



ILLUSTRATIVE MATHEMATICS

Stories of Grades K-5





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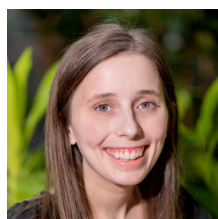




Introduction

At IM, we believe that the mathematical ideas introduced in the primary grades build the foundation for future years. The authors of IM K–5 Math deliberately crafted the curriculum to build mathematical understandings as well as community. Furthermore, IM K–12 Math™ was written with an intentional coherence both within and across grade levels.

In the following pages, you will read the stories of the mathematics of each course of IM K–5 Math, detailed by authors of the curriculum. These stories convey the coherent structure of grade-level content, how concepts are developed for students, and the deliberate choice of tools and representations used in each grade, from kindergarten through grade 5. You will also learn about key design features of IM K–5 Math, including structures to develop strong math communities, instructional routines to invite all students to think deeply about math concepts, and teacher reflection prompts to support responsive instruction.



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THE STORY OF Kindergarten

■ Alex Clayton

Kindergarten Lead, IM K-5 Math v.1



Many 5 year olds love to tell you (and show you) how high they can count. This skill of rote counting, saying the counting words in the same order each time, is a lot like singing the ABCs . While students may sing “l-m-n-o-p”, they may not know that “m” is a letter or what sound “m” makes or that “m” is the first letter in math. The same can be true with counting. Students may be able to say the numbers from 1 to 50, but not have a very good idea what “7” means.

Much of our work in kindergarten is about developing this understanding: Why do we count? What does counting tell us? What exactly is 7? Let’s look at how we develop these ideas in kindergarten.

COUNTING AND CARDINALITY IN UNIT 2: NUMBERS 1-10

One of the things you’ll notice throughout the kindergarten course is that we rarely tell students to count. Instead, we ask “How many are there?” This may seem like a small shift, but it is important for a few reasons:

- + We want students to feel like counting is a way to solve problems and gain information about their world, rather than a task to be performed. Students are naturally curious about how many things there are, and counting is one way to find out!
- + We want to develop students’ understanding of cardinality, that the last number when counting tells us how many objects there are and represents all of the objects counted. (K.CC.B.4.b). Without an understanding of cardinality, students may count a group of objects but not be able to tell you how many there are.

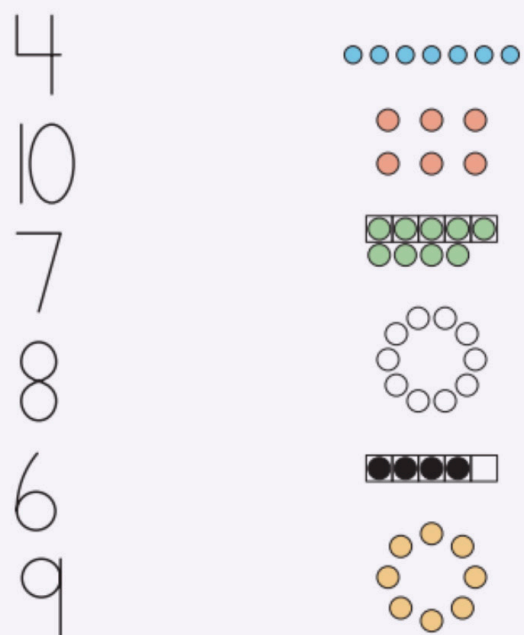


Objects, Images, and Numbers

The first section of Unit 2 is Section A: Count and Compare Groups of Objects. It is easier to work with objects because they can be moved around. Students can move each object as they count it, which helps them ensure that they count each object only one time. When comparing, the objects can be lined up and matched so that students can see which group has more. Then students work with images in Section B: Count and Compare Groups of Images. Counting each image only one time is more challenging because they cannot be moved around, so students develop methods of keeping track of what they've counted.

It is important for students to be able to make connections between many representations. 7 pattern blocks, 7 butterflies, the written number 7, and the spoken word “seven” are all representations of 7. These important connections are happening throughout Section C: Connect Quantities and Numbers. Students count out a given number of objects, which requires them to keep track of their goal (I need 9 counters) while counting. They match written numbers to groups of objects and images, like in this activity in Unit 2, Lesson 13:

13.1: Matching Groups of Images and Numbers



How does this activity bring together the work that students have been doing throughout the unit?

Comparing Numbers

Now that students have looked at many different representations of numbers, they use these different representations to compare written numbers in Section D: Compare Numbers. They may use objects or draw pictures to represent each number and then figure out which is more or less. Or they might compare the numbers using what they've learned about the count sequence over time—that each number is one more than the last (Common Core Math Standard K.CC.B.4.c).

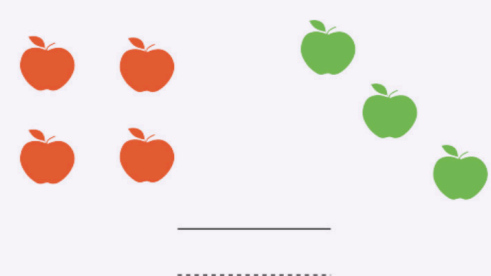
While the focus of Unit 3: Flat Shapes All Around Us is on geometry, students continue to practice counting and comparing through questions like “How many rectangles do you have?” and “Are there fewer triangles or circles?”

FROM COUNTING TO UNIT 4: UNDERSTANDING ADDITION AND SUBTRACTION

Unit 4 is titled “Understanding Addition and Subtraction,” but our work with counting is far from over. Look at this activity from Unit 4, Lesson 2. How are counting and addition connected in this activity?

2.2: How Many Apples?

1.



There are _____ apples.

Some of the first experiences that students are having with addition are counting two groups of objects. While students are using the counting skills that they have been developing, they are also working on important addition concepts. In the activity, they discuss how many red apples there are, how many green apples there are, and how many apples there are all together.

In Section B: Represent and Solve Story Problems, students work with Add To, Result Unknown and Take From, Result Unknown story problems. After making sense of a story problem, students need to represent it.

Let's look at this story problem from Unit 4, Lesson 10:

There were 3 apple slices at the picnic.
Tyler's dad brought 5 more apple slices to the picnic.
How many apples slices are there now?

Students might use objects or drawings to represent the story, building off their work from Unit 2. For example, students may count out 3 cubes to show the 3 apple slices at the picnic. Then they may take out 5 more cubes to show the apple slices Tyler's dad brought to the picnic.

Now that they've shown what is happening in the story, what is left for students to do? Count! Students count all of the counters to figure out that there are 8 apple slices at the picnic. While students may try other methods throughout the year (e.g. recognizing some sums, subitizing, knowing that when one is added the answer is the next counting number), the focus throughout kindergarten is on counting to solve addition and subtraction story problems.

