TAMALPAIS UNION HIGH
SCHOOL DISTRICT

Course of Study Precalculus 1-2

Overview Information

Title of Course: Precalculus 1-2	
Course Author(s): Math Department	**Schools where the course will be taught:** All schools in the TAM district
Length of Course: 1 year	**Subject Area and Discipline:** Mathematics
Grade Levels: 11th and 12th	**Is this course an integrated course?** No
Is this course being submitted for possible UC honors designation? No	**Are you seeking UC approval?** YES **If so, in what area (A-G)?** C, Mathematics
Prerequisites (required or recommended): Successful completion of Advanced Algebra or Advanced Algebra Honors	**Co-requisites (required or recommended):** None

Check all that apply:

- ✓ UC A-G (C)
- ❑ Graduation Requirement
- ✓ Elective
- ❑ Honors/AP
- ❑ ROP

Introduction to the Course

Course Overview:

Students synthesize their conceptual understanding of algebraic, logarithmic, exponential, and trigonometric functions and their inverses. Additional topics include complex numbers, polar coordinates, vectors, conic sections, parametric equations, and an introduction to limits. Sequence and series are an optional time permitting topic.

Throughout all topics studies, students apply their understanding to real world problems through the math models they build, solve, and interpret. All topics studied provide students the experience to deepen their understanding of the properties of mathematics and improve their practice with algebraic manipulations involved in algebraic expressions, inequalities, and equations. The course aims to engage students in the eight mathematical practices of the common core standards as the students develop their understanding of the course concepts.

Students should be engaged in meaningful culturally relevant instructional practices that develop conceptual understanding, procedural fluency and application to real life situations. Teachers will use high impact instructional strategies such as those outlined in the Tam4Ward Instructional toolkit.

Students should be provided ample opportunity to work collaboratively with others to communicate their mathematical thinking. They should be encouraged to share multiple solutions and strategies to solve complex problems. Students should be engaged in productive struggle so that they may learn to persevere and feel a sense of accomplishment in solving complex mathematical problems.

In addition to the content taught in this course, students should continue to work on developing their fluency in skills from previous mathematics courses. Students should develop an awareness of their own learning and level of understanding and be provided appropriate scaffolds and support to make progress towards mastery, regardless of their background, the language of origin, or foundational knowledge.

Ongoing daily formative assessments in the form of openers, warm ups, activities, group work, assignments, and teacher observations should be utilized to inform instruction.

Students will build fluency using a graphing calculator in order to be well prepared for AP Calculus. Various graphing calculator skills are built into each unit and students should have access to these calculators and use them regularly.

California Mathematics Framework
All Unit Outcomes are aligned with the PreCalculus Chapter of the California Framework.

Unit Title: POLYNOMIAL, POWER, & RATIONAL FUNCTIONS

Unit Summary: This unit will include the foundations of the study of functions. Included are the concepts of domain, range, composition, inverses, zeros, extrema and symmetry. Students will graph a variety of polynomial and rational functions, and transformations of parent functions. Key features of these functions will be identified. The properties of functions developed in this unit will be used in future units on exponential, logarithmic and trigonometric Functions.

Unit Outcomes:

- Identify the intervals on which a function is increasing, decreasing, or constant
- Find and interpret the zeros of a function in context
- Describe the key features of six parent functions: identity, absolute value, square root, quadratic, cubic, and reciprocal
- Identify whether functions are odd/even/neither
- Apply vertical and horizontal shifts and stretches and reflections to parent functions to graph the transformed functions
- Combine functions using arithmetic operations and compose functions
- Find an inverse function algebraically and graphically and understand one-to-one functions
- Evaluate and graph piecewise functions to describe scenarios that have different rules for certain intervals of the domain
- Solve polynomial and rational equations and inequalities including complex solutions
- Accurately graph a third or fourth degree polynomial function using the Factor theorem (and information gleaned from the Remainder theorem) and division to identify the x-intercepts. In addition, graph using the concepts of end behavior, multiplicity, domain and range, symmetry, Descartes Rule of Signs, and y-intercept

- Identify relative and absolute extrema using the intervals where the function is increasing and decreasing.
- Identify the location of all vertical and horizontal and/or slant asymptotes, and holes in a rational function, and its x- and y-intercepts. Identify the appropriate domain and range based on the asymptotic information
- Write and solve application problems involving polynomial and rational functions, including optimization problems
- Use a graphing calculator to find x-intercepts and maximum/minimum values of polynomial functions and determine an appropriate window

