

# Pequea Valley School District

## Math

Course: Calculus

Grade: 11, 12

### Unit 1: Limits

Unit Essential Question: How do limits form the foundation of Calculus?

CONCEPT
<b>Rates of Change and Limits</b>
VOCABULARY
Average Speed, Instantaneous Speed, Properties of Limits, One-Sided Limits, Left-Hand Limit, Right-Hand Limit, Substitution Methods, Sandwich Theorem
KNOW
<ul style="list-style-type: none"> <li>• Average speed</li> <li>• Instantaneous speed</li> <li>• Limits graphically</li> <li>• Properties of limits</li> </ul>
UNDERSTAND
<ul style="list-style-type: none"> <li>• Similarities and differences of average and instantaneous speed</li> <li>• Evaluating limits numerically, graphically, and algebraically</li> <li>• Definition of a limit using one-sided limits</li> </ul>

CONCEPT
<b>Limits Involving Infinity</b>
VOCABULARY
Finite Limits, Infinite Limits, End Behavior, Vertical Asymptote, Horizontal Asymptote
KNOW
<ul style="list-style-type: none"> <li>• Asymptotes</li> <li>• End behavior</li> </ul>
UNDERSTAND
<ul style="list-style-type: none"> <li>• How to use limits to describe behavior near an asymptote</li> </ul>

CONCEPT
<b>Continuity</b>
VOCABULARY
Continuity at a Point, Removable Discontinuity, Jump Discontinuity, Infinite Discontinuity, Intermediate Value Theorem
KNOW
<ul style="list-style-type: none"> <li>• Continuity</li> <li>• Intermediate Value Theorem</li> </ul>
UNDERSTAND
<ul style="list-style-type: none"> <li>• How to determine if a function is continuous at a point</li> <li>• How to apply the Intermediate Value Theorem</li> </ul>

CONCEPT
<b>Rates of Change and Tangent Lines</b>
VOCABULARY
Average Rate of Change (AROC), Instantaneous Rate of Change (IROC), Secant Line, Tangent Line, Normal Line, Slope of a Curve
KNOW
<ul style="list-style-type: none"> <li>• Average Rate of Change</li> <li>• Instantaneous Rate of Change</li> <li>• Forms of linear equations</li> </ul>
UNDERSTAND
<ul style="list-style-type: none"> <li>• How to calculate the slope of a curve</li> <li>• How to write the equation of a line</li> </ul>

LEARNING OUTCOME	LEARNING OUTCOME	LEARNING OUTCOME	LEARNING OUTCOME
<ul style="list-style-type: none"> <li>• Learners will be able to express limits symbolically using correct notation.</li> <li>• Learners will be able to interpret limits expressed symbolically.</li> <li>• Learners will be able to determine limits of functions.</li> <li>• Learners will be able to deduce and interpret behavior of functions using limits.</li> </ul>	<ul style="list-style-type: none"> <li>• Learners will be able to express limits for end behavior symbolically using correct notation.</li> <li>• Learners will be able to express limits near vertical asymptotes symbolically using correct notation.</li> <li>• Learners will be able to deduce and interpret behavior of functions using limits.</li> </ul>	<ul style="list-style-type: none"> <li>• Learners will be able to analyze functions for intervals of continuity or points of discontinuity.</li> <li>• Learners will be able to determine the applicability of important calculus theorems using continuity.</li> </ul>	<ul style="list-style-type: none"> <li>• Learners will be able to calculate the slope of a curve.</li> <li>• Learners will be able to write the equation of lines that are secant, tangent and normal to a curve.</li> </ul>

### College Board Curriculum Framework:

LO 1.1A(a)

Express limits symbolically using correct notation.

LO 1.1A(b)

Interpret limits expressed symbolically.

LO 1.1B

Estimate limits of functions.

LO 1.1C

Determine limits of functions.

LO 1.1D

Deduce and interpret behavior of functions using limits.

LO 1.2A

Analyze functions for intervals of continuity or points of discontinuity.

LO 1.2B

Determine the applicability of important calculus theorems using continuity.

### Assessments: Unit Assessment

### Pacing: 21 days (including assessment)

### Resources: Schoology, Pearson Calculus AP\* Edition, AP Central website

# Pequea Valley School District Math

Course: Calculus

Grade: 11, 12

## Unit 2: Basic Derivatives

Unit Essential Question: What are derivatives and how do they work?

