

Summit Public Schools

Summit, New Jersey

Grade Level: 10-12 **Content Area:** Mathematics

Length of Course: Full Academic Year

Curriculum: Precalculus Essentials

Developed by:
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Summer 2020

Course Description: Precalculus Essentials is an in-depth study of the concept of functions and trigonometry. Several function classes including linear, quadratic, polynomial, rational, exponential, logarithmic, and trigonometric functions are studied. Within each function class, characteristics of the function are emphasized including the basic form and graph, equations and inequalities associated with the function, and applications. Both algebraic and graphical techniques will be used throughout the course. Appropriate prerequisite skills and understandings for this course are required for success in this course.

Suggested Timeline

Marking Period 1

Functions and Relations	
Topic	Class Periods
Relations vs. Functions. Domain and Range.	2
Interval Notation, Domain and Range from a Graph	1
Function Operations (add, subtract, multiply, divide, composition)	1
Inverse Functions (Equations and Graphs)	3
Review/Test	2
Total	9

Graphs of Functions	
Topic	Class Periods
Parent Functions and Transformations	3
Radical Equations and Inequalities	2
Piecewise Functions	2
Odd and Even Functions	1
Review/Test	2
Total	10

Polynomial Functions	
Topic	Class Periods
Solving Quadratic Equations and inequalities/Types of Solutions	2
Equations of Higher Degree Polynomials	2
Finding all Roots of Higher Degree Polynomials	2
Sketching Graphs using Intercepts and End Behavior	2
Review/Test	2
Total	10

Marking Period 2

Rational Functions	
Topic	Class Periods
What is a Rational Function? Vertical and Horizontal Asymptotes	2
Slant Asymptotes and Removable Discontinuities	2
Graphing any Rational Function	2
Rational Equations and Inequalities	2
Review/Test	2
Total	10

Conic Sections	
Topic	Class Periods
What is a Conic Section? What are Their Uses?	1
Graphing and Writing Equations of Parabolas	2
Graphing and Writing Equations of Hyperbolas	2
Graphing and Writing Equations of Ellipses	2
Graphing any Conic Section	1
Review/Test	2
Total	10

Exponential and Logarithmic Functions	
Topic	Class Periods
Exponential Applications	2
Exponential Graphs. Growth vs. Decay. Base of e	2
Logarithm Properties	2
Graphs of Logarithmic Functions	1
Applications of Logarithms	1
Review/Test	2
Total	10

Marking Period 3

Angles and Trigonometric Ratios	
Topic	Class Periods
Solving Right Triangles	1
Trigonometric Functions of Any Angle	2
Radians and Degrees	1
The Unit Circle/Finding Trig Ratios of Special Angles	2
Inverse Trig Functions	2
Review/Test	2
Total	10

Graphs of Trigonometric Functions	
Topic	Class Periods
Graphs of Sine and Cosine/Amplitude and Period	2
Midline and Phase Shifts	2
Secant, Cosecant, Tangent, Cotangent	2
Graphing any Trig Function/Writing Equations Given a Graph	2
Trigonometric Graph Applications	2
Review/Test	2
Total	12

Trigonometric Identities	
Topic	Class Periods
Reciprocal and Pythagorean Identities	1
Simplifying Trig Expressions	2
Verifying Identities with Reciprocal and Pythagorean Identities	2
Practice with Reciprocal and Pythagorean Identities	1
Review/Test	2
Total	8

Marking Period 4

Trigonometric Equations	
Topic	Class Periods
Sum and Difference Identities	2
Double and Half Angle Identities	2
Solving Trigonometric Equations	2
Review/Test	2
Total	8

Solving Triangles	
Topic	Class Periods
Law of Sines and the Ambiguous Case	3
Law of Cosines	2
Areas of Triangles	1
Solving any Triangles/Applications	2
Review/Test	2
Total	10

Sequences and Series	
Topic	Class Periods
Arithmetic Sequences	1
Arithmetic Series	1
Geometric Sequences	1
Geometric Series	1
Infinite Series	1
Review/Test	2
Total	7

Unit 1: Functions and Relations

Standards and Outcomes F-IF Interpreting Functions F-BF Building Functions A-APR Arithmetic with Polynomial and Rational Functions	
Big Ideas: <i>Course Objectives/Content Statements</i> - Decide whether a relation is a function or not - Find the domain and range of a function graphically and algebraically. Write in interval notation. - Perform algebraic operations on functions - Find the inverse of a function	
Essential Questions <i>What provocative questions will foster inquiry, understanding, and transfer of learning?</i>	Enduring Understandings <i>What will students understand about the big ideas?</i>
- What is a domain and a range? How are they related? - How are a domain and range found? - What makes a relation a function? - What is Interval Notation, and what is it used for? - What operations can be performed on functions? - What is an inverse function? - How is an inverse function found?	Students will understand... - That a domain is the set of all input values in a relation, the range is the set of all output values - How domains and ranges can be found from a graph, how a domain can be found with an equation - That a function is a relation such that every input has only one output - That interval notation is a way to express a continuous set of numbers. How the [and) are used to show inclusivity or exclusivity - How functions can be added, subtracted, multiplied, divided, or composed - That an inverse function is made by switching the domain and range - How inverses can be found either graphically or algebraically Vocabulary: Relation, Function, Domain, Range, Interval Notation, Composition, Inverse
Areas of Focus <i>Cumulative Progress Indicators/Specific Standards</i>	Examples, Outcomes, Assessments

F-IF 1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x . The graph of f is the graph of the equation $y = f(x)$.