Sample Unit Assignments:

- Textbook practice - on paper and online
- Desmos activities to develop understanding of transformations and parent functions
- Group assignments on large whiteboards graphing rational functions or solving optimization problems and then presenting to the class.
- Examples of real life application problems

Sample Assessment:

- Identify parent functions by matching equations with graphs
- Describe transformations of parent functions from a graph or equation
- Combine 2 functions by addition/subtraction/multiplication or division and find the appropriate domain for the new function
- Compose a function within another function using appropriate function notation and find the appropriate domain for the new function.
- Find the inverse of a function graphically and algebraically and verify that a function is an inverse using composition of functions.
- Graph polynomial functions by finding x and y intercepts and appropriate end behavior (accurate extreme points not required)
- Graph rational functions by finding x and y intercepts, horizontal/slant & vertical asymptotes, holes and appropriate end behavior
- Use division and factoring as appropriate to find all factors and all zeros of 3rd, 4th or 5th degree polynomials
- Write an equation to represent an optimal quantity with a given constraint and use the graphing calculator to find the maximum/minimum values

Unit Title: EXPONENTIAL & LOGARITHMIC FUNCTIONS

Unit Summary: This unit explores the inverse relationship between logarithmic and exponential functions. Using that relationship, with their appropriate properties, students will solve problems and model real life situations. Students will have the ability to graphically, numerically, and symbolically represent exponential and logarithmic functions in a variety of contexts. Natural logarithms and the natural base are discussed as models for growth and decay problems, including logistic models. The properties and characteristics of exponents will be reviewed and used to translate an expression into equivalent forms.

Unit Outcomes:

- Sketch and analyze the graphs of exponential, logarithmic and logistic functions. Include finding the domain, range and asymptotes
- Expand and condense logarithmic expressions using log properties
- Solve exponential and log equations using a variety of strategies including changing form, taking the ln of both sides and using log properties

- Distinguish between situations that can be modeled with exponential, logarithmic and logistic functions
- Apply understanding of exponential, logarithmic and logistic functions to solve real world problems
- Use graphing calculator to solve problems using natural log and base

Sample Unit Assignments:

- Textbook practice - on paper and online
- Video explaining the wealth gap using exponential functions
- Examples of real life application problems

Recommended Common Assessment:

- Graph exponential and log functions using transformations, including finding the domain, range and asymptotes
- Expand and condense logarithmic expressions using log properties
- Solve exponential and log equations using a variety of strategies including changing form, taking the ln of both sides and using log properties
- Determine and use appropriate models to solve problems involving exponential, logarithmic and logistic functions

Unit Title: TRIGONOMETRIC FUNCTIONS

Unit Summary: This unit views the trigonometric functions in terms of the unit circle in the coordinate plane. This enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. Students will graph these functions in the coordinate plane, relying on their transformational skills of all graphs. Additionally the inverse trigonometric functions are defined, graphed and evaluated.

Unit Outcomes:

- Understand the unit circle in both radian and degree measure
- Calculate linear and angular velocity of a rotating object
- Define the six trigonometric functions in terms of coordinates on the unit circle
- Use special right triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$
- Evaluate trigonometric functions using their reference angles
- Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions
- Use the unit circle to express the values of sine, cosine, and tangent for x , $\pi + x$, and $2\pi - x$ in terms of their values for x , where x is any real number
- Graph the six parent functions, noting the periodic nature of each graph and their domains and ranges
- Transform the graphs using reflections, amplitude change, horizontal and vertical shifts, and period changes
- Determine the zeros, local maximum and minimums, and intervals of increase and decrease for trigonometric graphs
- Write the equation for a given trigonometric graph
- Interpret graphs of sine and cosine graphs that model real world situations
- Evaluate an inverse trig function
- Evaluate functions that include composition of trig functions
- Graph inverse functions of sine, cosine and tangent., noting their domain and range
- Use a graphing calculator to evaluate trigonometric functions and inverse trig functions

Sample Unit Assignments:

- Textbook practice - on paper and online
- Construction of unit circle using special right triangles

- Desmos exploration of transformations of trig functions
- Examples of real life application problems

Recommended Common Assessment:
- Evaluate the six trigonometric functions in radians using the unit circle in exact form
- Graph the six trigonometric functions using transformations, including finding the domain, range, asymptotes, amplitude and period
- Evaluate the inverse of sine, cosine and tangent., noting their domain and range

Unit Title: ANALYTIC TRIGONOMETRY

Unit Summary:
This unit will engage students in the use of the fundamental trigonometric identities to solve equations, simplify expressions, and to prove trigonometric identities. Students will use established trigonometric formulas to evaluate expressions involving trigonometric functions.