CONCEPT
<b>Derivative of a Function</b>
VOCABULARY
Limit Definition of a Derivative Function, Limit Definition of the Value of the Derivative at a Point, One-Sided Derivatives, Left-Hand Derivative, Right-Hand Derivative, Power Rule, Product Rule, Quotient Rule
KNOW
<ul style="list-style-type: none"> <li>Limit definitions of derivatives</li> <li>Differentiability of a function at a point</li> <li>Derivative rules</li> </ul>
UNDERSTAND
<ul style="list-style-type: none"> <li>When and how to use the two limit definitions of a derivative</li> <li>How to decide if a function is differentiable at a point</li> <li>How to use the rules of derivatives</li> </ul>
LEARNING OUTCOME
<ul style="list-style-type: none"> <li>Learners will be able to identify the derivative of a function as the limit of a difference quotient.</li> <li>Learners will be able to calculate derivatives.</li> </ul>

CONCEPT
<b>Velocity and Other Rates of Change</b>
VOCABULARY
Position, Velocity, Acceleration, Speed, Displacement, Total Distance, Rate of Change
KNOW
<ul style="list-style-type: none"> <li>Linear motion</li> <li>Rate of change</li> </ul>
UNDERSTAND
<ul style="list-style-type: none"> <li>The mathematical relationship between an object's position, velocity, acceleration, and speed.</li> </ul>
LEARNING OUTCOME
<ul style="list-style-type: none"> <li>Learners will be able to use derivatives to describe rectilinear motion.</li> </ul>

CONCEPT
<b>Trigonometric Derivatives</b>
VOCABULARY
Sine, Cosine, Tangent, Cosecant, Secant, Cotangent
KNOW
<ul style="list-style-type: none"> <li>Trigonometric functions</li> </ul>
UNDERSTAND
<ul style="list-style-type: none"> <li>How to determine the derivatives of trigonometric functions</li> </ul>
LEARNING OUTCOME
<ul style="list-style-type: none"> <li>Learners will be able to differentiate the trigonometric functions.</li> </ul>

- Learners will be able to determine higher order derivatives..
- Learners will be able to recognize the connection between differentiability and continuity.
- Learners will be able to solve problems involving the slope of a tangent line.



## **College Board Curriculum Framework:**

LO 2.1A

Identify the derivative of a function as the limit of a difference quotient.

LO 2.1B

Estimate derivatives.

LO 2.1C

Calculate derivatives.

LO 2.1D

Determine higher order derivatives.

LO 2.2A

Use derivatives to analyze properties of a function.

LO 2.2B

Recognize the connection between differentiability and continuity.

LO 2.3A

Interpret the meaning of a derivative within a problem.

LO 2.3B

Solve problems involving the slope of a tangent line.

## **Assessments: Unit Assessment**

**Pacing: 18 days (including assessment)**

**Resources: Schoology, Pearson Calculus AP\* Edition, AP Central website**

# Pequea Valley School District

## Math

Course: Calculus

Grade: 11, 12

### Unit 3: Advanced Derivatives

Unit Essential Question: What are derivatives and how do they work?

CONCEPT
Chain Rule
VOCABULARY
Chain Rule, Composed Functions
KNOW
<ul style="list-style-type: none"> <li>Chain Rule</li> <li>Function composition</li> </ul>
UNDERSTAND
<ul style="list-style-type: none"> <li>When and how to apply the Chain Rule</li> <li>Function composition</li> </ul>
LEARNING OUTCOME
<ul style="list-style-type: none"> <li>Learners will be able to differentiate composed functions.</li> </ul>

CONCEPT
Implicit Differentiation
VOCABULARY
Implicitly Defined Functions
KNOW
<ul style="list-style-type: none"> <li>Implicitly defined functions</li> </ul>
UNDERSTAND
<ul style="list-style-type: none"> <li>Which equations are implicitly defined functions</li> </ul>
LEARNING OUTCOME
<ul style="list-style-type: none"> <li>Learners will be able to solve for the derivative of implicitly defined functions.</li> </ul>

CONCEPT
Exponential and Logarithmic Derivatives
VOCABULARY
Inverse Function, Logarithm, Exponential Function, $e$
KNOW
<ul style="list-style-type: none"> <li>Logarithms</li> <li>Exponential functions</li> </ul>
UNDERSTAND
<ul style="list-style-type: none"> <li>How to determine the derivatives of logarithmic functions</li> <li>How to determine the derivatives of exponential functions.</li> </ul>
LEARNING OUTCOME
<ul style="list-style-type: none"> <li>Learners will be able to differentiate logarithmic functions.</li> <li>Learners will be able to differentiate exponential functions.</li> </ul>

