Linear Algebra Curricular Document

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Linear Algebra Course Information	
CURRICULUM/CONTENT AREA	COURSE LENGTH
Math	.5
GRADE LEVEL	DATE LAST REVIEWED
9-12	New dual credit course for 2024
PREREQUISITE(s) *if applicable*	BOARD APPROVAL DATE
B or higher in Calculus BC and/or Calculus III	January 16, 2024
PRIMARY RESOURCE *if applicable*	
Linear Algebra & Its Applications by Stephen Lay, 6th edition	

Desired Results

Linear algebra is the study of vector spaces and the linear transformations between such spaces. In this class we will work mostly with the vector space $\mathbb{R}^n$. We will learn how to represent linear transformations with matrices and study different types of linear transformations, such as diagonalizable transformations.

Linear algebra is very important for both pure and applied mathematics. The techniques of linear algebra are used in engineering, physics, natural sciences, computer science and economics. For example, when we combine calculus with linear algebra we can solve linear systems of differential equations.

This course may be taken for dual credit from a university partner.

Goals of the course:

- Students will learn the main concepts and techniques in linear algebra.
- Students will learn some applications of linear algebra.
- Students will be prepared for more advanced courses in mathematics, computer science, physics and any other subjects that require linear algebra.

LEARNING OBJECTIVES/STANDARDS	ESSENTIAL QUESTIONS
Objective 1: Vectors and Vector Operations. Algebraically manipulate vectors and vector equations. Linear combinations. Objective 2: Matrices and Linear Transformations. Algebraically manipulate matrices and abstract linear transformations. Common operations on matrices. The matrix corresponding to a linear transformation. Objective 3: Systems of Linear Equations. Solve systems of linear equations using Gaussian Elimination. Recognize solution sets of linear systems. Objective 4: Matrix Inverses. Compute the inverse of a given matrix. Characterize the invertibility of a matrix in terms of solutions to linear systems. Understand the effects of elementary row operations. Objective 5: Vector Spaces. Definitions of vector spaces, subspaces, spans, linear (in)dependence, bases, and dimension. Objective 6: Rank and Nullity. Compute the column and null space of a matrix. The rank-nullity theorem and applications. Objective 7: Coordinate Systems. Representations of vectors, change-of-basis matrices, similarity. Objective 8: Determinants. Definitions and basic properties of determinants. Compute determinants of matrices to solve linear systems. Objective 9: Eigenvalues and Eigenvectors. Definitions and basic computations. The characteristic polynomial. Geometric and algebraic multiplicity. Objective 10: Diagonalization. Diagonalization of a matrix. Relation to similarity. Canonical forms.	1. What strategies and tools transcend all mathematical problems, and how can I apply those strategies/tools in unique settings? 2. How can I use mathematics, specifically Linear Algebra to make sense of the physical world? 3. How can mathematics be used to provide models that help us interpret data and make predictions? 4. What is a mathematically precise solution? 5. How are mathematical concepts related? 6. How will the topics of Linear Algebra be applied to real life applications?

UNIT 1: Linear Equations in Linear Algebra

UNIT 1 DESIRED RESULTS

ESSENTIAL QUESTION:

1. What strategies and tools transcend all mathematical problems, and how can I apply those strategies/tools in unique settings?
2. How can I use mathematics, specifically Linear Algebra to make sense of the physical world?
3. How can mathematics be used to provide models that help us interpret data and make predictions?
4. What is a mathematically precise solution?
5. How are mathematical concepts related?
6. How will the topics of Linear Algebra be applied to real life applications?

LEARNING TARGETS: *Students will know and be able to…*

1.1 Systems of Linear Equations

- ➔ Use elementary row operations to solve systems of linear equations
- ➔ Determine if a system of linear equations is consistent
- ➔ Determine the conditions for which a linear system is consistent
- ➔ Determine the validity of statements about systems of linear equations, row operations or matrices
- ➔ Identify the row operation that transforms one matrix into another matrix
- ➔ Solve applications involving systems of linear equations

1.2 Row Reduction and Echelon Forms

- ➔ Identify matrices in echelon form and reduced echelon form
- ➔ Row reduce matrices to reduced echelon form
- ➔ Find the general solution to a system with a given augmented matrix
- ➔ Determine if a system is consistent given a description of the corresponding coefficient matrix
- ➔ Determine the conditions for which a linear system has specified types of solutions
- ➔ Determine the validity of statements about row reduction and echelon forms
- ➔ Find interpolating polynomials for given data

1.3 Vector Equations

- ➔ Compute sums and scalar products of vectors, both algebraically and geometrically
- ➔ Convert between vector equations and systems of equations
- ➔ Determine if a vector is a linear combination of other vectors
- ➔ Characterize the span of a set of vectors algebraically or geometrically
- ➔ Determine the validity of statements about vectors and vector equations
- ➔ Solve applications involving vector equations.