UNIT 5: COMPOSING AND DECOMPOSING NUMBERS TO 10

In the Common Core Standards, kindergartners are expected to "understand addition as putting together and adding to, and understand subtraction as taking apart and taking from." Sometimes teachers focus on adding to and taking from, and skip over putting together and taking apart. In Unit 5 of Kindergarten (Composing and Decomposing Numbers to 10), we pay special attention to putting together and taking apart.

If you think back to the activity where students are counting apples, what they are really doing is putting together red apples and green apples to figure out how many apples there are. Students work on making (composing) and breaking apart (decomposing) numbers throughout this unit. They use objects and drawings to discover different ways to make and break apart numbers, including in story problems like this one from Unit 5, Lesson 7:

Jada made 6 paletas with her brother.
They made two flavors, lime and coconut.
How many of the paletas were lime?
Then how many of the paletas were coconut?



WORKING WITH LARGER NUMBERS IN UNIT 6: NUMBERS 0-20

In Unit 6, students apply the methods that they developed in Unit 2 and throughout the year to count larger groups of objects and images. When counting larger groups, it can be more challenging to count accurately, so students need to attend to counting each thing only one time and keeping track of what they've already counted.

In Unit 7, students also revisit their work with counting, comparing, adding, and subtracting in the context of geometry.

NEXT STEPS: OH, THE PLACES THEY'LL GO!

What begins as rote counting takes us on a journey through much of the major work of the grade. In kindergarten, we have the amazing gift and responsibility to begin building a foundation that students will work on for years to come. How will the work that students do with counting in kindergarten impact them in future years?

ALEXANDRA CLAYTON

Kindergarten Lead, IM K-5 Math v.1 at Illustrative Mathematics. Alex believes that both teachers and instructional materials should empower students to develop their ideas and mathematical identities. She worked as a kindergarten teacher in New Orleans before joining the IM team as the Kindergarten Lead. Now based in Philadelphia, Alex is using her passion for supporting the youngest learners as an Early Childhood Curriculum Specialist at IM. She especially loves working with her very youngest learner, her son Graham, who was born in December 2020.



THE STORY OF Grade 1

■ **Brianne Durst**

*Grade K-2 Writer and Coherence
Reviewer, IM K-5 Math v.1*



Grade 1 teachers have the awesome responsibility of introducing their students to, and helping them build an understanding of, the structure of our number system. This is no small task! Just think about how many tasks in later years depend on an understanding of place value. This understanding that begins to develop in grade 1 is the foundation of number sense.

So, how should we begin students' journey to understanding place value? We want to build on students' prior learning and give them sufficient time and experiences to extend and solidify their understanding. Grade 1 does this by following the Invitation-Deep Dive-Consolidation/Application structure of the IM curriculum.

EXTENDING AN INVITATION IN UNIT 1 (ADDING, SUBTRACTING, AND WORKING WITH DATA)

Unit 1 Section A is an invitational welcome for both students and teachers.

Students are invited to get to know one another and experience joy in their new math community by playing games that focus on addition and subtraction within 10, something familiar from kindergarten. Beginning the year with fun, low-stakes activities can quell nerves that students may have as they enter a new school year. This can go a long way in creating a math community where all students feel capable, comfortable





sharing ideas, and valued as an important member of the community. Unit 1 Section A also launches the year-long work of developing fluency with addition and subtraction within 10. Giving students time to work with these smaller numbers helps set them up for success with what is to come in later units.

Section A is an invitation to teachers to learn about the students in their new class in an authentic way. As teachers, we start a school year eager to get to know our students as mathematicians. While students play games, the teacher has time to listen to students explain their ideas and reasoning. Through observation and conversation, teachers learn a lot about their students as mathematicians and how they are working with numbers through 10. The data work in Sections B and C allows for continued community building as students learn about each other as they create surveys and representations of class data. They ask and answer questions about their data, continuing their work with addition and subtraction within 10. Future learning will be built on these understandings.

BUILDING ONTO THE FOUNDATION IN UNIT 2 (ADDITION AND SUBTRACTION STORY PROBLEMS) AND UNIT 3 (ADDING AND SUBTRACTING WITHIN 20)

In Unit 2, the focus shifts to making sense of and solving different types of story problems. The numbers used in the problems are intentionally kept within 10 to allow students to continue to build fluency with addition and subtraction in this range.

Then, Unit 3 Section A offers students the chance to assess their fluency within 10 and identify any facts they need to continue working with to become fluent. Through this work from the beginning of grade 1, students develop a strong foundation on which to build. What would be helpful to know about place value to find the value of this sum?

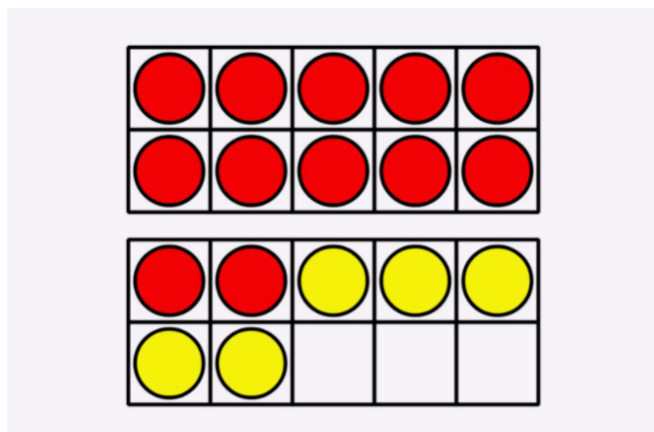
$$37 + 25$$

The expression above is one that students will encounter in Unit 5. Asked to find the value of this expression in the beginning of grade 1, students may

count out 37 cubes and 25 cubes and count them all to find the total. Others may start with 37 and count on 25 more. Neither strategy is very efficient when working with these numbers, but both are reflective of the work first graders have done up to this point.

In kindergarten, students worked with teen numbers and saw that all teen numbers have 10 and some more. Without labeling them, kindergarteners were working with groups of 10 ones. In Unit 3 Section B, first graders build on this work and learn that the group of 10 ones makes a new unit called a ten. They continue working with teen numbers and see that each teen number has 1 ten and some more ones. Throughout this work students see teen numbers represented on double 10-frames. This representation allows students to see both the unit of ten, and the 10 ones inside that unit. This is essential as students develop a true understanding of the unit of ten.