2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F-BF 1. b) Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

4. Find inverse functions.

a) Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse.

A-APR 1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

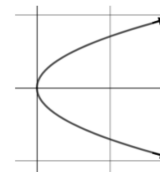
Sample Assessment Questions:

- Decide if a graph, table, or equation is a function. Explain your answer.

Find the domain and range of $f(x) = \frac{x+2}{3x-9}$

$g(x) = \sqrt{3x-9}$

$h(x) = \frac{x+2}{\sqrt{3x-9}}$

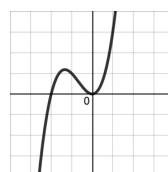


- Using the following function definitions, evaluate each expression

$f(x) = 3x^2 + 2, g(x) = 3x + 2, h(x) = \sqrt{x+2}$

a) $f(1)$ b) $g(3)$ c) $(h \circ f)(x)$

- Find the inverse

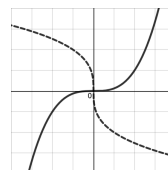


a) $f(x) = 5\sqrt{x} + 1$

b) $f(x)2x^2 + 3$

c) $f(x) = (2x + 3)^2$

- Are these functions inverses of each other?



Types of Assessments:

- Quick quizzes for each topic
- Half period quiz
- Full period test

Instructional Strategies/Technology Suggestions:

- Desmos/Graphing Calculators to show relationships between inverse functions
- Kahoot/Quizzes for practice questions

Unit 2: Graphs of Functions

Standards and Outcomes F-IF Interpreting Functions A-CED Creating Functions F-BF Building Functions A-REI Reasoning with Equations and Inequalities	
Big Ideas: <i>Course Objectives/Content Statements</i> - Using parent function graphs to graph more complex functions - Apply translations and compressions to functions - Interpret and graph a piecewise function - Solve equations and inequalities with radicals	
Essential Questions <i>What provocative questions will foster inquiry, understanding, and transfer of learning?</i>	Enduring Understandings <i>What will students understand about the big ideas?</i>
- What do each of the parent functions look like? What are their properties? - How does adding or subtracting a constant in the equation change the graph of a function? - How does multiplying by a number in an equation change the graph? - How can equations and inequalities with radicals be solved? - What makes a function odd or even? - What are piecewise functions? How are they graphed?	Students will understand... - How to graph cubic, quadratic, linear, square root, cube root, and absolute value parent functions - How adding a value before the operation (inside parenthesis) translates the graph left or right, while adding after the operations translates up or down - How multiplication stretches or shrinks the graph - How to solve equations and inequalities with variables inside of a square or cube root - That odd functions are symmetric across the origin, and even functions are symmetric across the y-axis - That piecewise functions have different equations for different values of x, and how to graph them Vocabulary: Parent Function, Linear, Quadratic, Cubic, Cube Root, Square Root, Absolute Value, Translation, Compression, Stretch, Odd Function, Even Function, Piecewise
Areas of Focus <i>Cumulative Progress Indicators/Specific Standards</i>	Examples, Outcomes, Assessments
F-IF C 7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases." (a) Graph linear and quadratic functions and show intercepts, maxima, and minima. (b) Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.	Sample Assessment Questions: - Sketch a graph of each parent function - Sketch a graph of each function along with the parent function a) $f(x) = 2(x - 3)^2$ b) $f(x) = \sqrt[3]{(1 - x)} + 1$ c) $f(x) = -(x + 1)^2 + 2$ d) $f(x) = \frac{1}{4}\sqrt{x} - 2$ - Solve each equation

F-IF 9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

A-REI A Understand solving equations as a process of reasoning and explain the reasoning 1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. 2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

A-CED 1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

a) $\sqrt{-1 + 2x} = x$

b) $x - 5 = \sqrt{2x - 2}$

c) $\sqrt{x + 1} - 1 = \sqrt{x - 2}$

- Sketch a graph of each function

a) $f(x) = \begin{cases} (x - 4)^2 & \text{if } x < 4 \\ x - 1 & \text{if } x \geq 4 \end{cases}$

b) $f(x) = \begin{cases} (x + 3)^2 & \text{if } x < -2 \\ 2x - 2 & \text{if } -2 \leq x < 3 \\ -x + 1 & \text{if } x \geq 3 \end{cases}$

Types of Assessments:

- Quick quizzes for each topic
- Half period quiz
- Full period test

Instructional Strategies/Technology Suggestions:

- Desmos/Graphing Calculators to show how graphs change when adding constants into an equation
- Kahoot/Quizzes for practice questions