Unit Outcomes:
- Learn the Fundamental Identities, (Reciprocal, Pythagorean, Quotient, Even-Odd, Cofunction), Sum and Difference and Double Angle Formulas
- Simplify a trigonometric expression using the identities/formulas above
- Verify an identity using the identities/formulas above
- Find the exact value of a trigonometric expression using the identities/formulas above
- Solve trigonometric equations for general solutions and solutions on a specified interval
- Use the identities/formulas above to solve trigonometric equations for general solutions and solutions on a specified interval
- When time allows, include the Half Angle, Product to Sum and Sum to Product Formulas

Sample Unit Assignments:
- Textbook practice - on paper and online
- Group assignments on large whiteboards proving trig identities then presenting to the class.
- Examples of real life application problems

Recommended Common Assessment:

- Find the exact value of a trig expression using identities and formulas
- Solve a variety of trig equations and specify the general solutions, as well as solutions on a specified interval
- Use known trig identities to verify other identities to be true

Unit Title: APPLICATIONS OF TRIGONOMETRY & POLAR COORDINATES

Unit Summary: This unit applies and further develops skills learned in prior units to model periodic behavior both graphically and symbolically. The Law of Sines and Law of Cosines will be discussed and applied to scenarios that involve distances and navigation. The latter part of the unit introduces polar coordinates as an alternative to rectangular coordinates. Students investigate the graphs of equations that involve polar coordinates.

Unit Outcomes:
- Model and solve problems involving bearing and simple harmonic motion
- Use the Law of Sines to solve oblique triangles, triangles in the ambiguous case, and triangles in applied problems

- Find the area of an oblique triangle
- Use the Law of Cosines to solve oblique triangles and triangles in applied problems
- Use a graphing calculator to store values to be used in subsequent calculations
- Use polar coordinates to locate a point in the plane
- Convert polar coordinates to rectangular and vice-versa
- Identify polar equations: lines, circle, cardioid, limacon, rose, lemniscate
- Understand the symmetry of polar equations
- Graph accurate graphs of polar equations by plotting points and/or using a graphing utility

Sample Unit Assignments:
- Textbook practice - on paper and online
- Examples of real life application problems

Sample Assessment:
- Use the Law of Sines and Law of Cosines to solve oblique triangles
- Find the Area of an oblique triangle
- Plot a point given in polar coordinates
- Convert between rectangular coordinates and polar coordinates
- Graph accurate graphs of polar equations: circles, cardioids, limacons, roses, lemniscates by both plotting points and/or using a graphing utility

Unit Title: VECTORS & PARAMETRIC EQUATIONS

Unit Summary: This unit develops the notion of a vector to describe a quantity that has both magnitude and direction. Students learn how vectors can be used to model applied problems in physics and engineering. An introduction to the use of parametric equations to describe movement along a curve is also explored.

Unit Outcomes:
- Graph a vector and its scalar multiple
- Add and subtract vectors algebraically and graphically
- Find the magnitude and direction of a given vector, and vice versa
- Find the unit vector for a given vector
- Find the angle between two vectors
- Determine the horizontal and vertical components of a vector
- Solve real world applications that can be modeled using vectors
- Graph parametric equations
- Find parametric equations for a curve

Optional, if time allows:
- Find the projection of a vector onto other vectors and its application in Work problems
- Use three dimensional coordinate geometry and vectors in three dimensions
- Find the cross product between vectors and use it to find a vector perpendicular to other vectors
- Plot points in the Complex Plane. Convert a Complex number from rectangular form to polar form. Find products and quotients of complex numbers in polar form. Use DeMoivre's Theorem. Find Complex Roots

Sample Unit Assignments:
- Textbook practice - on paper and online
- Examples of real life application problems

Sample Assessment:
- Graph vectors and decompose a vector into its vertical and horizontal components
- Find a vector magnitude and a unit vector
- Find the angle between vectors

- Decompose a vector into its vertical and horizontal components
- Add and subtract vectors, find the resulting vectors and graph all vectors
- Solve real world applications that can be modeled using vectors
- Graph the curve whose parametric equations are given.