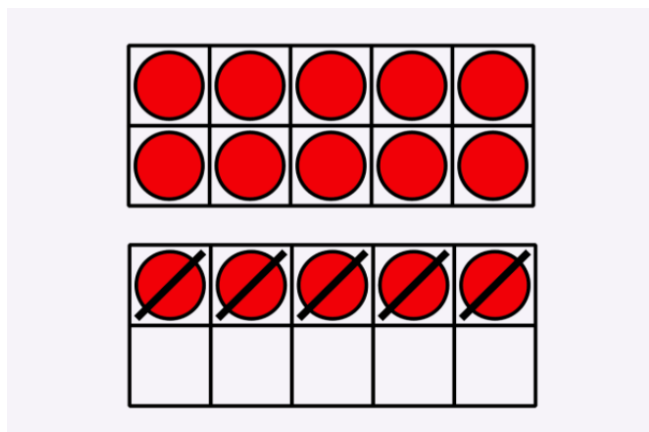
In grade 1, students have their first experiences using place value to add and subtract. They add and subtract teen numbers and one-digit numbers, beginning with expressions that will not require making a new unit of ten, such as $14 + 3$ or $12 + 5$. They notice that the unit of ten doesn't change, and relate the sum to adding and subtracting of ones.



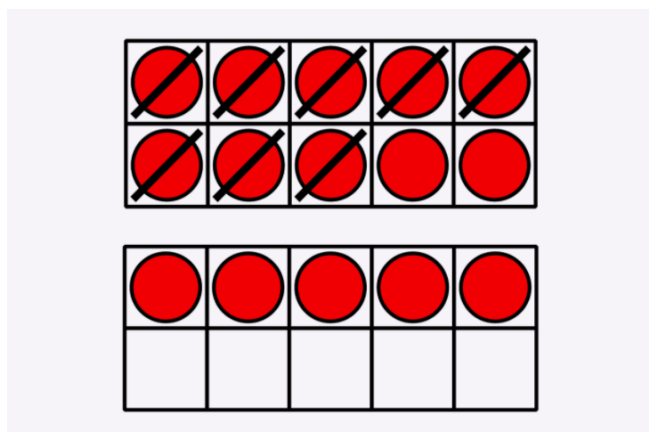
Students then progress to adding and subtracting teen numbers and one-digit numbers that will require making a new unit of ten or breaking apart the unit of ten. Students rely heavily on their previous work with numbers within 10 to support them in finding the value of these sums and differences.

How do both of these strategies rely on fluency with numbers within 10?

*Diego is playing Number Card Subtraction.
He started with 15 and then picked an 8.
He started out by doing this:*



*What could Diego do next to find the difference?
Andre was also finding the value of $15 - 8$.
He started out by doing this:*



What could Andre do next to find the difference?

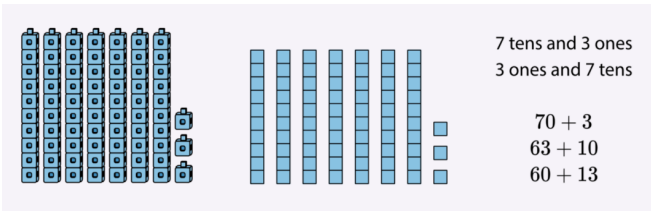
IM K–5 Math: Unit 3 Lesson 23

DEEP DIVE INTO NEW LEARNING IN UNIT 4 (NUMBERS TO 99) AND UNIT 5 (ADDING WITHIN 100)

In Unit 4, students use what they have learned about teen numbers and the unit of ten to generalize the structure of two-digit numbers, relating the two digits to the number of tens and ones. They interpret and use multiple representations of numbers up to 99, such as connecting cubes, base-ten diagrams, words,

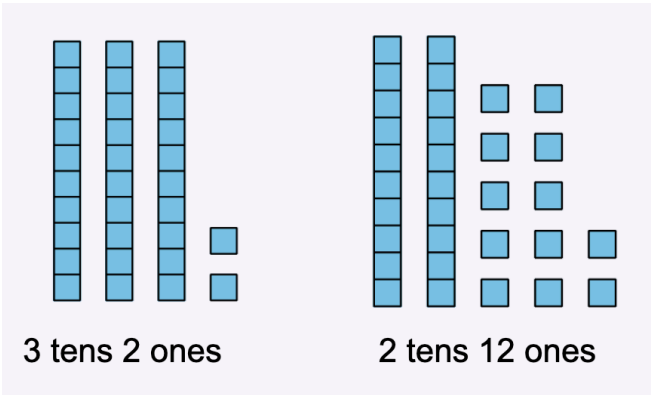
and expressions. Connecting cubes in towers of 10 and singles are used throughout grade 1, rather than base-ten blocks, so that units of ten can be physically composed and decomposed with the cubes.

Although students work physically with connecting cubes, they interpret base-ten diagrams, recognizing the diagram as a simplified image of the connecting cubes. This helps students make sense of a more efficient way of drawing diagrams to match their connecting cubes. As students develop their understanding of place value and work with each of these representations, they are able to compare any two-digit numbers by comparing the number of tens, and, if needed, the number of ones.



IM K-5 Math: Unit 4 Overview

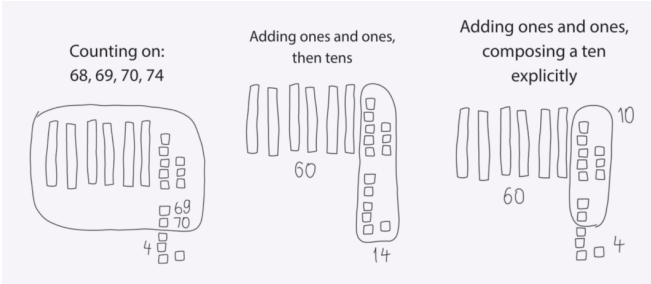
In Unit 5 students will add 2 two-digit numbers that require making a new unit of ten. In preparation for this, students interpret and build two digit numbers with different amounts of tens and ones. For example, students learn to recognize 32 as both 3 tens 2 ones and 2 tens 12 ones.



IM K-5 Math: Unit 4 Section D Overview

Students then begin adding within 100 following a similar learning progression. They add 2 two-digit numbers that do not require making a new unit of ten, such as 42 + 35. They use their place value understanding to add tens and tens and ones and ones, or add on adding tens first then ones.

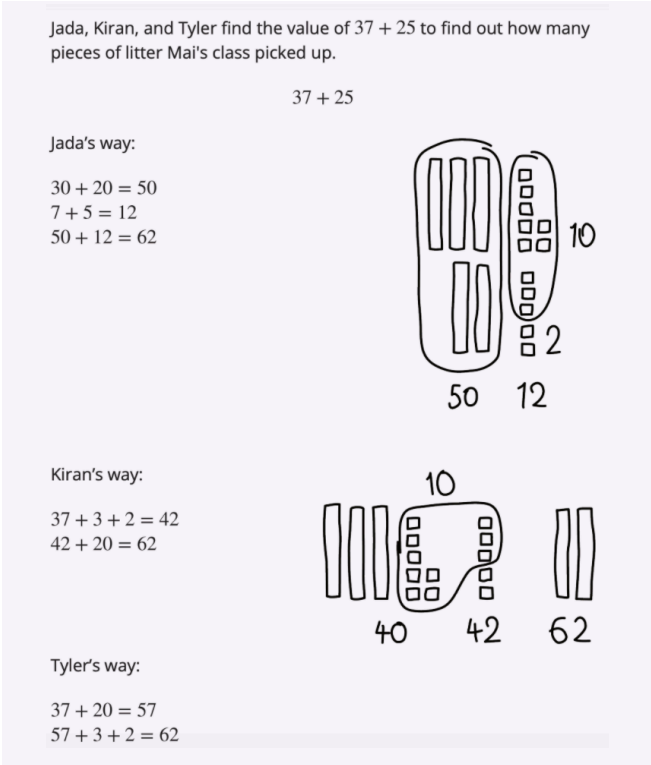
Next, students add a two-digit and a one-digit number that requires making a new unit of ten, such as 68 + 6.