Unit 3: Polynomial Functions

Standards and Outcomes A-SSE Seeing Structure in Expressions A-APR Arithmetic with Polynomials and Rational Expressions F-IF Interpreting Functions A-Rei Reasoning with Equations and Inequalities Students will be able to find the zeros and end behavior of any polynomial function, and sketch a graph based on the roots and end behavior. They will also be able to write equations of polynomials based on roots and end behavior, and solve equations and inequalities using polynomial equations.	
Big Ideas: Course Objectives/Content Statements - Solving quadratic equations and inequalities - Roots of polynomial functions - End behavior of polynomial functions - Graphing any polynomial function	
Essential Questions <i>What provocative questions will foster inquiry, understanding, and transfer of learning?</i>	Enduring Understandings <i>What will students understand about the big ideas?</i>
- What is the most effective way to solve a quadratic equation? How does this relate to a graph? - What is an imaginary solution? - What values satisfy a quadratic inequality? - How many x-intercepts does a higher degree polynomial have? How can we find them? - What is end behavior, and how does it relate to the graph of a polynomial? - How can we sketch a graph of any given polynomial?	Students will understand... - How to check if a quadratic is factorable, and if not use either completing the square or the quadratic formula to find solutions - That imaginary numbers are solutions to an equation that will not appear on a graph - How to find the solution set to a quadratic inequality by graphing or algebraically - That the number of roots is directly related to the degree. How to use the rational root theorem along with synthetic division and solving quadratics to completely factor a polynomial and find its roots - How end behavior is related to the lead coefficient of an polynomial - How to combine information about end behavior, roots and multiplicity to create a sketch. Vocabulary: Polynomial, Factor, Root, Zero, Solution, x-Intercept, y-intercept, Imaginary, Rational Root Theorem, Synthetic Division, End Behavior, Even Multiplicity, Odd Multiplicity
Areas of Focus <i>Cumulative Progress Indicators/Specific Standards</i>	Examples, Outcomes, Assessments
A-SSE 3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression." a) Factor a quadratic expression to reveal the zeros of the function it defines.	Sample Assessment Questions: - Find the x intercepts a) $f(x) = 15x^2 - x - 2$ b) $f(x) = 3x^2 - 12x - 3$ c) $f(x) = 12x^2 - 5x - 2$ d) $f(x) = -4x^2 - 5x + 4$ - Solve each inequality a) $21x^2 + 15x + 56 > 20x^2$

b) Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

A-APR 2. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a , the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.

3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

F-IF 7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

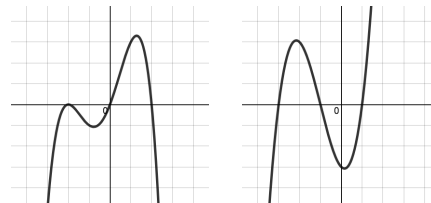
A-REI 4. Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b .

b) $6x^2 - 9x + 7 \geq 4x + 1$

c) $x^2 + 3x + 12 \geq 19$

d) $3x^2 - 10x + 25 \geq 2x^2$

- Explain the relationship between a factor and a root
- Write the equation of a polynomial with the roots 2 and $-4i$.
- Write an equation of a polynomial in standard form that has the same end behavior and roots of the graph



- Sketch a graph based on the roots and end behavior

a) $f(x) = x^4 - 2x^3 - 3x^2 + 4x + 4$

b) $f(x) = -x^4 - 6x^3 - 12x^2 - 10x - 3$

Types of Assessments:

- Quick quizzes for each topic
- Half period quiz
- Full period test

Instructional Strategies/Technology Suggestions:

- Desmos/Graphing Calculators to check work and practice
- Exploration activity of how end behavior relates to the equation of a polynomial
- Kahoot/Quizzes for practice questions

Unit 4: Rational Functions

Standards and Outcomes

F-IF Interpreting Functions

A-APR Arithmetic with Polynomial and Rational Expressions

Students will be able to solve equations and inequalities involving rational functions. They will also be able to use their skills from the previous unit to find information about, and graph rational functions

Big Ideas: *Course Objectives/Content Statements*

- Solve rational equations and inequalities
- Sketch a graph of a rational function

Essential Questions

What provocative questions will foster inquiry, understanding, and transfer of learning?

- What are asymptotes?
- How are asymptotes found?
- What makes up a graph of a rational function?
- How is the domain of a rational function related to its graph?
- What techniques can we use to solve rational equations and inequalities?

Enduring Understandings

What will students understand about the big ideas?

Students will understand...

- That an asymptote is a line or curve that a graph approaches, but is not actually part of the graph
- How to find horizontal, vertical, and slant asymptotes using polynomial division and other strategies
- How to graph a rational function with intercepts, asymptotes and additional points if needed.
- Determine which x values that are not part of the domain represent asymptotes, and which represent holes

Vocabulary: Rational Function, Horizontal Asymptote, Vertical Asymptote, Slant Asymptote, Removable Discontinuity (Hole), Polynomial Division

Areas of Focus

Cumulative Progress Indicators/Specific Standards

F-IF 7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases."

d) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a) Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

9. Compare properties of two functions each represented in a different way (algebraically,

Examples, Outcomes, Assessments

Sample Assessment Questions:

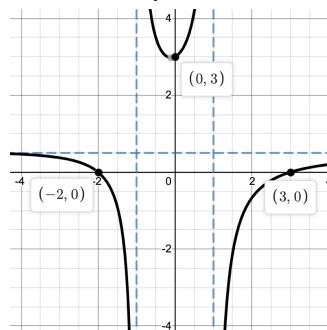
- Describe how a horizontal and vertical asymptote are related to a graph.
- Sketch a graph of each function using the horizontal, vertical, and slant asymptotes.
 - a) $f(x) = \frac{x-3}{x-2}$
 - b) $f(x) = \frac{x+1}{x^2+6x+5}$
 - c) $f(x) = \frac{2x^3-5x^2-3x}{x^2-2x}$
 - d) $f(x) = \frac{x^3-4x^2+3x}{x^3-x^2-2x}$

graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

A-APR 6. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system

D. Represent and solve equations and inequalities graphically 10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line)

- Write an equation of the given function



- Why is it ok for a graph to cross its horizontal asymptote, but not the vertical asymptote?
- Explain why sometimes a factor in the denominator represents a hole, and sometimes it represents an asymptote.
- Solve the equation or inequality
 - a) $\frac{1}{3x} - \frac{1}{6} = \frac{x^2 - 9x + 18}{6x}$
 - b) $\frac{x-1}{x^2-4x} = \frac{3}{x^2-4x} + \frac{1}{x^2-7x+12}$
 - c) $\frac{x^2+7x+10}{x^2+2x-15} > 0$
 - d) $\frac{-2x-3}{x+5} > -1$
- How are asymptotes and holes related to the solution of a rational inequality?

Types of Assessments:

- Quick quizzes for each topic
- Half period quiz
- Full period test

Instructional Strategies/Technology:

- Desmos/Graphing Calculators to check work and practice
- Exploration activity of how the numerator and the denominator change the graph
- Kahoot/Quizzes for practice questions

Unit 5: Conic Sections

Standards and Outcomes

A-CED Creating Equations

G-GPE Expressing Geometric Properties with Equations

Students will be able to use conic sections to solve real life problems, sketch graphs of ellipses, parabolas, hyperbolas and circles, and write equations given a graph of a conic section.

Big Ideas: *Course Objectives/Content Statements*

- Graph a parabola
- Graph an ellipse
- Graph a hyperbola
- Write an equation of a conic section

Essential Questions

What provocative questions will foster inquiry, understanding, and transfer of learning?

- What are the important characteristics of a parabola? How can we find them?
- What are the important characteristics of a hyperbola? How can we find them?
- What are the important characteristics of an ellipse? How can we find them?
- What are the similarities between all of the conic sections? How can we identify the type of conic section from an equation?

Enduring Understandings

What will students understand about the big ideas?

Students will understand...

- That a parabola can be graphed using its directrix and focus. How to find the directrix and focus from an equation in any form, and sketch a graph.
- That a hyperbola can be graphed using its foci and center. How to find the foci, center and the semi major, and semi minor axis to sketch a graph from an equation in any form
- That an ellipse can be graphed using its foci and center. How to find the foci, center and the semi major, and semi minor axis to sketch a graph from an equation in any form
- How each of the conic sections is derived from a cone, and they are all based on keeping a distance consistent from the focus or foci. How to tell which type of conic to graph based on the equation and the relationship to the properties of the conic section

Vocabulary: Conic Section, Parabola, Ellipse, Hyperbola, Circle, Focus/Foci, Directrix, Center, Vertex, Major Axis, Semi Major Axis, Minor Axis, Semi Minor Axis

Areas of Focus

Cumulative Progress Indicators/Specific Standards

A-CED 2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

G-GPE A. Translate between the geometric description and the equation for a conic section

Examples, Outcomes, Assessments

Sample Assessment Questions:

- Explain how the foci are related to the graph of an ellipse and a hyperbola
- Sketch a graph of each conic section
 - a) $-16x^2 + 32x + y^2 + 2y - 31 = 0$
 - b) $x^2 - 4x - 16y - 12 = 0$
 - c) $x^2 - 6x + 4y^2 - 8y - 25 = 0$

d) $x^2 - 2x + y^2 - 2y - 7 = 0$

- Where are the foci in a circle?
- Write an equation of an ellipse with a center of (-2, 1), one major axis vertex at (3, 1), and on focus at (2, 1)

Types of Assessments:

- Quick quizzes for each topic
- Half period quiz
- Full period test

Instructional Strategies/Technology:

- Desmos/Graphing Calculators to help show the relationship between the equation and graph of a conic section
- Exploration activity to find the center and axis measurements of hyperbolas and ellipses
- Kahoot/Quizzes for practice questions