1.4 The Matrix Equation Ax=b

- ➔ Compute the product of a matrix and a vector

- ➔ Convert between matrix equations, vector equations and systems of equations
- ➔ Solve matrix equations using augmented matrices
- ➔ Characterize the span of the column vectors of a matrix
- ➔ Determine whether a matrix equation has no, one or many solutions
- ➔ Solve problems involving the properties of the matrix equation Ax=b
- ➔ Determine the validity of statements about vectors and vector equations

1.5 Solution Sets of Linear Systems

- ➔ Determine if a system of equations has a nontrivial solution
- ➔ Solve a system of equations or a matrix equation and write the solution in parametric form
- ➔ Describe the solution sets of systems of equations geometrically
- ➔ Find the parametric equation of a line
- ➔ Determine the validity of statements about solution sets of linear equations
- ➔ Determine if a matrix equation has a unique or nontrivial solution and if nontrivial, find the solutions to the matrix equation Ax=0 by inspection
- ➔ Find matrix A such that the matrix equation Ax=0 has an indicated solution set

1.7 Linear Independence

- ➔ Determine if a set of vectors is linearly independent and determine if a vector is in a given span.
- ➔ Determine conditions for which vectors are linearly independent or have a given span.
- ➔ Determine the validity of statements about linear independence.
- ➔ Describe possible echelon forms of a matrix given information about the independence of the columns.
- ➔ Find nontrivial solutions of the matrix equation Ax=0 that has given characteristics.
- ➔ Determine the conditions for which matrix equations have a specified solution.

1.8 Introduction to Linear Transformations

- ➔ Algebraically find the image of a given vector under a linear transformation.
- ➔ Given a linear transformation T(x)=Ax, find x for a given b in the image of T.
- ➔ Determine the conditions for which a linear transformation has a given domain and codomain.
- ➔ Determine if a vector is in the range of a given linear transformation.
- ➔ Geometrically describe the image of a vector under a linear transformation.
- ➔ Use the linearity of transformations to find the images of vectors under the transformations.
- ➔ Find matrices that define given transformations.
- ➔ Determine the validity of statements about linear transformations.
- ➔ Explain or prove properties of linear and nonlinear transformations.

1.9 The Matrix of a Linear Transformation

- ➔ Find the standard matrix of a linear transformation.
- ➔ Use a graph to find the image of a given vector.
- ➔ Find vectors whose images under a linear transformation are given.
- ➔ Determine the validity of statements about properties of linear transformations.
- ➔ Determine if linear transformations are one-to-one or onto.
- ➔ Find possible echelon forms of standard matrices for linear transformations with given properties.

UNIT 1 ASSESSMENT EVIDENCE

Performance is evaluated in terms of… Students will show their learning by…

Performance Assessment Options *May include, but are not limited to the following:* ● Formative and summative assessments. ● Feedback & Scoring Rubric based on Priority Standards	Other assessment options *May include, but are not limited to the following:* ● Unit Personal Progress Checks

UNIT 2: Matrix Algebra

UNIT 2 DESIRED RESULTS

ESSENTIAL QUESTION:

1. What strategies and tools transcend all mathematical problems, and how can I apply those strategies/tools in unique settings?
2. How can I use mathematics, specifically Linear Algebra to make sense of the physical world?
3. How can mathematics be used to provide models that help us interpret data and make predictions?
4. What is a mathematically precise solution?
5. How are mathematical concepts related?
6. How will the topics of Linear Algebra be applied to real life applications?

LEARNING TARGETS *Students will know and be able to…*

2.1 Matrix Operations

- ➔ Compute sums, products, and scalar products of matrices.
- ➔ Given properties of a product of matrices and one factor, determine properties of the other factor.
- ➔ Find values or matrices such that products of matrices have given properties.
- ➔ Express a matrix as a product of matrices.
- ➔ Solve applications involving matrix operations.
- ➔ Determine the validity of statements about matrix operations.
- ➔ Given properties of a matrix, determine properties of the product of that matrix and another matrix.
- ➔ Prove theorems or derive formulas related to matrix operations.
- ➔ Evaluate and compare inner and outer products.

2.2 The Inverse of a Matrix

- ➔ Find the inverse of a 2x2 matrix using the formula.
- ➔ Use the inverse of a matrix to solve a linear system.
- ➔ Determine the validity of statements about inverses of matrices.

- ➔ Solve equations involving invertible matrices.
- ➔ Find the inverse of a matrix using row reduction.
- ➔ Construct matrices such that products involving them have given properties.
- ➔ Solve applications involving inverses of matrices.