IM K-5 Math: Unit 5 Section B Overview

Now, we are ready to come back to the expression presented earlier: 37 + 25. Students are now equipped to add within 100 in more efficient ways. They discuss different strategies based on place value, such as adding tens and tens and ones and ones. They might add on to make a new ten, then add the rest. Students represent their thinking using connecting cubes in towers of 10 and singles, base-ten drawings, and equations.

Mai and her classmates volunteer to clean up the local park. They pick up 37 plastic bottles and 25 paper wrappers. How many pieces of litter did they pick up all together?



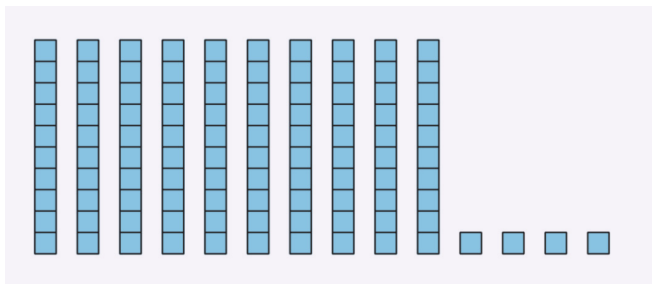
IM K-5 Math: Unit 5 Lesson 11

Consolidate and Apply Learning in Unit 6 (Length Measurements Within 120 Units)

According to standard 1.NBT.A.1, students should count to 120, read and write numbers in this range, and represent a number of objects in this range with a written numeral. Working toward this standard allows students to apply their new understanding of place value.

While the focus on linear measurement in Unit 6 may feel unrelated to the place value work in the previous units, it is actually a wonderful opportunity to consolidate and apply this learning. After learning proper measurement techniques, students use base-ten cubes to measure lengths up to 99 length units, then measure lengths even longer than 99 length units. In order to determine the total number of length units, they organize the cubes into groups of 10. They see that 10 tens is 100. A hundred is not discussed as a unit in grade 1, but the written notation is introduced so students can read and write the numbers 100–120.

10 tens 4 ones is 104.



IM K–5 Math: Unit 6 Lesson 8

Grade 1 teachers have the awesome responsibility of introducing their students to, and helping them build an understanding of, the structure of our number system. The IM Grade 1 materials support teachers in doing this by taking time to build the necessary foundational skills and strategically building upon and applying these skills.

NEXT STEPS

Consider the work you do to help students build an understanding of place value. In what ways do you support building on foundational skills to allow for a strong conceptual understanding of the base-ten structure of numbers?



BRIANNE DURST

Grade K–2 Writer and Coherence Reviewer, IM K–5 Math v.1
Brianne believes that all students deserve the chance to learn mathematics as a subject that makes sense, that this sense-making should be done individually as well as within a math community, and that students need to believe in the value of sharing and reflecting upon their ideas as well as the ideas of others in their math community. She was a classroom teacher in grades 2 and 4 for a total of 12 years before becoming a math coach. She spent 5 years as a math coach and interventionist for grades K–2 at Community Roots Charter School in Brooklyn, NY. She is also a member of the Math Collective, a group that offers professional learning materials and experiences to teachers in New York City. Brianne led the Grade 1 revisions for v.1 and continues to work at IM as a PK–5 Curriculum Specialist. Through her work with IM, she is able to help provide students with a mathematics experience that makes sense and encourages learning with a community of peers.



THE STORY OF Grade 2

■ Mike Henderson

IM K–5 Math Curriculum Specialist



Teaching addition and subtraction in grade 2 is a challenging balancing act. As students move from numbers within 100 to numbers within 1,000, they need to use approaches that involve directly representing the quantities they are adding or subtracting and be supported in reasoning more abstractly. However, we don't want students to feel they always have to represent the quantities they add and subtract with objects or drawings, especially within 20. Although we want students to use increasingly abstract and efficient strategies based on place value, we don't want to push them too fast toward the algorithms they will use in grades 3 and 4 or position these strategies as the "right" way to add or subtract.

When considering different ways to tell the mathematical story of grade 2, we wondered:

- + How do we make sure students are given time to consolidate their understanding of numbers and operations within 100 before extending their place value understanding within 1,000?
- + How do we make sure students spend time making sense of place value strategies without feeling forced to use procedures that do not make sense to them?
- + How do we make sure students (and their teachers) do not feel pressure to master a laundry list of different strategies and representations for adding and subtracting?



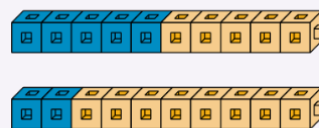


We know students enter grade 2 with a variety of approaches for reasoning about adding and subtracting numbers. We want them to know that the ways they think about addition and subtraction continue to make sense even as the numbers get substantially larger. Although understanding place value and using place value understanding to add and subtract is the major work of the grade, a goal in writing the story of grade 2 was to make sure students focus on foundational math concepts without feeling funneled toward procedural ways of thinking.

THE INVITATION TO GRADE 2 MATH

The initial lessons of grade 2 invite students to learn more about each other and the ways they do math through activities and games that focus on addition and subtraction within 20. They continue to learn more about each other and their interests by asking questions and organizing data with picture graphs and bar graphs. The activities provide opportunities to build a joyful math community where students are encouraged to share ideas and celebrate different ways of thinking.

Teachers learn about the ways students think about addition and subtraction. An important focus in the early lessons is on combinations of 10 and making 10. These early activities from kindergarten and grade 1 help students not only recall their work with combinations of numbers that make 10, but also recall their work with place value. Later activities in the section give teachers an opportunity to learn about how students think about tens and ones as they represent numbers within 50 and add them.



$$\begin{aligned} 10 - 5 &= \underline{\quad} \\ 5 + \underline{\quad} &= 10 \\ 2 + \underline{\quad} &= 10 \\ 10 - 8 &= \underline{\quad} \end{aligned}$$

IM K-5 Math: Grade 2 Unit 1 Lesson 2 Activity 2

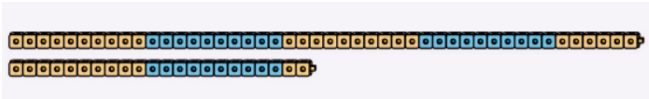
The picture graphs and bar graphs also provide an accessible way for students to show different ways they may add or subtract. As they answer questions about the graphs, students show different ways to add

and compare quantities within 20, including strategies like counting on. Although counting strategies as students' go-to method will fade throughout the year, it is a useful strategy that makes sense to many students and one we want students to continue to use with flexibility. The number range is intentionally kept within 20 in this section to make the work accessible to all students, while nudging students to reason more abstractly about the graphs.

