Intermediate Algebra
7.5 Inverse Functions

Name: _____

Inverse Functions

Relation: a pairing between input and output values.

example

input/ domain →

x	1	2	3
y	2	4	6

output/ range →

Inverse Relation: switches input and output values

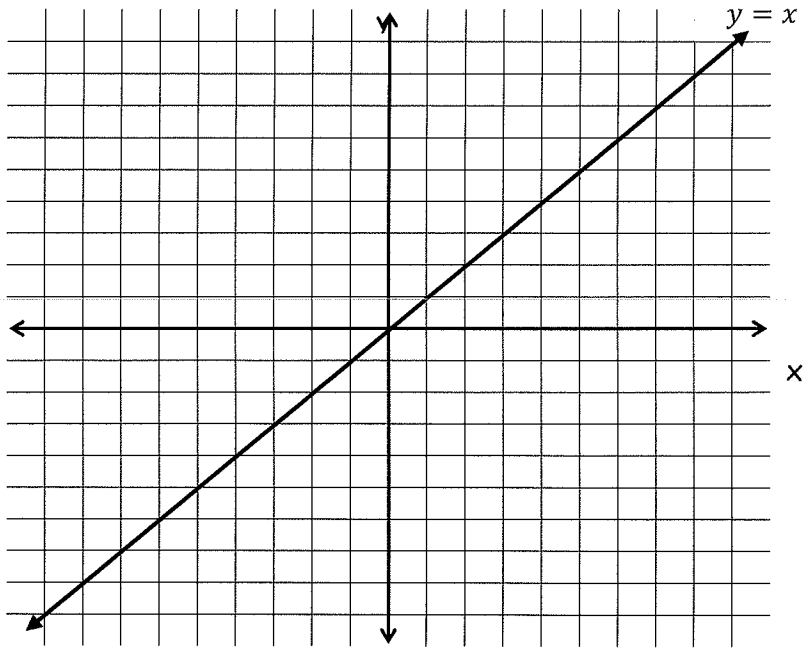
→ switch x and y!!

Original Relation

Input (x)	-2	-1	0	1	2	3
Output (y)	-9	-7	-5	-3	-1	1

Inverse Relation

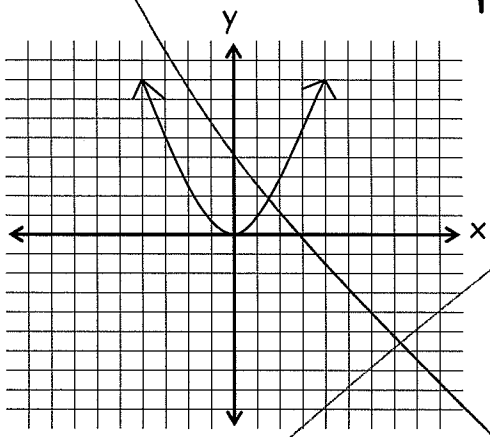
Input (x)	-9	-7	-5	-3	-1	1
Output (y)	-2	-1	0	1	2	3



Vertical Line Test:

Skipped

Ex.



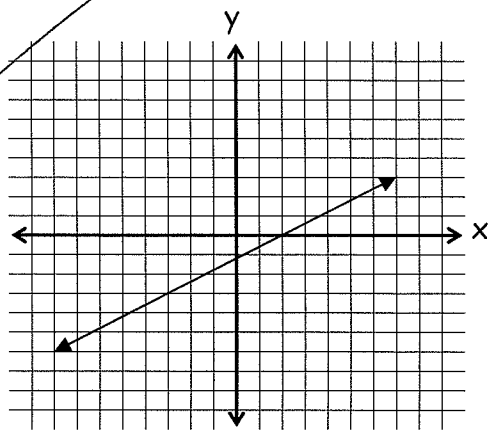
Is the graph shown a function?

Is the inverse a function?

How do you know?

Horizontal Line Test:

Ex.



Is the graph shown a function?

Is the inverse a function?

How do you know?

Verifying Inverse Functions.

Functions f and g are inverse functions if:

$f(g(x)) = x$ and $g(f(x)) = x$

composition

*The inverse of f is often denoted by f^{-1}

Example 1:

Verify that $f(x) = 2x - 5$ and $g(x) = \frac{1}{2}x + \frac{5}{2}$ are inverse functions

$f(g(x)) = 2\left(\frac{1}{2}x + \frac{5}{2}\right) - 5$

$= x + 5 - 5$

$= x$

$g(f(x)) = \frac{1}{2}(2x - 5) + \frac{5}{2}$

$= x - \frac{5}{2} + \frac{5}{2}$

$= x$

Yes, they are inverses.

Example 2:

Verify that $f(x) = x + 3$ and $g(x) = x - 3$ are inverse functions.

$f(g(x)) = (x-3)+3$

$= x-3+3$

$= x$

$g(f(x)) = (x+3)-3$

$= x+3-3$

$= x$

Yes! They are inverses.

Now your turn!

1. Verify that $f(x) = 2x - 4$ and $g(x) = \frac{1}{2}x + 2$ are inverse functions.

$f(g(x)) = 2\left(\frac{1}{2}x+2\right)-4$

$= x+4-4$

$= x$

$g(f(x)) = \frac{1}{2}(2x-4)+2$

$= x-2+2$

$= x$

Yes! They are inverses.

Finding Inverse Functions.

1. Rewrite f(x) as y
2. Switch x and y
3. Solve for y

Examples:

1. Find the inverse of $f(x) = 3x - 6$.

$y = 3x-6$

Switch x and y → $x = 3y-6$

$+6 \quad +6$

$\frac{x+6}{3} = \frac{3y}{3}$

$y = \frac{x+6}{3}$

$y = \frac{x}{3} + \frac{6}{3}$

$y = \frac{x}{3} + 2$

2. Find the inverse of $f(x) = 4x$

$y = 4x$

$\frac{x}{4} = \frac{4y}{4}$

$\boxed{y = \frac{x}{4}}$

3. Find the inverse of $f(x) = x^3 + 1$

$y = x^3 + 1$

$x = y^3 + 1$

$-1 \quad -1$

$\sqrt[3]{x-1} = \sqrt[3]{y^3}$

$\boxed{y = \sqrt[3]{x-1}}$

Now your turn!

4. Find the inverse of $f(x) = 3x - 2$

$y = 3x - 2$

$x = 3y - 2$

$+2 \quad +2$

$\frac{x+2}{3} = \frac{3y}{3}$

$\boxed{y = \frac{x+2}{3}}$

5. Find the inverse of $f(x) = \frac{2x+3}{5}$

$y = \frac{2x+3}{5}$

$5 \cdot x = \frac{2y+3}{5} \cdot 5$

$5x = 2y + 3$

$-3 \quad -3$

$\frac{5x-3}{2} = \frac{2y}{2}$

$\boxed{y = \frac{5x-3}{2}}$