CONCEPT
L'Hopital's Rule
VOCABULARY
L'Hopital's Rule
KNOW
<ul style="list-style-type: none"> <li>L'Hopital's Rule</li> </ul>
UNDERSTAND
<ul style="list-style-type: none"> <li>How to evaluate a limits of the indeterminate form <math>0/0</math> and <math>\infty/\infty</math>.</li> </ul>
LEARNING OUTCOME
<ul style="list-style-type: none"> <li>Learners will be able to use L'Hopital's Rule to evaluate limits of the indeterminate form <math>0/0</math> and <math>\infty/\infty</math>.</li> </ul>

## **College Board Curriculum Framework:**

LO 2.1B

Estimate derivatives.

LO 2.1C

Calculate derivatives.

LO 2.1D

Determine higher order derivatives.

LO 2.2A

Use derivatives to analyze properties of a function.

LO 2.2B

Recognize the connection between differentiability and continuity.

LO 2.3A

Interpret the meaning of a derivative within a problem.

LO 2.3B

Solve problems involving the slope of a tangent line.

## **Assessments: Unit Assessment**

## **Pacing: 20 days (including assessment)**

## **Resources: Schoology, Pearson Calculus AP\* Edition, AP Central website**

# Pequea Valley School District Math

Course: Calculus

Grade: 11, 12

## Unit 4: Applications of Derivatives–Part 1

Unit Essential Question: How can derivatives be applied to solve real life problems?

CONCEPT
The Extreme Value Theorem
VOCABULARY
Absolute extrema, Local extrema, Critical Points, The Extreme Value Theorem
KNOW
<ul style="list-style-type: none"> <li>Extrema</li> <li>The Extreme Value Theorem</li> </ul>
UNDERSTAND
<ul style="list-style-type: none"> <li>How to identify the extrema of a function</li> <li>How to apply the Extreme Value Theorem</li> </ul>
LEARNING OUTCOME
<ul style="list-style-type: none"> <li>Learners will be able to use derivatives to identify the extrema of a function</li> </ul>

CONCEPT
The Mean Value Theorem
VOCABULARY
The Mean Value Theorem
KNOW
<ul style="list-style-type: none"> <li>The Mean Value Theorem</li> </ul>
UNDERSTAND
<ul style="list-style-type: none"> <li>How to apply the Mean Value Theorem</li> </ul>
LEARNING OUTCOME
<ul style="list-style-type: none"> <li>Learners will be able to use derivatives to apply the Mean Value Theorem</li> </ul>

CONCEPT
Connecting $f'$ and $f''$ with the graph of $f$
VOCABULARY
Increasing, Decreasing, Concavity, Point of Inflection, First Derivative Test, Second Derivative Test
KNOW
<ul style="list-style-type: none"> <li>Increasing/decreasing behavior</li> <li>Concavity</li> </ul>
UNDERSTAND
<ul style="list-style-type: none"> <li>How to use the graphs of <math>f'</math> and <math>f''</math> to develop the graph of <math>f</math>.</li> </ul>
LEARNING OUTCOME
<ul style="list-style-type: none"> <li>Learners will be able to use derivatives to determine where a function is increasing or decreasing.</li> <li>Learners will be able to use derivatives to determine where a function is concave up or concave down.</li> </ul>

## **College Board Curriculum Framework:**

LO 2.1B

Estimate derivatives.

LO 2.1C

Calculate derivatives.

LO 2.1D

Determine higher order derivatives.

LO 2.2A

Use derivatives to analyze properties of a function.

LO 2.3A

Interpret the meaning of a derivative within a problem.

LO 2.3B

Solve problems involving the slope of a tangent line.

LO 2.3D

Solve problems involving rates of change in applied contexts.

LO 2.3E

Verify solutions to differential equations.

LO 2.3F

Estimate solutions to differential equations.

LO 2.4A

Apply the Mean Value Theorem to describe the behavior of a function over an interval.

## **Assessments: Unit Assessment**

## **Pacing: 17 days (including assessment)**

## **Resources: Schoology, Pearson Calculus AP\* Edition, AP Central website**



# Pequea Valley School District Math

Course: Calculus

Grade: 11, 12

## Unit 5: Applications of Derivatives–Part 2

Unit Essential Question: How can derivatives be applied to solve real life problems?