Teachers also have opportunities to learn about how students add within 100 with a focus on how they think about adding using their understanding of place value. The numbers used are intentionally within 100 to give a purpose for introducing a more abstract representation to make sense of the quantities in Compare problems—the tape diagram. Although the tape diagram is not a representation used to make sense of performing computations, it does help students make sense of the relationships between quantities in story problems and apply what they know about the relationship between addition and subtraction to solve problems.

SUBTRACT WITH STRATEGIES BASED ON PLACE VALUE

Where Unit 1 elicits what students already know about addition and subtraction, Unit 2 extends their understanding of subtraction within 100 using strategies based on place value and other ways that make sense to them. Students build on counting strategies to consider how they might count on by tens and ones.



- + Make trains with cubes.
- + Find the total number of cubes you and your partner used. Show your thinking.
- + Find the difference between the number of cubes you and your partner used. Show your thinking.

IM K-5 Math: Grade 2 Unit 2 Lesson 1 Activity 2

Students are then introduced to base-ten blocks as a way to represent tens and ones. Students explore these blocks and compare them to their use of connecting cubes. Although students may continue to use connecting cubes throughout the unit, as the numbers increase in size, they begin to see the benefit of using base-ten blocks to quickly show tens and ones.

What is the same and what is different between these tools?

Find the number that makes the equation true. Show your thinking using cubes, blocks, or drawings.

41 + ____ = 84

IM K-5 Math: Grade 2 Unit 2 Lesson 2 Activity 1

This leads to more explicit conversations about place value. Students show their understanding of the value of each digit as they explain the ways they represent situations using base-ten blocks and expressions.

Diego started with 5 tens and 5 ones. Represent Diego's blocks with base-ten blocks. How many does he have?	Diego took away 2 tens. Draw a representation to show what happened to Diego's blocks.	Write an equation to show how many Diego has now.
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IM K-5 Math: Grade 2 Unit 2 Lesson 5 Activity 2

Students make sense of base-ten drawings. Students first see and use these drawings in grade 1 when they are a direct representation of the connecting cube towers of 10 and single cubes. Now, students may interpret them as connecting cubes or as base-ten

blocks. Over the course of the year, students continue to use base-ten blocks and drawings to represent decomposing tens in different ways.

Diego and Elena drew base-ten diagrams to find the value of $82 - 9$.

Diego

Elena

IM K-5 Math: Grade 2 Unit 2 Lesson 6 Activity 1

Although the unit introduces base-ten blocks and decomposing a ten as a strategy for subtraction within 100, both are built on ways students have reasoned about and represented operations with two-digit numbers in grade 1. Students are asked to make sense of and try out different ways to represent decomposing a ten when subtracting by place, but they are not expected to use one particular strategy or representation at the end of the Unit 2 to subtract. Students should continue to use the approaches that make sense to them.

DEEP DIVES ON LENGTH AND A HUNDRED
Length and the number line

In Unit 3, students extend their understanding of measuring length as they make sense of and use standard length units. Students measure, compare, and add lengths using a variety of concrete and abstract tools. This helps all students continue to consolidate their understanding of the structure of the number system and prepares them to make sense of the number line. Students' understanding of length units and the way length units are represented on a ruler is essential to work with the number line for addition and subtraction in Unit 4 and lays a foundation for students' understanding of fractions as numbers in grade 3.

In Unit 4, students continue to deepen their number sense within 100 as they locate numbers and reason about their relative size. They also use the number line to deepen their understanding of addition and subtraction. In particular, the number line helps students think about subtraction as the distance or length between two numbers on the number line. This understanding helps students flexibly choose computation strategies within 100 and decide when some strategies might be more useful than others. For example, compare these different approaches for finding $81 - 79$ that students investigate in Unit 7 Lesson 1.

Tyler

$81 - 70 = 11$
 $11 - 9 = 2$

Elena

$81 - 79 = 2$

IM K-5 Math: Grade 2 Unit 7 Lesson 1 Activity 1

Although the number line is not a representation that easily shows composing or decomposing base-ten units, it does help students show and explain strategies based on counting on or counting back by place. Students also see the strategies they used when adding and subtracting within 20—such as making or getting to a multiple of 10—continue to work with larger numbers.

Diego is finding the value of $33 + 45$. He says he can count on by tens, then by ones. He used a number line to show what he means.

Diego and Tyler found the value of $53 - 29$. Their work is shown.

Diego's Method:

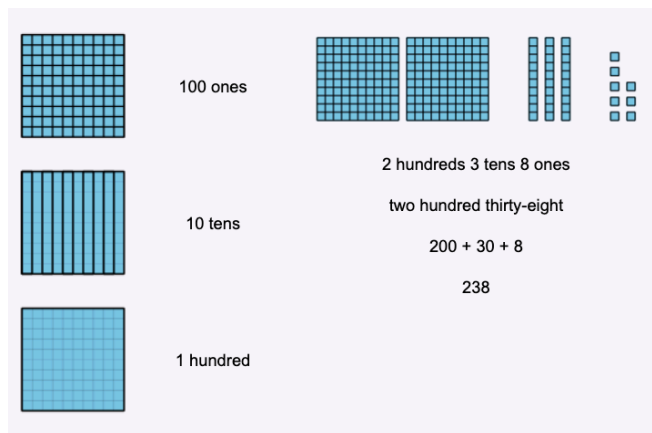
Tyler's Method:

IM K-5 Math: Grade 2 Unit 4 Lesson 10 Activity 2 and Unit 4 Lesson 11 Activity 1

A hundred

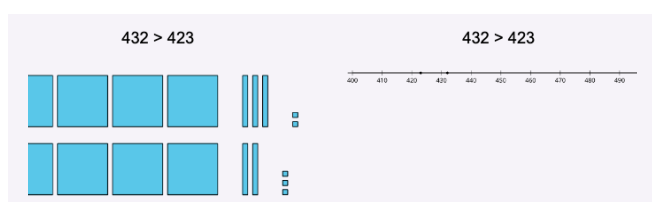
In Unit 5 students make sense of a hundred as a base-ten unit.

Finally! Three-digit numbers and place value! Many teachers may recognize the content of this unit as the traditional focus of early units of grade 2. Although we know that many students are ready to extend their understanding of place value to include a hundred at the beginning of grade 2, we also believe that in building on students' understanding and strengths with working within 100 in earlier units, all students will be able to make sense of a hundred as a unit and will be less likely to feel like strategies based on place value are the only (or "right") ways to add and subtract.