2.3 Characterizations of Invertible Matrices

- ➔ Determine if a given matrix is invertible.
- ➔ Determine the validity of statements about characterizations of invertible matrices.
- ➔ Solve conceptual problems involving characterizations of invertible matrices.
- ➔ Solve problems involving transformations and their matrices.
- ➔ Find condition numbers of coefficient matrices.

UNIT 2 ASSESSMENT EVIDENCE

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Performance Assessment Options
May include, but are not limited to the following:

- Formative and summative assessments.
- Feedback & Scoring Rubric based on Priority Standards

Other assessment options
May include, but are not limited to the following:

- Unit Personal Progress Checks

UNIT 3: Determinants

UNIT 3 DESIRED RESULTS

ESSENTIAL QUESTION:

1. What strategies and tools transcend all mathematical problems, and how can I apply those strategies/tools in unique settings?
2. How can I use mathematics, specifically Linear Algebra to make sense of the physical world?
3. How can mathematics be used to provide models that help us interpret data and make predictions?
4. What is a mathematically precise solution?
5. How are mathematical concepts related?
6. How will the topics of Linear Algebra be applied to real life applications?

LEARNING TARGETS *Students will know and be able to…*

3.1 Introduction to Determinants

- ➔ Compute determinants using cofactor expansions.
- ➔ Compute 3x3 determinants by multiplying entries on six specific diagonals.
- ➔ Determine the effect of elementary row operations on a determinant.
- ➔ Compute determinants of elementary matrices.
- ➔ Verify properties of determinants using elementary matrices.
- ➔ Calculate determinants of scalar multiples of matrices.
- ➔ Determine the validity or reasonableness of statements about determinants.
- ➔ Relate determinants to the area of a parallelogram.

3.2 Properties of Determinants

- ➔ Identify properties of determinants.
- ➔ Find determinants by row reduction to echelon form.
- ➔ Find determinants by combining row reduction and cofactor expansion.
- ➔ Use properties of determinants to evaluate determinants.
- ➔ Use determinants to determine if a matrix is invertible or a set of vectors is linearly independent.
- ➔ Determine the validity of statements about properties of determinants.
- ➔ Verify properties of determinants for specified matrices.

3.3 Cramer's Rule, Volume and Linear Transformations

- ➔ Use Cramer's rule to compute the solutions of systems of equations.
- ➔ Compute the adjugate of a matrix and use it to find the inverse of the matrix.
- ➔ Find the area of a parallelogram or volume of a parallelepiped using matrix determinants.
- ➔ Use determinants to determine geometric properties of linear transformations.
- ➔ Derive formulas for areas and volumes using matrix determinants.

UNIT 3 ASSESSMENT EVIDENCE

Performance is evaluated in terms of… Students will show their learning by…

Performance Assessment Options

May include, but are not limited to the following:

- Formative and summative assessments.
- Feedback & Scoring Rubric based on Priority Standards

Other assessment options

May include, but are not limited to the following:

- Unit Personal Progress Checks

UNIT 4: Vector Spaces

UNIT 4 DESIRED RESULTS

ESSENTIAL QUESTION:

1. What strategies and tools transcend all mathematical problems, and how can I apply those strategies/tools in unique settings?
2. How can I use mathematics, specifically Linear Algebra to make sense of the physical world?
3. How can mathematics be used to provide models that help us interpret data and make predictions?
4. What is a mathematically precise solution?
5. How are mathematical concepts related?
6. How will the topics of Linear Algebra be applied to real life applications?

LEARNING TARGETS *Students will know and be able to…*

4.1 Vector Spaces and Subspaces

- ➔ Show that a given set is not a vector space or a subspace.
- ➔ Determine whether a given set is a subspace.
- ➔ Show that a given set is a vector space or a subspace.
- ➔ Determine whether a vector or matrix is in a given subspace.
- ➔ Given a set of vectors, find a spanning set for it or explain why it is not a vector space.
- ➔ Solve applications involving vector spaces and subspaces.
- ➔ Determine the validity of statements about vector spaces and subspaces.

4.2 Null Spaces, Column Spaces, Row Spaces, and Linear Transformations.

- ➔ Determine whether a vector is in the null or column space of a matrix.
- ➔ List vectors that span a null space.
- ➔ Determine whether a given set is a vector space.
- ➔ Find a matrix whose column space is a given set.
- ➔ Find k such that the null and column spaces of a given matrix are subspaces of R^k.
- ➔ Find a nonzero vector in the null, column, or row space of a matrix.
- ➔ Determine the validity of statements about linear transformations and null, column, and row spaces.
- ➔ Use the theory of null and column spaces to find solutions of linear systems.
- ➔ Identify and prove properties of linear transformations and null, column, and row spaces.