CONCEPT
<b>Optimization</b>
VOCABULARY
Optimization, Maximize, Minimize
KNOW
<ul style="list-style-type: none"><li>Mathematical Models</li></ul>
UNDERSTAND
<ul style="list-style-type: none"><li>How to create a mathematical model for a particular situation</li></ul>
LEARNING OUTCOME
<ul style="list-style-type: none"><li>Learners will be able to use derivatives to optimize a quantity in a particular situation.</li></ul>

CONCEPT
<b>Linearization</b>
VOCABULARY
Linearization, Local Linearity
KNOW
<ul style="list-style-type: none"><li>Forms of linear equations</li></ul>
UNDERSTAND
<ul style="list-style-type: none"><li>Differentiable functions are locally linear</li></ul>
LEARNING OUTCOME
<ul style="list-style-type: none"><li>Learners will be able to use linearization to approximate a function's values.</li></ul>

CONCEPT
<b>Related Rates</b>
VOCABULARY
Related Rates
KNOW
<ul style="list-style-type: none"><li>Rates of Change</li></ul>
UNDERSTAND
<ul style="list-style-type: none"><li>Quantities can change at different rates.</li></ul>
LEARNING OUTCOME
<ul style="list-style-type: none"><li>Learners will be able to use related rates to solve a problem with multiple changing quantities.</li></ul>

## College Board Curriculum Framework:

LO 2.1B

Estimate derivatives.

LO 2.1C

Calculate derivatives.

LO 2.1D

Determine higher order derivatives.

LO 2.2A

Use derivatives to analyze properties of a function.

LO 2.3A

Interpret the meaning of a derivative within a problem.

LO 2.3B

Solve problems involving the slope of a tangent line.

LO 2.3C

Solve problems involving related rates, optimization, and rectilinear motion.

LO 2.3D

Solve problems involving rates of change in applied contexts.

LO 2.3E

Verify solutions to differential equations.

LO 2.3F

Estimate solutions to differential equations.

## **Assessments: Unit Assessment**

## **Pacing: 16 days (including assessment)**

## **Resources: Schoology, Pearson Calculus AP\* Edition, AP Central website**

# Pequea Valley School District

## Math

Course: Calculus

Grade: 11, 12

### Unit 6: The Definite Integrals

Unit Essential Question: How does integral calculus relate to differential calculus?

CONCEPT
<b>Definite Integrals</b>
VOCABULARY
Left Rectangular Approximation Method (LRAM), Right Rectangular Approximation Method (RRAM), Midpoint Rectangular Approximation Method (MRAM), Riemann Sum
KNOW
<ul style="list-style-type: none"> <li>Area of a rectangle</li> <li>Definite integrals</li> <li>Properties of Definite Integrals</li> </ul>
UNDERSTAND
<ul style="list-style-type: none"> <li>How to approximate the area under a curve using rectangles</li> <li>The relationship between the area under a curve and an integral</li> </ul>

CONCEPT
<b>Antiderivatives</b>
VOCABULARY
Antiderivative, Average Value, Mean Value Theorem, General Solution, Particular Solution, Constant of Integration
KNOW
<ul style="list-style-type: none"> <li>Antiderivatives of common expressions</li> <li>The Mean Value Theorem</li> <li>Average value</li> </ul>
UNDERSTAND
<ul style="list-style-type: none"> <li>The relationship between derivatives and antiderivatives</li> <li>The average value of a function over a given interval</li> </ul>

CONCEPT
<b>The Fundamental Theorem of Calculus</b>
VOCABULARY
Fundamental Theorem of Calculus Part 1 and Part 2, Total Area
KNOW
<ul style="list-style-type: none"> <li>The Fundamental Theorem of Calculus</li> <li>Total area</li> </ul>
UNDERSTAND
<ul style="list-style-type: none"> <li>How to differentiate an integral</li> <li>How to evaluate a definite integral using an antiderivative</li> <li>The difference between area under a curve and total area</li> </ul>

CONCEPT
<b>The Trapezoidal Rule</b>
VOCABULARY
Trapezoidal Approximation
KNOW
<ul style="list-style-type: none"> <li>Area of a Trapezoid</li> <li>The trapezoidal rule</li> </ul>
UNDERSTAND
<ul style="list-style-type: none"> <li>How to approximate the area under a curve using trapezoids</li> </ul>