Unit 6: Exponential and Logarithmic Functions

Standards and Outcomes F-IF Interpreting Functions F-BF Building Functions F-LE Linear and Exponential Models A-SSE Seeing Structure in Expressions Students will be able to apply exponential and logarithmic properties to solve real life problems. They will understand that a logarithmic function is the inverse of an exponential function, and how exponent properties translate into logarithmic properties.	
Big Ideas: <i>Course Objectives/Content Statements</i> • Apply exponential and logarithmic functions to real life • Graph exponential and logarithmic functions	
Essential Questions <i>What provocative questions will foster inquiry, understanding, and transfer of learning?</i>	Enduring Understandings <i>What will students understand about the big ideas?</i>
• What are applications of exponential functions? How can we use them to model real life scenarios? • What are the important characteristics of an exponential graph? How can we find them? • How is a logarithm related to an exponent? • What are the important characteristics of a logarithmic graph? How can we find them? • How can logarithms be used to help solve real life problems?	Students will understand... • How exponential functions are used to model growth and decay of populations, or monetary value. How the value e is used in these applications? • That an exponential graph continues to get steeper as x increases, and has a horizontal asymptote. How to find the asymptotes and steepness from the equation • That a logarithmic function is the inverse of an exponential function, they have the similar properties as exponents. How to use logarithmic properties to make expressions look simpler • That a logarithmic graph looks like an exponential graph reflected over the line $y = x$. How to find the asymptote and steepness from the equation. • Logarithms can be applied in real life in the same scenarios as exponential functions, but they are used to solve for different values Vocabulary: Exponential Function, Logarithm, Logarithmic Function, Horizontal Asymptote, Exponential Growth, Exponential Decay, Principle
Areas of Focus <i>Cumulative Progress Indicators/Specific Standards</i>	Examples, Outcomes, Assessments
F-IF C. Analyze functions using different representations 7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.	Sample Assessment Questions: • Evaluate if possible a) $\log_3 9$ b) $\log 1000$ c) $\log_2 \frac{1}{16}$

e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

F-BF B. Build new functions from existing functions

5. (+) Use the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents

F-LE A. Construct and compare linear and exponential models and solve problems

1. Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. 4. Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to $abct = d$ where a , c , and d are numbers and the base b is 2, 10, or e ; evaluate the logarithm using technology.

A-SSE B. Write expressions in equivalent forms to solve problems

c. Use the properties of exponents to transform expressions for exponential functions. For example the expression $1.15t$ can be rewritten as $(1.151/12)^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.

- Sketch a graph of each function

a) $f(x) = 2^x + 1$

b) $f(x) = 2 \cdot \left(\frac{1}{e}\right)^x$

c) $f(x) = x + 2$

d) $f(x) = (x - 1)$

- \$500 is deposited in a savings account at a 2.5% interest rate.

a) Compounded **monthly**, what will their balance be after 5 years?

b) Compounded **continuously**, what will their balance be after 5 years?

c) Compounded **annually**, how long will it take for the balance to reach \$1,000?

Types of Assessments:

- Quick quizzes for each topic
- Half period quiz
- Full period test

Instructional Strategies/Technology:

- Desmos/Graphing Calculators to help show the relationship between the equation and graph of a exponential and logarithmic functions
- Kahoot/Quizzes for practice questions

Unit 7: Angles and Trigonometric Ratios

Standards and Outcomes G-SRT Similarity, Right Triangles, and Trigonometry F-TF Trigonometric Functions	
Students will be able to apply trigonometry to angles within a right triangle, and any other possible angle. They will be able to use radians or degrees to measure angles, and be able to evaluate the six trigonometric functions for special angles in any quadrant	
Big Ideas: Course Objectives/Content Statements - Solve right triangles - Apply trigonometry to any angle - Evaluate trigonometric functions and their inverses using angles around the unit circle	
Essential Questions <i>What provocative questions will foster inquiry, understanding, and transfer of learning?</i>	Enduring Understandings <i>What will students understand about the big ideas?</i>
- How is trigonometry used to solve a right triangle? - How are trigonometric ratios found for angle that do not fit into a triangle? - What is a radian? How is it related to a degree? - How can we use special right triangles to find certain trigonometric ratios? - How can we find angles based on trigonometric ratios?	Students will understand... - How to use sine, cosine, and tangent to solve for missing sides and angles in a right triangle - How to find trigonometric ratios of any angle in standard position in a coordinate plane - That a radian is the angle whose arc length is one radius, and it is another way to measure angles besides degrees - How 30-60-90 and 45-45-90 or $\pi/6$-$\pi/3$-$\pi/2$ and $\pi/4$-$\pi/4$-$\pi/2$ triangles can be used along with the unit circle and/or reference angles to find trigonometric ratios of angles in every quadrant - How to use inverse trigonometric functions to find possible angles given a ratio Vocabulary: Trigonometric Function/Ratio, Sine, Cosine, Tangent, Secant, Cosecant, Cotangent, Radian, Degree, Reference Angle, Standard Position, Initial Side, Terminal Side, Inverse Trigonometric Function, Principle Value (for inverses), Unit Circle
Areas of Focus <i>Cumulative Progress Indicators/Specific Standards</i>	Examples, Outcomes, Assessments
G-SRT C. Define trigonometric ratios and solve problems involving right triangles 6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. 7. Explain and use the relationship between the sine and cosine of complementary angles. 8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.	Sample Assessment Questions: - What is one radian? Explain with a diagram - Convert the angle from degrees to radians, or from radians to degrees. a) $-\frac{20\pi}{9}$ b) 100° - Draw each angle in standard position and find the reference angle. a) 200° b) $-\frac{10\pi}{3}$

F-IF1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

2.Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

3.(+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$

- Evaluate each trig function. If the angle is not in the first quadrant, draw it and find the reference angle.