4.3 Linearly Independent Sets: Bases

- ➔ Determine whether a set is linearly independent or is a basis for or spans a vector space.
- ➔ Find bases for the null, column, or row space of a matrix.
- ➔ Find a basis for the span of a set of vectors.
- ➔ Determine the validity of statements about linearly independent sets and bases.
- ➔ Explain or prove statements related to linearly independent sets and bases.
- ➔ Find a basis for a subspace spanned by real-valued functions.
- ➔ Solve applications involving linearly independent sets and bases.

4.4 Coordinate Systems

- ➔ Map a coordinate vector in a basis B to a vector in the standard basis.
- ➔ Map a vector to its coordinate vector in a basis B.
- ➔ Find the change-of-coordinates matrix from a basis B to the standard basis.
- ➔ Find the coordinate vector of a polynomial in a basis B.

- ➔ Determine the validity of statements about coordinate systems.
- ➔ Use coordinate vectors to determine if a set of polynomials is linearly independent.
- ➔ Determine whether a set of polynomials forms a basis for a vector space of polynomials.
- ➔ Solve applications involving coordinate systems.

4.5 The Dimension of a Vector Space

- ➔ Find the dimension of a subspace.
- ➔ Find the dimensions of the null, column, and row spaces for a matrix.
- ➔ Determine the validity of statements about the dimensions of vector spaces.
- ➔ Find coordinate vectors in polynomial spaces.
- ➔ Use the rank theorem to find dimensions of vector spaces.
- ➔ Extend a set of vectors in R^n to a basis for R^n.

4.6 Change of Basis

- ➔ Map a coordinate vector in one base to a coordinate vector in another base.
- ➔ Identify formulas used to map coordinate vectors from one base to another base.
- ➔ Find the change-of-coordinates matrix between two nonstandard bases.
- ➔ Determine the validity of statements about the change of basis.
- ➔ Change coordinates in polynomial space.

UNIT 4 ASSESSMENT EVIDENCE

Performance is evaluated in terms of… Students will show their learning by…

Performance Assessment Options *May include, but are not limited to the following:*	Other assessment options *May include, but are not limited to the following:*
• Formative and summative assessments. • Feedback & Scoring Rubric based on Priority Standards	• Unit Personal Progress Checks

UNIT 5: Eigenvalues and Eigenvectors

UNIT 5 DESIRED RESULTS

ESSENTIAL QUESTION:

1. What strategies and tools transcend all mathematical problems, and how can I apply those strategies/tools in unique settings?
2. How can I use mathematics, specifically Linear Algebra to make sense of the physical world?
3. How can mathematics be used to provide models that help us interpret data and make predictions?

4. What is a mathematically precise solution?
5. How are mathematical concepts related?
6. How will the topics of Linear Algebra be applied to real life applications?

LEARNING TARGETS *Students will know and be able to…*

5.1 Eigenvectors and Eigenvalues
- ➔ Determine if a vector or number is an eigenvector or eigenvalue of a given matrix.
- ➔ Find a basis for the eigenspace corresponding to an eigenvalue.
- ➔ Find the eigenvalues of matrices.
- ➔ Determine the validity of statements about eigenvectors and eigenvalues.
- ➔ Answer conceptual questions involving eigenvectors and eigenvalues.
- ➔ Use eigenvalues to find images of eigenvectors under linear transformations.

5.2 The Characteristic Equation
- ➔ Find the characteristic polynomial and eigenvalues of a 2x2 and 3x3 matrix.
- ➔ Find eigenvalues of triangular matrices.
- ➔ Find the value for an element of a matrix given criteria for the eigenspace.
- ➔ Determine the validity of statements about the characteristic equation.

5.3 Diagonalization
- ➔ Use the Diagonalization Theorem to find the eigenvalues of a matrix and a basis for each eigenspace.
- ➔ Determine the validity of statements about diagonalization.
- ➔ Determine whether matrices are diagonalizable and if so, diagonalize them.

5.4 Eigenvectors and Linear Transformations
- ➔ Find and use matrices for linear transformations relative to given bases.
- ➔ Find the B-matrix for a transformation.
- ➔ Find a basis for R^2 such that the matrix for a linear transformation relative to it is diagonal.
- ➔ Verify statements related to similar matrices, eigenvectors, and linear transformations.
- ➔ Find eigenvectors and eigenvalues of linear transformations.

5.5 Complex Eigenvalues
- ➔ Find eigenvalues and bases for eigenspaces in C^2.
- ➔ Use complex eigenvalues to find angles of rotation and scale factors for transformations.
- ➔ Find factorizations of the form PCP^{-1} for 2x2 matrices with complex eigenvalues.

UNIT 5 ASSESSMENT EVIDENCE

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- Formative and summative assessments.
- Feedback & Scoring Rubric based on

Other assessment options
May include, but are not limited to the following:

- Unit Personal Progress Checks

Priority Standards	