IM K–5 Math: Grade 2 Unit 5 Section A Overview

Because the introduction of three-digit numbers comes after extensive work with numbers on the number line, students are also able to use this tool as a way to compare three-digit numbers. Although the focus in the unit is on using place value reasoning to compare numbers, the number line supports students' ongoing development of number sense and counting within 1,000. The development of a strong sense of the relative size of three-digit numbers helps later when students consider the relationship between numbers in an expression before selecting the approach for finding its value.



How do both representations show that $432 > 423$? How does each representation connect to students' development of strategies for addition and subtraction within 1,000?

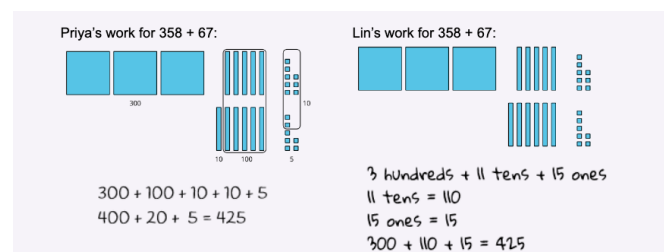
Adapted from IM K–5 Math: Grade 2 Unit 5 Lesson 9 Activity 2

CONSOLIDATION

In Units 6, 7, and 8, students consolidate their understanding about addition and subtraction within 100 and their understanding of place value within 1,000. In Unit 6, students extend their understanding of number lines and skip-counting to tell time to the nearest hour and minute on analog clocks. They also add and subtract within 100 as they solve problems with money. In Unit 8, students skip-count and use expressions with equal addends to determine whether quantities are even or odd, and to find the total number of objects in an array. This is the foundation for their work with multiplication in grade 3.

In Unit 7, students add and subtract within 1,000. No new strategies or representations are introduced in this unit. Students build on the ways they have added and subtracted numbers since Unit 1 with the ways they represented numbers within 1,000 in Unit 4. This includes inviting students to think about ways they can use the relationship between addition and subtraction and counting on strategies.

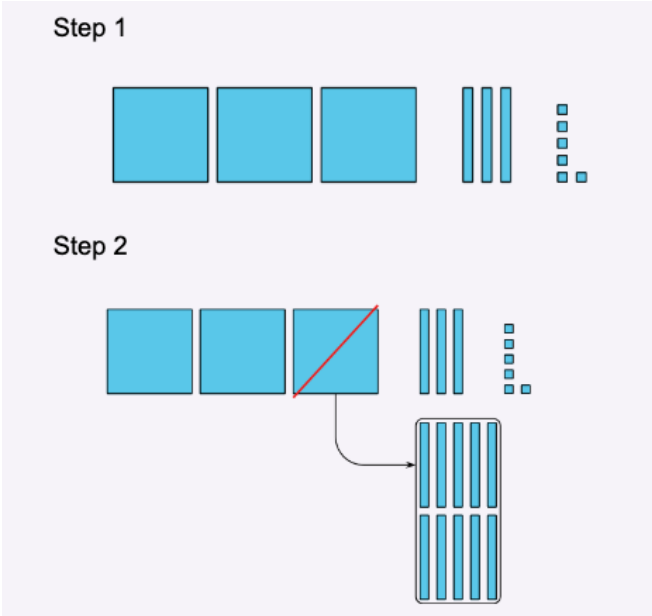
They make sense of ways to compose and decompose units, and explore different ways to represent their thinking. No one approach is positioned as the "right" way to add or subtract by place, and the emphasis is intentionally on place value concepts rather than procedures to avoid moving too fast toward the standard algorithm.



IM K–5 Math: Grade 2 Unit 7 Lesson 8 Activity 2

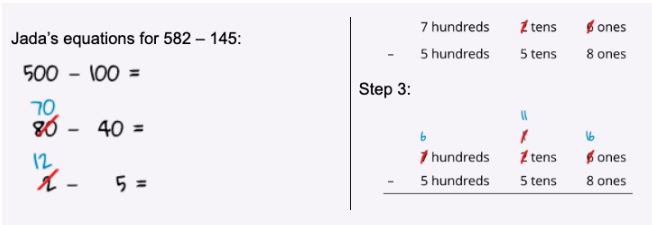
The unit lays a foundation for understanding and using algorithms in later grades. For example, the way we present drawings to students in the materials mirrors the annotation used in many subtraction algorithms.

Mai used base-ten blocks to find the value of $336 - 52$. Then, she started making a diagram to show her work. Explain what Mai did in Step 2. Show what Mai could do next to find the difference.



IM K-5 Math: Grade 2 Unit 7 Lesson 16 Activity 1

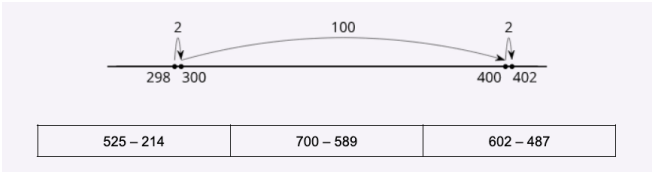
Students also see recording methods that are precursors to the algorithms they will see in grade 3 and build off the ways they represented three-digit numbers in Unit 4. However, they are not expected to use or master these recording methods. The mathematical purpose is for students to make sense of these methods and to invite students to try them. They connect the ways that make sense to them to these more abstract recording methods. They also have opportunities throughout the unit to share their own thinking, including any invented algorithms, and reflect on ways to make their thinking clear for themselves and others.



IM K-5 Math: Grade 2 Unit 7 Lesson 15 Activity 1

Throughout each section of the unit, students flexibly choose strategies and explain why they chose a particular strategy based on the numbers in the expressions. They continue to see that the strategies they learned in earlier grades continue to work with larger numbers.

Jada's thinking for $402 - 298$



What ways might you expect students to find the value of these differences? How might they show that they are thinking about the relationship between the numbers and choosing strategies flexibly and strategically? How do these strategies connect to the strategies they shared as early as Unit 1?

IM K-5 Math: Grade 2 Unit 7 Lesson 16

NEXT STEPS

From the beginning of grade 2, students use the approaches that make sense to them as they add and subtract within 100. They are introduced to a limited selection of new representations—base-ten blocks and number lines—as ways to represent their operational thinking. Each strategy they use is an extension of strategies they have used before with smaller numbers. How do these choices in our story help students develop an understanding that math makes sense? How does it prepare students for the work of adding and subtracting with algorithms in grade 3 and the standard algorithm in grade 4?