a) $\sin\left(-\frac{2\pi}{3}\right)$ **b)** $\cos(150^\circ)$ **c)** $\tan\left(-\frac{5\pi}{3}\right)$

d) $\csc\left(\frac{\pi}{3}\right)$ **e)** $\cot(-45^\circ)$ **f)** $\sec(240^\circ)$

- You are stuck in a tree. If you look down at an angle of depression of 37° you can see your cat. If you know that your cat is 35 feet from the base of the tree, how high up are you?
- Evaluate each inverse trig function in radians and degrees.

a) $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ **b)** $\arctan\left(\frac{\sqrt{3}}{3}\right)$ **c)** $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

Types of Assessments:

- Quick quizzes for each topic
- Half period quiz
- Full period test

Instructional Strategies/Technology:

- Calculators to work with inverse trig functions using principle values
- Kahoot/Quizzes for practice questions

Unit 8: Graphs of Trigonometric Functions

Standards and Outcomes F-TF Trigonometric Functions F-IF Interpreting Functions	
Students will be able to graph the parent functions for sine, cosine, tangent, cosecant, secant, and cotangent. They will be able to apply transformations to graph any trigonometric function, and use the graphs to answer real life questions.	
Big Ideas: <i>Course Objectives/Content Statements</i> - Graphing trigonometric functions - Solving real word problems using sine and cosine functions and graphs	
Essential Questions <i>What provocative questions will foster inquiry, understanding, and transfer of learning?</i>	Enduring Understandings <i>What will students understand about the big ideas?</i>
- What are important characteristics of sine and cosine parent functions? What are characteristics of periodic functions? - How do different constants in an equation change the graph of a trigonometric function? - What are important characteristics of secant, cosecant, cotangent and tangent parent functions? How can they change? - How can trigonometric functions and graphs be applied to the real world?	Students will understand... - How the sine and cosine graphs are related to the unit circle. How to graph one cycle of sine or cosine, and how the graph continues to repeat. What a midline, amplitude and period are and how to find them. The domain of any periodic function is all real numbers - That the midline, phase shift, amplitude and period are related to transformations from previous parent functions. How to find each one from an equation. - How to graph secant, cosecant parent functions and how they are closely related to sine and cosine. How to graph tangent and cotangent parent functions and how they are related to each other. That the same transformations can be applied to all 6 trigonometric functions. - How to use sine and cosine graphs to model periodic behavior in real life, and make predictions. Vocabulary: Periodic Function, Period, Cycle, Amplitude, Midline, Phase Shift, Sinusoid(al)
Areas of Focus <i>Cumulative Progress Indicators/Specific Standards</i>	Examples, Outcomes, Assessments
F-TF A. Extend the domain of trigonometric functions using the unit circle 1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. 2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of	Sample Assessment Questions: Graph two cycles of each function. Find the amplitude, period, phase shift and midline if applicable. Label at least 4 points in one of the cycles.

angles traversed counterclockwise around the unit circle.

3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosines, and tangent for $\pi-x$, $\pi+x$, and $2\pi-x$ in terms of their values for x , where x is any real number.

4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

B. Model periodic phenomena with trigonometric functions

5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

F-IF 7 e) Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

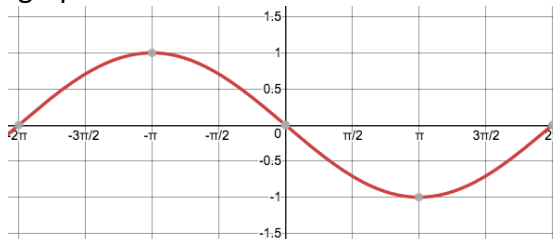
a) $f(x) = -2\sin\left(\frac{1}{2}x - \pi\right) + 1$

b) $f(x) = \frac{1}{2}\cos\left(2x + \frac{\pi}{4}\right)$

c) $f(x) = \tan\left(x - \frac{\pi}{2}\right)$

d) $f(x) = \cot(2x + \pi) - 2$

- Write a sine and cosine equation for the graph



- You took your best friend to an amusement park to ride a Ferris wheel so you could work on your trig homework. You got on the ride 3 feet off of the ground. After 2 minutes you reached the top, which was 37 feet off of the ground.

a) Draw a graph of at least one cycle if $t = 0$ represents the start of the ride. Label at least 5 points on the graph.

b) Find an equation for the graph.

How long will it take to be

c) 20 feet off of the ground?

d) 25 feet off of the ground?

