8.4 Logarithms and Logarithmic Functions

Find x in the following problems.

a) $2^x = 4$ $x = 2$

b) $2^x = .5$ $x = -1$

c) $2^x = 8$ $x = 3$

d) $2^x = 6$ $x = ?$

Let b and y be positive numbers, $b \neq 1$.

The *logarithm of y with base b* is denoted by $\log_b y$ and is defined as follows:

$$\log_b y = x \quad \text{if and only if} \quad b^x = y$$

Converting Logarithmic Equations and Exponential Equations

$b^x = a$ ⟷ $\log_b a = x$

Exponential Form | Logarithmic Form

Rewrite the equation in either exponential or logarithmic form.

Exponential Form	Logarithmic Form
$2^5 = 32$	$\log_2 32 = 5$
$6^{-2} = \frac{1}{36}$	$\log_6 \frac{1}{36} = -2$
$10^4 = 10{,}000$	$\log_{10} 10{,}000 = 4$
$5^3 = 125$	$\log_5 125 = 3$
$7^0 = 1$	$\log_7 1 = 0$
$4^0 = 1$	$\log_4 1 = 0$

Evaluating Logarithmic Expressions

a) $\log_{10} 100 =$ 2

$10^x = 100$

b) $\log_5 \frac{1}{5} =$ -1

$5^x = \frac{1}{5}$

c) $\log_{15} 1 =$ 0

$15^x = 1$

d) $\log_3 \frac{1}{27} =$ -3

$3^x = \frac{1}{27}$

e) $\log_4(-64) =$ does not exist

$4^x = -64$

f) $\log_2 8 =$ 3

$2^x = 8$

Common Logarithm

$\log_{10}$

↑

base 10

Natural Logarithm

$\log_e \rightarrow \ln$

Use your calculator to evaluate each expression. *Round to 3 decimal places.*

a) $\log 7 \approx$ .845

↑

base 10

b) $\log 0.25 \approx$ $-.602$

↑

base 10

c) $\ln 2 \approx$ .693

$\log_e 2$

Change of Base:

$$\log_a a = \frac{\log_{10} a}{\log_{10} b}$$

← change to base 10 so you can use calculator

a) $\log_3 81 =$ 4

$\frac{\log 81}{\log 3}$

b) $\log_{1/4} 256 =$ -4

$\frac{\log 256}{\log(.25)}$

c) $\log_{10} 0.001 =$ -3

↑

base 10

$\frac{\log(.001)}{\log 10}$