

Solve the rational equations.

1.  $\frac{5x}{x-2} = \frac{10}{x-2} + \frac{7(x-a)}{x-a}$

$5x = 10 + 7(x-a)$

$5x = 10 + 7x - 14$

$5x = 7x - 4$

$-7x = -7x$   
 $\frac{-2x}{-2} = \frac{-4}{-2}$

$x = 2$

no solution

3.  ~~$\frac{2}{x-3} = \frac{3}{x+1}$~~

$2(x+1) = 3(x-3)$

$2x + 2 = 3x - 9$   
 $-2x + 9 = -2x + 9$

$11 = x$

$x = 11$

2.  ~~$\frac{x-5}{-3} = \frac{4}{x+2}$~~

$(x-5)(x+2) = -12$

$x^2 - 3x - 10 = -12$   
 $+12 \quad +12$

$x^2 - 3x + 2 = 0$

$(x-2)(x-1) = 0$

$x = 2, 1$

4.  ~~$\frac{4(x+3)}{x-3} = \frac{2(x-3)}{x+3} + \frac{2x+2}{x^2-9}$~~   
 $(x-3)(x+3)$

$4(x+3) + 2(x-3) = 2x + 2$

$4x + 12 + 2x - 6 = 2x + 2$

$6x + 6 = 2x + 2$   
 $-2x \quad -6 \quad -2x \quad -6$

$4x = -4$

$x = -1$

5. Given  $f(x) = \frac{x-2}{6}$  and  $g(x) = \frac{x-2}{x-1}$ , for what value(s) of  $x$  are  $f(x) = g(x)$ ? Show the algebraic work supporting your answer.

Set them equal to each other

~~$\frac{x-2}{6} = \frac{x-2}{x-1}$~~

$(x-2)(x-1) = 6(x-2)$

$x^2 - 3x + 2 = 6x - 12$   
 $-6x + 12 \quad -6x + 12$

$x^2 - 9x + 14 = 0$

$(x-7)(x-2) = 0$

$x = 7$   
and  
 $x = 2$

Find the key information and sketch the graph.

$$6. f(x) = \frac{-1}{x+3}$$

Vertical asymptote:  $x = -3$

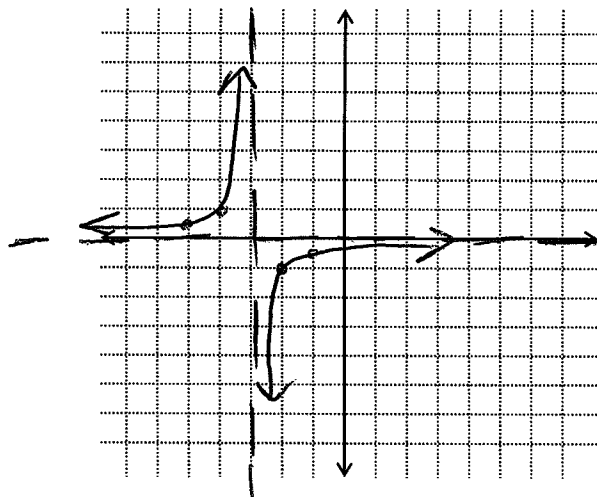
Domain:  $(-\infty, -3) \cup (-3, \infty)$

Horizontal asymptote:  $y = 0$

Range:  $(-\infty, 0) \cup (0, \infty)$

Reflected? yes

y-intercept:  $(0, -1/3)$



$$y = \frac{-1}{x}$$

x	y
-2	1/2
-1	1
0	und
1	-1
2	-1/2

Find the key information and sketch the graph.

$$7. f(x) = \frac{2}{x-1} + 3$$

Vertical asymptote:  $x = 1$

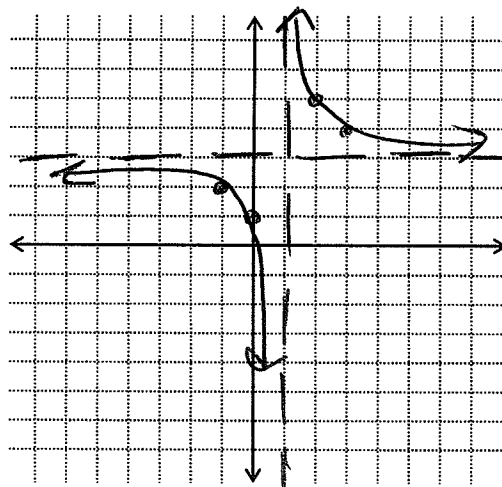
Domain:  $(-\infty, 1) \cup (1, \infty)$

Horizontal asymptote:  $y = 3$

Range:  $(-\infty, 3) \cup (3, \infty)$

Reflected? no

y-intercept:  $(0, 1)$



$$y = \frac{2}{x}$$

x	y
-2	-1
-1	-2
0	und
1	2
2	1

Manipulate the function into the transformation form. Determine the information.

$$8. f(x) = \frac{x+7}{x-2}$$

$$x-2 \left| \begin{array}{r} 1 + \frac{9}{x-2} \\ x+7 \\ -(x-2) \\ \hline 9 \end{array} \right.$$

Transformation form:

$$f(x) = \frac{9}{x-2} + 1$$

VA:  $x = 2$

HA:  $y = 1$

x-int:  $(-7, 0)$

y-int:  $(0, -7/2)$

$$9. f(x) = \frac{4x-2}{x+1}$$

$$x+1 \overline{) \begin{array}{r} 4 - \frac{6}{x+1} \\ 4x-2 \\ -(4x+4) \\ \hline 0-6 \end{array}}$$