Unit Title: ANALYTIC GEOMETRY: CONICS

Unit Summary: This unit starts with the geometric definition for conic sections as the cross-sections of a cone, resulting in the parabola, circle, ellipse and hyperbola. The conics will be analyzed algebraically using the distance formula to obtain their equations and learn about their various properties. Students will write equations and graph the various conics sections using a variety of information.. Previous learned algebra skills such as factoring and completing the square will be utilized in this work.

Unit Outcomes:

- Graph a parabola from an equation or from given information about its vertex, focus and/or directrix
- Graph an ellipse from an equation or from given information about its center, foci, vertices and major and minor axes
- Graph a hyperbola from an equation or from information about its center, foci, vertices and/or asymptotes
- Write the equation of a conic given its features or its graph
- Identify the type of conic given an equation in standard form
- Complete the square to rewrite an equation in vertex form
- Solve applied problems involving conic sections

Optional, if time allows:

- Systems of conics, graphically and algebraically
- Graph a conic that has a rotation of axes
- Analyze an equation using a rotation of axes

Sample Unit Assignments:

- Textbook practice - on paper and online
- Examples of real life application problems

Sample Assessment:

- Write an equation for a given graph of a conic
- Write an equation for the conic with the given features
- Graph and identify all the features of a given conic equation
- Identify the type of conic given
- Complete the square and rewrite the equation in vertex form
- Solve the applied problem involving a conic section

Unit Title: INTRODUCTION TO LIMITS

Unit Summary: This unit introduces the concept of a limit. Appropriate notation will be stressed for this new concept. The limit at a given point and at positive/negative infinity will be explored. The limit will be determined from a graph, a table of values and an explicit function. Students will compare the slope of a secant line over a given interval (average rate of change) to the slope of a tangent line at a given point (instantaneous rate of change).

Unit Outcomes:

- Use limit notation correctly
- Find the limit of a function given the graph, or using the table on a graphing calculator
- From a graph, determine whether or not the limit exists, and find one-sided limits
- Determine whether a function is continuous given its graph and by use of limits

- Find the limit of an explicit function by substitution
- Find the limit of a function by simplifying algebraically (i.e. factoring, use of conjugates)
- Find the average rate of change of a function on a given interval
- Find the instantaneous rate of change of a function at an input value

Sample Unit Assignments:

- Textbook practice - on paper and online
- Desmos activity - Introduction to Limits
- Examples of real life applicatihttps://docs.google.com/document/d/1SWFkdT8BZ9qWJ8qbS8Hn2aW5js02O03cYkWs5EXN71s/edit?usp=sharingon problems

Recommended Common Assessment:

- Find the limit of a function given the graph of a function, or using the table function on a graphing calculator
- Determine whether the limit of a function exists, and find one-sided limits given the graph of the function
- Determine whether a function is continuous given the graph of the function
- Find the limit of a function by substitution, or by simplifying algebraically
- Find the average rate of change of a function on a given interval and the instantaneous rate of change of a function at an input value

Unit Title: SEQUENCES AND SERIES (optional)

Unit Summary: This unit will investigate both algebraic and geometric sequences and series. Individual terms and sums will be determined. This unit will consider the meaning of convergence and divergence. The concept of a limit will be related to the convergence of a sequence.

Unit Outcomes:

- Write the terms of a sequence, and find the nth term of a sequence
- Use sum notation and find the sum or partial sum of a series
- Determine if a series converges and identify the sum of an infinite series.

Sample Unit Assignments:

- Textbook practice - on paper and online
- Examples of real life application problems

Recommended Common Assessment:

- Write the terms of a sequence, and find the nth term of a sequence
- Use sum notation and find the sum or partial sum of a series
- Determine if a series converges and identify the sum of an infinite series

Recommended Texts and Resources: *pending pilot*

- Precalculus 7th edition, Blitzer (Pearson), using MyMathLab as the online component

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