LEARNING OUTCOME	LEARNING OUTCOME	LEARNING OUTCOME	LEARNING OUTCOME
<ul style="list-style-type: none"> <li>Learners will be able to calculate a definite integral using areas and properties of definite integrals.</li> <li>Learners will be able to interpret the definite integral as the limit of a Riemann sum.</li> <li>Learners will be able to express the limit of a Riemann sum in integral notation.</li> <li>Learners will be able to approximate a definite integral.</li> </ul>	<ul style="list-style-type: none"> <li>Learners will be able to recognize antiderivatives of basic functions.</li> <li>Learners will be able to apply definite integrals to problems involving the average value of a function.</li> <li>Learners will be able to apply definite integrals to problems involving motion.</li> </ul>	<ul style="list-style-type: none"> <li>Learners will be able to analyze functions defined by an integral</li> <li>Learners will be able to find the derivative of an integral.</li> </ul>	<ul style="list-style-type: none"> <li>Learners will be able to approximate a definite integral.</li> </ul>

### College Board Curriculum Framework:

LO 3.1A

Recognize antiderivatives of basic functions.

LO 3.2A(a)

Interpret the definite integral as the limit of a Riemann sum.

LO 3.2B

Approximate a definite integral.

LO 3.2C

Calculate a definite integral using areas and properties of definite integrals.

LO 3.3A

Analyze functions defined by an integral.

LO 3.3B(a)

Calculate antiderivatives.

LO 3.3B(b)

Evaluate definite integrals.

LO 3.4A

Interpret the meaning of a definite integral within a problem.

LO 3.4B

Apply definite integrals to problems involving the average value of a function.

LO 3.4C

Apply definite integrals to problems involving motion.

LO 3.4E

Use the definite integrals to solve problems in various contexts.

### Assessments: Unit Assessment

Pacing: 24 days (including assessment)

Resources: Schoology, Pearson Calculus AP\* Edition, AP Central website

# Pequea Valley School District Math

Course: Calculus

Grade:11,12

## Unit 7: Differential Equations and Mathematical Modeling

Unit Essential Question: **How are differential equations used to model real life problems?**

CONCEPT
<b>Slope Fields and Differential Equations</b>
VOCABULARY
Differential Equation, Slope Field, Initial Value, Indefinite Integrals, Constant of Integration, General Solution, Particular Solution
KNOW
<ul style="list-style-type: none"> <li>Slope</li> <li>Solutions to differential equations</li> </ul>
UNDERSTAND
<ul style="list-style-type: none"> <li>How to determine a general solution to a differential equation</li> <li>How to determine a particular solution to a differential equation given an initial value.</li> </ul>
LEARNING OUTCOME
<ul style="list-style-type: none"> <li>Learners will be able to construct a slope field from a differential equation.</li> <li>Learners will be able to analyze differential equations to obtain general and specific solutions.</li> <li>Learners will be able to interpret, create, and solve differential</li> </ul>

CONCEPT
<b>Integration by Substitution</b>
VOCABULARY
U-Substitution
KNOW
<ul style="list-style-type: none"> <li>Chain Rule</li> </ul>
UNDERSTAND
<ul style="list-style-type: none"> <li>When and how to use u-substitution when integrating.</li> </ul>
LEARNING OUTCOME
<ul style="list-style-type: none"> <li>Learners will be able to know when and how to use u-substitution when integrating.</li> </ul>

CONCEPT
<b>Separable Differential Equations</b>
VOCABULARY
Separable Differential Equation, Exponential Growth, Exponential Decay
KNOW
<ul style="list-style-type: none"> <li>Exponential Growth</li> <li>Exponential Decay</li> </ul>
UNDERSTAND
<ul style="list-style-type: none"> <li>When and how to use separation of variables when integrating.</li> </ul>
LEARNING OUTCOME
<ul style="list-style-type: none"> <li>Learners will be able to know when and how to use separation of variables when integrating.</li> </ul>

equations from problems in context.

### **College Board Curriculum Framework:**

LO 3.1A

Recognize antiderivatives of basic functions.

LO 3.2B

Approximate a definite integral.

LO 3.2C

Calculate a definite integral using areas and properties of definite integrals.

LO 3.3A

Analyze functions defined by an integral.

LO 3.3B(a)

Calculate antiderivatives.

LO 3.3B(b)

Evaluate definite integrals.

LO 3.4A

Interpret the meaning of a definite integral within a problem.

LO 3.4E

Use the definite integrals to solve problems in various contexts.

LO 3.5A

Analyze differential equations to obtain general and specific solutions.

LO 3.5B

Interpret, create, and solve differential equations from problems in context.

### **Assessments: Unit Assessment**

### **Pacing: 18 days (including assessment)**

### **Resources: Schoology, Pearson Calculus AP\* Edition, AP Central website**