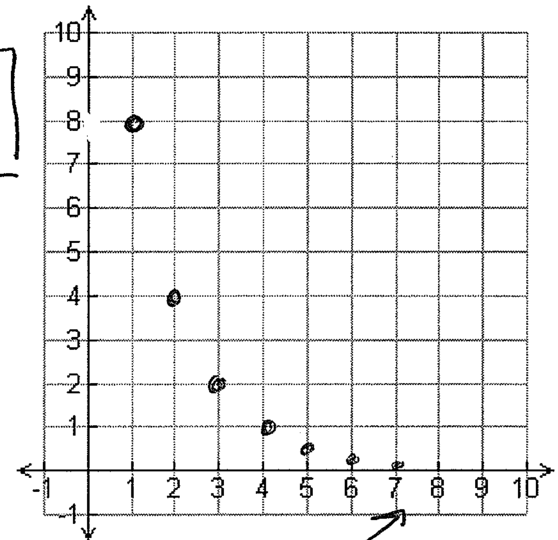
$$\frac{\frac{1}{2}}{.0625} = \frac{a_1 (.0625)}{.0625}$$

$$8 = a_1$$

$$\boxed{a_n = 8\left(\frac{1}{2}\right)^{n-1}}$$

Write the first 7 terms of the sequence. Then graph the sequence.

$$8, 4, 2, 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}$$



numbers get really, really small

Find the sum of the first 15 terms of the geometric series.

$n = 15$

$a_1 = 8$

$r = 1/2$

$$S_{15} = 8\left(\frac{1-\left(\frac{1}{2}\right)^{15}}{1-\left(\frac{1}{2}\right)}\right) = 15.9995 \approx 16$$

Now, find the sum of the first 100 terms of the geometric series. Sum of the first 500 terms?

$$S_{100} = 8\left(\frac{1-\left(\frac{1}{2}\right)^{100}}{1-\left(\frac{1}{2}\right)}\right)$$

$$= 16$$

$$S_{500} = 8\left(\frac{1-\left(\frac{1}{2}\right)^{500}}{1-\left(\frac{1}{2}\right)}\right)$$

$$= 16$$

* as n gets larger the sum approaches 16

You can find the partial sum, S_n, of the first n terms of an infinite geometric series by adding the terms. However, some partial sums may approach a certain value.

The sum of an infinite series with first term a_1 and common ration r is given by:

$$S = \frac{a_1}{1-r} \quad \text{where } |r| < 1$$

If $|r|>1$. the infinite series has no sum

1. Find the sum of the infinite geometric series, if it exists.

a. $a_1 = 1,\ r = -\frac{1}{2}$

$S = \frac{1}{1-(-1/2)} = \boxed{\frac{2}{3}}$

b. $2 + \frac{2}{3} + \frac{2}{9} + \frac{2}{27} + \cdots$

$a_1 = 2$

$r = 1/3$

$S = \frac{2}{1-\left(\frac{1}{3}\right)} = \boxed{3}$

c. $a_1 = 5,\ r = 2$

sum does not exist

d. $3 - \frac{3}{4} + \frac{3}{16} - \frac{3}{64} + \cdots$

$a_1 = 3$

$r = -\frac{1}{4}$

$S = \frac{3}{1-\left(-\frac{1}{4}\right)} = \boxed{2.4}$

2. What is the common ratio of the infinite geometric series with the given information?

a. $S = 8,\ a_1 = 3.$

$\frac{8}{1} = \frac{3}{1-r}$

$8(1-r) = 3$

$8 - 8r = 3$
$-8 \qquad -8$

$-8r = -5$

$\boxed{r = .625}$

b. $S = 6,\ a_1 = 2$

$6 = \frac{2}{1-r}$

$6(1-r) = 2$

$6 - 6r = 2$
$-6 \qquad -6$

$\frac{-6r}{-6} = \frac{-4}{-6}$

$\boxed{r = 2/3}$