MIKE HENDERSON

Mike is passionate about creating learning environments and resources that support teachers as they listen and learn from the ideas and experiences of their students. Mike began his teaching career as a grade 1 teacher in Gallup, NM and taught grade 3 in Phoenix, AZ. After earning his National Board Certification, he began work as a math instructional coach and academic consultant. He has supported teachers, coaches, principals, and superintendents across the country in their work toward creating math classrooms that support the learning of all students. At Illustrative Mathematics, Mike wrote and reviewed lessons for grades 1-4 for IM K-5 Math v.1.



THE STORY OF Grade 3

■ Mike Henderson

IM K–5 Math Curriculum Specialist



As a former grade 3 teacher, I know first hand how daunting teaching math at this level can be. On top of developing fluency with addition and subtraction within 1,000, students need to learn about new topics like multiplication, division, and fractions—topics that may feel big and challenging. To add to the stress, grade 3 is often the first year with end-of-year, high stakes standardized testing. Pressure can build up for both teachers and their students and create a vicious cycle where teachers may look for gaps to close rather than look for the brilliance all students bring into the classroom.

When planning the story of grade 3 math, we had a goal of intentionally disrupting this cycle by:

- + ensuring that all students experience the “new” topics of multiplication, division, and fractions as clearly connected to what they already know
- + ensuring that all students are given time to develop deep conceptual understanding of multiplication and division, build fluency from this understanding, and have ample opportunities to apply their understanding in a variety of contexts throughout the year

In order to reach these goals, we wrote a math story that builds students’ conceptual understanding of multiplication at the very beginning of the school year.





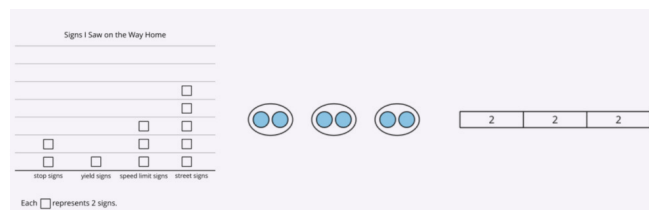
THE INVITATION TO GRADE 3 MATH

Much like the early units in Grades K–2, Unit 1 in Grade 3 begins by inviting students and their teachers to learn more about one another by asking questions, collecting data, and organizing the data with graphs. As students learn more about each other, their interests, and the different ways they do math, they build the foundations of their math community.

The scaled picture and bar graphs also provide a natural opportunity to think about ways to group and count objects other than by counting by 1. This is an extension of students’ work in grade 2 with skip-counting by 2, 5, and 10. The activities help students connect their understanding of different ways to group or count objects to build a conceptual understanding of multiplication. Rather than present multiplication as something new, the activities provide opportunities for students to see multiplication as a new way of thinking about math they’ve done before.

Students make sense of different ways to represent equal groups. The group sizes are intentionally 2, 5, and

10 throughout this unit to continue to build on students’ strengths and allow them to focus on sense making rather than on calculation. They are introduced to different representations they may use throughout the year to represent their thinking, including equal groups drawings, tape diagrams, arrays, and expressions.



IM K–5 Math: **Grade 3 Unit 1 Lesson 9 Activity 1** and **Lesson 10 Activity 1**

*To learn more about how the work with arrays in Unit 1 builds coherence with students’ work from grade 2, check out **Fluency Development Within and Across the Grades** in IM K–5 Math™, part 3: Multiplication and Division.*

In Unit 2, Area and Multiplication, students deepen their conceptual understanding of multiplication through the study of area. Area concepts are

introduced early on in the story of grade 3 because they build on the geometry, measurement, and multiplication foundations from K–2. As early as kindergarten, students learned that they can build a shape by putting together other shapes and can make new shapes by decomposing a given shape. In grade 2, students partitioned rectangles into equal-size squares. So, although defining area and measuring area are new, area concepts build on what students already know, and the activities in Unit 2 intentionally help make these connections.

Students use unit squares to measure the area of rectangles and relate the structure of gridded rectangles to the structure of arrays. They begin to find the value of products that include group sizes other than 2, 5, and 10. As they progress from gridded rectangles to rectangles with only the sides labeled, they make sense of the area diagram that will be used in later units.

$$6 \times 3 = 18$$



IM K–5 Math: Grade 3 Unit 2 Narrative

By the end of Unit 2, students have developed a stronger conceptual understanding of multiplication. They have engaged in sense-making about representations that they will use throughout the course. This conceptual foundation allows students to build fluency with multiplication and division throughout the year. For more, check out [**By the End of Grade 3: Developing Fluency with Multiplication**](#) by Grade 3 Lead Writer Zack Hill and [**Fluency Development Within and Across the Grades in IM K–5 Math™, part 3: Multiplication and Division**](#) by 2–5 Lead Writer Dionne Aminata and Grade 5 Lead Writer Sarah Caban.

BUILDING ON FOUNDATIONS

Addition and subtraction

In Unit 3, the focus shifts to Wrapping Up Addition and Subtraction Within 1,000, as well as rounding within 1,000. The unit is placed here, rather than at the

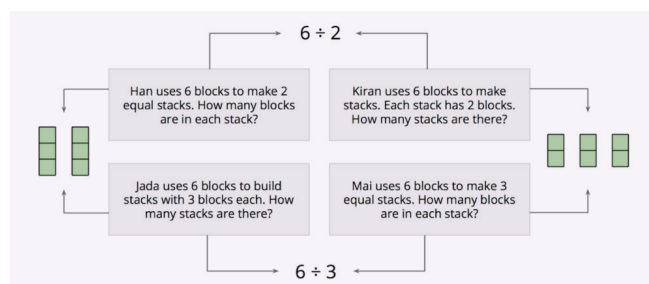
beginning of the year, for a few reasons. First, we want students and their teachers to make sense of multiplication in accessible ways as early as possible to give time to build fluency across the year. If addition and subtraction were the beginning of the story, we worry that any unfinished learning students have from K–2 may result in decisions that delay students' opportunities to engage with grade-level work.

Second, the work of Unit 3 allows students to revisit addition and subtraction with familiar base-ten representations. This focus on place value allows teachers to identify student strengths that can be used as they begin to multiply and divide with larger factors and dividends in later units, including multiples of 10. Unit 3 also revisits the number line as students think about rounding to the nearest multiple of 10 and 100 before they use the number line to make sense of fractions as numbers in Unit 5.

Last, the work of Unit 3 includes early opportunities for students to make sense of two-step word problems that include addition, subtraction, or multiplication. Students will continue to have opportunities to build their fluency with addition and subtraction throughout the year in warm-ups, centers, and their work with measurement in Unit 6.

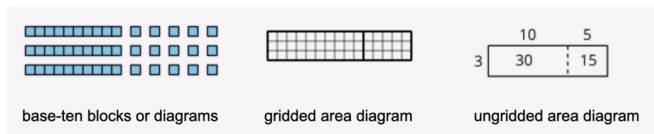
Making sense of division

In Unit 4, students build on their conceptual understanding of multiplication to make sense of division. They connect division to multiplication with an unknown factor to make sense of and solve “how many in each group” and “how many groups” situations.