Types of Assessments:

- Quick quizzes for each topic
- Half period quiz
- Full period test

Instructional Strategies/Technology:

- Students use the unit circle to sketch parent functions
- Desmos/Graphing Calculators
- Kahoot/Quizzes for practice questions

Unit 9: Trigonometric Identities

Standards and Outcomes F-TF Trigonometric Functions Students will be able to derive and use fundamental trigonometric identities to manipulate expressions and prove new identities.	
Big Ideas: <i>Course Objectives/Content Statements</i> - Deriving and applying reciprocal trigonometric identities - Deriving and applying Pythagorean trigonometric identities - Verifying and proving trigonometric identities	
Essential Questions <i>What provocative questions will foster inquiry, understanding, and transfer of learning?</i>	Enduring Understandings <i>What will students understand about the big ideas?</i>
- What relationships do the six trigonometric functions have to each other? - How can we use the relationships between different trigonometric functions to prove new identities?	Students will understand... - How all 6 of the trig functions can be written in terms of sine and/or cosine. Each one has a reciprocal relationship with another function. How the cofunctions are related to each other. How to derive the Pythagorean identities. - How to use the identities to rewrite expressions in a simpler way, or use them to prove new identities Vocabulary: Reciprocal Identity, Pythagorean Identity, Cofunction
Areas of Focus <i>Cumulative Progress Indicators/Specific Standards</i>	Examples, Outcomes, Assessments
F-TF C. Prove and apply trigonometric identities 8. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.	Sample Assessment Questions: - Explain why the fundamental Pythagorean identity is true using a diagram - Explain the relationship between two cofunctions - Simplify as much as possible a) $\frac{\cot^2 x}{\tan^2 x}$ b) $\frac{\sin^2 x - 1}{\cot^2 x}$ c) $\cos x \sin^2 x + \cos^3 x$ d) $\sin^2 x \cot^2 x + \sin^2 x$ - Verify each identity using the Pythagorean or reciprocal identities a) $\sec x + \tan x = \frac{1 + \sin x}{\cos x}$ b) $\sin x \cot x + \cos x = 2 \cos x$ c) $\frac{\cot x \sin x}{\cos x} = 1$ d) $\frac{\sin x \cos x \tan x}{\csc x} = \sin^3 x$ Types of Assessments: - Quick quizzes for each topic - Half period quiz - Full period test

	Instructional Strategies/Technology:
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|--|---|
| | <ul style="list-style-type: none">● Kahoot/Quizzes for practice questions● Desmos/graphing calculator to see the relationships between each function graphically |
|--|---|

Unit 10: Trigonometric Equations

Given that $\sin A = -\frac{2}{3}$ and A is in $QIII$,
 $\cos B = \frac{3}{4}$ and B is in QI .

a) Draw both angles. Then find the sine, cosine, and tangent of each one.

Evaluate each

b) $\sin(A + B)$ c) $\cos(B - A)$

d) $\cos(2A)$ e) $\tan(2B)$

Standards and Outcomes

F-TF Trigonometric Functions

Students will be able to use sum and difference equations to find trigonometric ratios of any linear combination of known angles. They will be able to solve equations involving trigonometric functions using the unit circle.

Big Ideas: Course Objectives/Content Statements

- Solving trigonometric equations
- Sum, Difference, Double, and Half angle identities for sine, cosine, and tangent

Essential Questions

What provocative questions will foster inquiry, understanding, and transfer of learning?

Enduring Understandings

What will students understand about the big ideas?

• How can we find the sine or cosine of angles that do not have a reference angle of $\pi/6$, $\pi/4$, $\pi/3$, or $\pi/2$? • What are the possible solutions to equations involving trigonometric functions? How can we use verified identities to help solve complex equations?	Students will understand... • How to use sum and difference identities for sine and cosine to find trigonometric ratios of linear combinations of known angles. How to use those sum and difference identities to derive double and half angle identities for sine, cosine, and tangent • How to use Pythagorean, reciprocal, cofunction, sum, difference, double, and half angle identities along with the unit circle to simplify and solve complex trigonometric equations Vocabulary: Sum/Difference Identity, Double/Half Identity
Areas of Focus <i>Cumulative Progress Indicators/Specific Standards</i>	Examples, Outcomes, Assessments
F-TF A. Extend the domain of trigonometric functions using the unit circle 3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosines, and tangent for $\pi-x$, $\pi+x$, and $2\pi-x$ in terms of their values for x, where x is any real number. 4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. 8. Prove the Pythagorean identity $\sin^2(x) + \cos^2(x) = 1$ and use it to find $\sin(x)$, $\cos(x)$, or $\tan(x)$ given $\sin(x)$, $\cos(x)$, or $\tan(x)$ and the quadrant of the angle. 9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.	Sample Assessment Questions: • • What is the sine of 15 degrees? Prove it. • Solve each equation a) $2 \cos x - 1 = 0$ b) $\tan^2 x + 3 = 6$ c) $\cos x + \cos^2 x = 1 - \cos^2 x$ d) $\cos x \sin 2x - \sin x = 0$ • Why does $\tan x = 1$ have only 2 solutions between 0° and 360°, but $\tan^2 x = 1$ has 4? Types of Assessments: • Quick quizzes for each topic • Half period quiz • Full period test Instructional Strategies/Technology: • Kahoot/Quizzes for practice questions