Transformation form:

$$f(x) = -\frac{6}{x+1} + 4$$

VA:  $x = -1$       HA:  $y = 4$   
 x-int:  $(\frac{1}{2}, 0)$       y-int:  $(0, -2)$

Manipulate the function into standard form.

$$10. f(x) = \frac{3}{x+1} + \frac{4(x+1)}{1(x+1)}$$

$$f(x) = \frac{4x+7}{x+1}$$

$$11. f(x) = \frac{2}{x-3} - \frac{1(x-3)}{1(x-3)}$$

$$= \frac{2 - 1(x-3)}{x-3}$$

$$f(x) = \frac{-x+5}{x-3}$$

12. Find all of the key information of the function:  $f(x) = \frac{4x-8}{2x+1}$

Vertical asymptote:  $x = -\frac{1}{2}$

Horizontal asymptote:  $y = 2$

x-intercept:  $(2, 0)$

Domain:  $(-\infty, -\frac{1}{2}) \cup (-\frac{1}{2}, \infty)$

Range:  $(-\infty, 2) \cup (2, \infty)$

y-intercept:  $(0, -8)$

13. Compare the functions. Find the key information and write a paragraph describing similarities and differences between the two graphs. Show work! (Hint: manipulate the forms to allow easy comparison!)

$$f(x) = \frac{3}{x+1} + 2$$

$$g(x) = \frac{2x+7}{x+1}$$

$$\begin{array}{r} 2 + \frac{5}{x+1} \\ x+1 \overline{) \begin{array}{r} 2x+7 \\ -(2x+2) \\ \hline 0+5 \end{array}} \end{array}$$

$$g(x) = \frac{5}{x+1} + 2$$

- both have a vertical asymptote at  $x = -1$
- both have a horizontal asy at  $y = 2$
- both have a vertical stretch, one by 3 and one by 5.
- different x-int and y-int

16. The population density in a large city is related to the distance from the center of the city. It can be modeled by  $D = \frac{5000x}{x^2 + 36}$  where D is the population density (in people per square mile) and x is the distance (in miles) from the center of the city.

a) Find the population density 10 miles from the center of the city.

$$D(10) = \frac{5000}{(10)^2 + 36} = 367.65 \approx \boxed{368 \text{ people}}$$

b) Find the population density 20 miles from the center of the city.

$$D(20) = \frac{5000(20)}{(20)^2 + 36} = 229.36 \approx \boxed{230 \text{ people}}$$

~~c) When is the population density 400 people per square mile? Use calculator to find intersection~~  
 ~~$400 = \frac{5000x}{x^2 + 36}$~~   
 ~~$4x = 400$~~

d.) What is the average rate of change in the population density as the number of miles from the center increases from 10 miles to 20 miles?  
 From 10 to 20 miles the population density decreases on average 13.8 people per mile.

$$D(10) = 368$$

$$D(20) = 230$$

$$\frac{230 - 368}{20 - 10} = \boxed{-13.8 \text{ people/mile}}$$

17. Suppose the cost per ton, C(x), to build an oil platform of x thousand tons is approximated by

$$C(x) = \frac{312500}{x + 625}$$

a) What is the cost per ton for 30 thousand tons?

$$C(30) = \frac{312500}{30 + 625} = \boxed{\$477.10}$$

b) What is the cost per ton for 50 thousand tons?

$$C(50) = \frac{312500}{50 + 625} = \boxed{\$462.96}$$

c) How many thousand tons was the platform if the cost per ton was \$483?

$$483 = \frac{312500}{x + 625} \quad \text{Ans.} \approx \boxed{22 \text{ thousand tons}}$$

d) What is the average rate of change in the cost per ton as the number of tons increases from 30 thousand tons to 50 thousand tons?

$$C(30) = 477.10$$

$$C(50) = 462.96$$

$$\frac{462.96 - 477.10}{50 - 30} = \boxed{-0.707 \text{ dollars per ton}}$$

e) What is the horizontal asymptote and what does it mean in the context of this problem?

horiz asy:  $y = 0$

The cost approaches zero

Perform the indicated operation.

$$18) \frac{3x^2y^3 \cdot 25y^4}{10xy^5 \cdot 12xy}$$

$$\frac{75x^2y^7}{120x^2y^6}$$

$$\boxed{\frac{5y}{8}}$$

$$19) \frac{x^2 - 9}{x^2 + 4x - 21} \cdot \frac{x^2 - 2x - 35}{x^2 + 8x + 15}$$

$$\frac{\cancel{(x-3)}\cancel{(x+3)}}{(x+7)\cancel{(x-3)}} \cdot \frac{(x-7)\cancel{(x+5)}}{\cancel{(x+3)}(x+5)}$$

$$\boxed{\frac{x-7}{x+7}}$$

$$20) \frac{x^2 - 4}{x^2 + x - 6} \div \frac{x^2 - 2x - 8}{x + 3}$$

Keep, change, flip

$$\frac{\cancel{(x-2)}\cancel{(x+2)}}{(\cancel{x+3})\cancel{(x-2)}} \cdot \frac{\cancel{x+3}}{(x-4)\cancel{(x+2)}}$$

$$\boxed{\frac{1}{x-4}}$$