Intermediate Algebra
Chapter 11 Review

Name: Key

$$a_n = a_1 + d(n-1) \qquad a_n = a_1(r)^{n-1}$$

$$S = n\left(\frac{a_1 + a_n}{2}\right) \qquad S = a_1\left(\frac{1-r^n}{1-r}\right) \qquad S = \frac{a_1}{1-r}$$

1) Determine whether the sequence is arithmetic, geometric, or neither. **Explain** your reasoning using full sentences.

a) 4, 8, 16, 32,... (×2, ×2, ×2) geometric – b/c it is multiplied by a common ratio each term

b) 6, 12, 18, 24, ..., (+6, +6) arithmetic – b/c you are adding a constant to each term

c) 4, 10, 18, 28, 40, ... neither – There is no common ratio or common difference

2) Write the first four terms of the sequence, starting with n = 1.

a) $a_n = 2^n + 1$

$a_1 = 2^1 + 1 = 3$
$a_2 = 2^2 + 1 = 5$
$a_3 = 2^3 + 1 = 9$
$a_4 = 2^4 + 1 = 17$

b) $a_n = -3n + 7$

$a_1 = -3(1) + 7 = 4$
$a_2 = -3(2) + 7 = 1$
$a_3 = -3(3) + 7 = -2$
$a_4 = -3(4) + 7 = -5$

3) Write the formula that would give the following sequences, then find the 100th term.

a) 23, 27, 31, 35, ... (+4, +4) arithmetic

$d = 4$
$a_1 = 23$

$a_n = 23 + 4(n-1)$
$= 23 + 4n - 4$
$a_n = 4n + 19$

$a_{100} = 4(100) + 19 = 419$

b) 5, 15, 45, 135, ... (×3, ×3) geometric

$r = 3$
$a_1 = 5$

$a_n = 5(3)^{n-1}$

$a_{100} = 5(3)^{100-1}$
$= 8.59 \times 10^{47}$ (scientific notation)

4) The 4th term of an arithmetic sequence is 15 and the 9th term of the sequence is 45. Find the common difference.

$a_4 = 15$

$a_9 = 45$

$d = \frac{45-15}{9-4} = \frac{30}{5} = \boxed{6}$

5) The 8th term of an arithmetic sequence is 35 and common difference is 3. Find the formula that would generate this sequence.

$a_8 = 35$

$d = 3$

arithmetic

$35 = a_1 + 3(8-1)$

$35 = a_1 + 21$

$-21 \quad -21$

$14 = a_1$

$a_n = 14 + 3(n-1)$

$= 14 + 3n - 3$

$\boxed{a_n = 3n + 11}$

6) Write the formula that would generate the following sequence: 80, 20, 5, $\frac{5}{4}$, $\frac{5}{16}$ ...

$\times\frac{1}{4}$ $\times\frac{1}{4}$

geometric

$r = \frac{1}{4}$

$a_1 = 80$

$\boxed{a_n = 80\left(\frac{1}{4}\right)^{n-1}}$

7) Write the formula that would generate the sequence with $a_2=10$ and $r=2$.

geometric

① find a_1

$10 = a_1(2)^{2-1}$

$\frac{10}{2} = \frac{a_1(2)}{2}$

$5 = a_1$

② write the rule

$\boxed{a_n = 5(2)^{n-1}}$

8) Find the sum of the first 60 terms of the series: 8 + 14 + 20 + 26 + ...

arithmetic

$d = 6$

$a_1 = 8$

$a_{60} = 8 + 6(60-1) = 362$

$+6 \quad +6$

$S_{60} = 60\left(\frac{8+362}{2}\right)$

$= \boxed{11,100}$

arithmetic

9) $\sum_{n=1}^{50} 7n-3$

$n=50$

$a_1 = 7(1)-3 = 4$

$a_{50} = 7(50)-3 = 347$

$S_{50} = 50\left(\frac{4+347}{2}\right)$

$= \boxed{8,775}$

10) Find the sum of the first 10 terms of the series (Use a formula!): $3 + 12 + 48 + 192 + \ldots$

×4 ×4

geometric

$r = 4$

$n = 10$

$a_1 = 3$

$S_{10} = 3\left(\frac{1-4^{10}}{1-4}\right)$

$= \boxed{1,048,575}$

11) Find the sum of the infinite series (if it exists): $90 + 30 + 10 + \frac{10}{3} + \ldots$

×$\frac{1}{3}$

$r = \frac{1}{3}$

$a_1 = 90$

$S = \frac{90}{1-1/3} = \boxed{135}$

12) Evaluate the sum of the infinite geometric series, if it exists.

a) $\sum_{n=1}^{\infty} 5(0.2)^{n-1}$

infinite

$r = .2$

$a_1 = 5$

$S = \frac{5}{1-.2}$

$= \boxed{6.25}$

b) $\sum_{n=1}^{\infty} 6(1.5)^{n-1}$

infinite

$r = 1.5$

$a_1 = 6$

Sum does not exist b/c $|r| > 1$

13) You are trying to find the maximum weight you can lift in a weightlifting excersie. You start with a single lift of 125 pounds. Then you increase the weight by 2 pounds and try again. You repeat this procedure until you reach a weight that you are unable to lift.

arithmetic

a. Write a rule for the total weight of your *nth* lifting attempt.

$d=2$

$a_1=125$

$a_n = 125 + 2(n-1)$

$\boxed{a_n = 2n + 123}$

b. You are unable to lift the weight on your sixth lift. So, based on your fifth lift, what is the maximum amount of weight you can lift in this exercise?

$a_5 = 2(5) + 123$

$= \boxed{133 \text{ lbs}}$

c. Find the sum of the weights lifted in your five successful lifts.

$a_1 = 125$

$a_5 = 133$

$S_5 = 5\left(\frac{125+133}{2}\right) = \boxed{645}$

14) A theater ~~has 18 seats in the first~~ row, 21 seats in the second row, 24 seats in the third row and so on and 105 seats in the last row. Find the total seating capacity of the theater.

arithmetic

what row is this?? solve for n!

sum

$d=3$

$a_1=18$

$n=30$

$a_{30}=105$

$105 = 18 + 3(n-1)$

$-18 \quad -18$

$\frac{87}{3} = \frac{3(n-1)}{3}$

$29 = n-1$

$+1 \quad +1$

$n=30$

$S_{30} = 30\left(\frac{18+105}{2}\right)$

$= \boxed{1845 \text{ seats}}$