Unit 11: Solving Triangles

Standards and Outcomes G-SRT Similarity, Right Triangles, and Trigonometry Students will be able to find any missing side length or angle measurement or the area of any triangle. They will be able to apply these skills to real life applications.	
Big Ideas: Course Objectives/Content Statements • Law of sines • The ambiguous case of law of sines • Law of cosines • Areas of any triangle	
Essential Questions <i>What provocative questions will foster inquiry, understanding, and transfer of learning?</i>	Enduring Understandings <i>What will students understand about the big ideas?</i>
• How can we solve triangles that are not right triangles? • How can we find the area of any triangle without knowing the height? • When will solving triangles help with real world problems?	Students will understand... • How to verify and use the law of sines to solve triangles, and when there may be more than one solution. How to use the law of cosines to solve triangles, and how extraneous solutions relate to principle values of sine inverse and cosine inverse • How to use heron's formula to find the area of any triangle • How to apply these skills to the real world Vocabulary: Law of Sines, Law of Cosines, Ambiguous Case (of law of sines)
Areas of Focus <i>Cumulative Progress Indicators/Specific Standards</i>	Examples, Outcomes, Assessments
G-SRT C. Define trigonometric ratios and solve problems involving right triangles 6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. 7. Explain and use the relationship between the sine and cosine of complementary angles. 8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.  D. Apply trigonometry to general triangles 9. (+) Derive the formula $A = \frac{1}{2} ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. 10. (+) Prove the Laws of Sines and Cosines and use them to solve problems. 11. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements	Sample Assessment Questions: • When is it possible to create more than one triangle with three given measurements? Explain why with a diagram. • Solve each triangle if possible. If more than one triangle can be made, solve both a) $a = 14, b = 9, c = 6$ b) $m\angle C = 63^\circ, b = 9, c = 12$ c) $m\angle A = 138^\circ, b = 15, c = 25$ d) $m\angle A = 118^\circ, m\angle B = 22^\circ, c = 24$ • What can go wrong when solving a triangle if you are given an obtuse angle? How can you know you solved the triangle correctly? • How is the law of cosines related to the Pythagorean theorem?

in right and non-right triangles (e.g., surveying problems, resultant forces).

Types of Assessments:

- Quick quizzes for each topic
- Half period quiz
- Full period test

Instructional Strategies/Technology:

- Kahoot/Quizzes for practice questions
- Exploration activity to find out why SSA creates the ambiguous case
- Calculator to solve for missing measurements

Unit 12: Sequences and Series

Standards and Outcomes

F-BF Building Functions

F-LE Linear and Exponential Models

A-SSE Seeing Structure in Expressions

Students will be able to recognize geometric and arithmetic sequences and series. They will be able to write equations

Big Ideas: Course Objectives/Content Statements

Essential Questions <i>What provocative questions will foster inquiry, understanding, and transfer of learning?</i>	Enduring Understandings <i>What will students understand about the big ideas?</i>
- What is an arithmetic sequence? How are they used? - What is a geometric sequence? How are they used? - What is the difference between a sequence and a series? - What is an infinite series? What is sigma notation, and how is it helpful?	Students will understand... - That an arithmetic sequence is a sequence where a constant is added to one term to find the next term. How to find specific terms, and how to write both recursive and explicit formulas for arithmetic sequences. - That a geometric sequence is a sequence where a term is multiplied by a constant to find the next term. How to find specific terms, and how to write both recursive and explicit formulas for geometric sequences. - That a series is the sum of a set of consecutive terms in a sequence. How to quickly evaluate a series given specific terms, a formula, or information about a sequence. - How to evaluate an infinite series, or tell if it converges. How to use sigma notation to write a series in a compact format. Vocabulary: Sequence, Series, Arithmetic, Geometric, Recursive, Explicit, Sigma Notation, Converge(nt), Diverge(nt)
Areas of Focus <i>Cumulative Progress Indicators/Specific Standards</i>	Examples, Outcomes, Assessments
F-BF Build a function that models a relationship between two quantities 2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. F-LE A. Construct and compare linear and exponential models and solve problems 2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	Sample Assessment Questions: - Find an explicit and recursive formula for the arithmetic sequences. a) 4, 16, 28, ... b) $a_1 = -12$, $a_4 = -21$ - Find the sum of the first 50 terms of the sequence in 1b. - Find the sum of the first 100 even integers (starting at 2). - Find an explicit and recursive formula for the geometric sequences. a) 5, -15, 45, ... b) $a_2 = 1$, $a_5 = 8$

A-SSE B. Write expressions in equivalent forms to solve problems

4. Derive and/or explain the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments

- What does it mean for a series to diverge? How can you tell if an arithmetic or geometric series diverges?
- Find the value of the infinite series or state that it diverges. If it diverges, explain why.
 - a) $1 + 1.1 + 1.2 + \dots$
 - b) $2 + 3 + \frac{9}{2} + \dots$
 - c) $3 - 1 + \frac{1}{3} - \dots$

- Write out the first three terms of the series. Find the sum, or state that it diverges.

$$\sum_{n=1}^{\infty} 5n - 3 \qquad \sum_{n=1}^{\infty} 2 \left(-\frac{1}{2}\right)^n$$

- Write each series below using sigma notation. Find the sum, or state that it diverges.

a) $27 + 9 + 3 + 1 + \frac{1}{3} + \dots$ b) $\frac{1}{5} + \frac{2}{5} + \frac{3}{5} + \frac{4}{5} + 1$

Types of Assessments:

- Quick quizzes for each topic
- Full period quiz

Instructional Strategies/Technology:

- Kahoot/Quizzes for practice questions
- Desmos/Graphing Calculator to see graphs of divergent or convergent sequences and series