IM K–5 Math: Grade 3 Unit 4 Section A Narrative

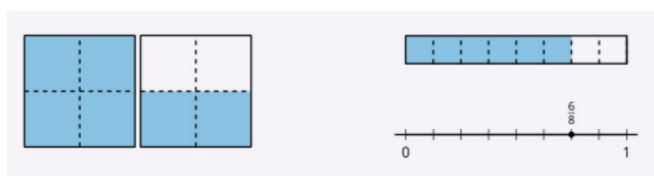
Students also build on work with area and area diagrams to use the distributive property to decompose factors to make it easier to find the value of products.



IM K-5 Math: Grade 3 Unit 4 Section C Narrative

Fractions

Unit 5 is called “Fractions as Numbers.” In grades K–2, students learned about composing and decomposing shapes from equal-size pieces. They learned names of these equal-size pieces—halves and fourths—in grade 1 and thirds in grade 2. Now, in grade 3, students learn that they are not simply names of equal-size pieces or parts of shapes, but are numbers. Students partition fraction strips and rectangles as they deepen their understanding of fractions and fraction notation. Later they use the number line to locate fractions and compare fractions with the same numerator or denominator.

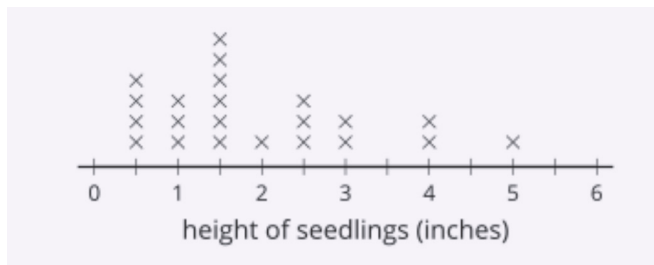


IM K-5 Math: Grade 3 Unit 5 Narrative

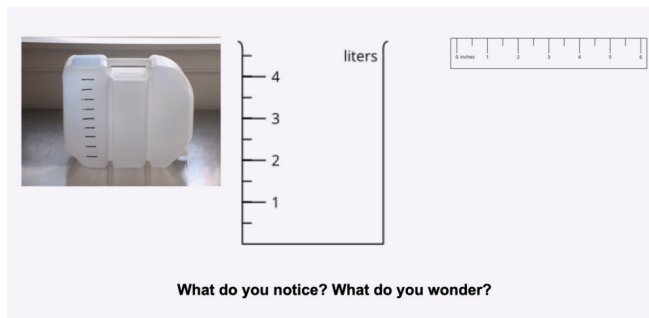
Consolidating learning with four operations

In Units 6 and 7 students apply learning from previous units and make connections within and across mathematical concepts.

In Unit 6, “Measuring Length, Time, Liquid Volume, and Weight,” students connect fractions to measurement. They make connections between whole numbers and fractions on number lines and measurement tools. This builds a foundation for work with addition and subtraction of fractions and fraction equivalence in grade 4.



IM K-5 Math: Grade 3 Unit 6 Lesson 4 Activity 1



IM K-5 Math: Grade 3 Unit 6 Lesson 8 Activity 2

The unit also provides opportunities for students to continue developing fluency with addition and subtraction within 1,000 as they solve problems involving time and measurement. They have new opportunities to work with one and two-step problems that involve all four operations.



A lemonade stand at the fair makes 132 liters of lemonade a day. When Clare and Tyler stopped by the stand, there were 90 liters left.

At the end of the day, there were 56 liters of lemonade left, which the vendor put on sale in 4-liter jugs.

IM K-5 Math: Grade 3 Unit 6 Lesson 14 Activity 2

Although the overall focus of Unit 7, “Two Dimensional Shapes and Perimeter,” is on geometry, students also revisit area and apply understandings about the four operations as they solve problems with area and perimeter. Unit 8, “Putting It Together,” includes additional opportunities for students to practice their fluency with multiplication and division within 100.

NEXT STEPS

From the very beginning, the story of grade 3 works to use the coherence in the standards to ensure that students do not see multiplication, division, and fractions as brand-new concepts, but as natural extensions of what they already know about math. We position topics that were a major focus in K–2—addition, subtraction, and place value—so that it supports the major work of grade 3, rather than derails it. How do these choices help

give all students opportunities to access grade-level work throughout the year? How do they help students make meaningful connections to the math they already know and understand?

MIKE HENDERSON

Mike is passionate about creating learning environments and resources that support teachers as they listen and learn from the ideas and experiences of their students. Mike began his teaching career as a grade 1 teacher in Gallup, NM and taught grade 3 in Phoenix, AZ. After earning his National Board Certification, he began work as a math instructional coach and academic consultant. He has supported teachers, coaches, principals, and superintendents across the country in their work toward creating math classrooms that support the learning of all students. At Illustrative Mathematics, Mike wrote and reviewed lessons for grades 1–4 for IM K–5 Math v.1.





THE STORY OF Grade 4

■ Patti Drawdy

K–5 Curriculum Specialist



“Why not start the year with place value?” Kaneka Turner, Grade 4 Lead Writer, hears this question often. Isn’t making sense of and operating on large numbers pretty essential in grade 4? Indeed, they are. Yet students begin Grade 4 in IM K–5 Math by playing with rectangles. In the first few lessons, they use square tiles to build rectangles of certain side lengths or areas. Why?

THE INVITATION TO GRADE 4 MATH

As in all IM courses, the flow of units in Grade 4 was considered with care—with both the mathematics and the students in mind. And as is typical with an IM course, Grade 4 opens with an invitation: to play, observe, and engage with content that is both relatable and fresh, and that improves students’ access to the major work of the grade. The opening unit also strives to generate conversations—around ideas, ways of thinking, and the experiences of the thinkers. These conversations serve as a basis for building a positive and inclusive learning community, an integral part of **all first units IM K–5 Math.**

■ Yenche Tioanda

3–12 Curriculum Specialist



The exploration of rectangles in the opening unit, Factors and Multiples, follows the design principles above. It activates concepts of multiplication of whole numbers and area of rectangles from grade 3. Then, it prompts students to make new observations about the relationship between pairs of side lengths and area, which paves the way for the introduction of factor pairs and multiples. The geometric context is also purposeful in that it grounds prime and composite numbers in concrete representations before





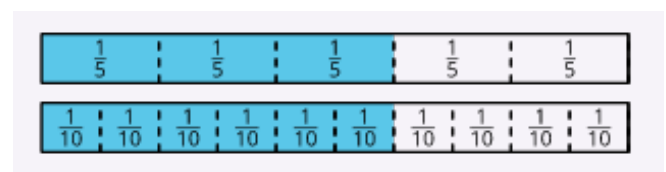
students make sense of such numbers in situations that require more abstraction. The experiences here develop students' multiplicative reasoning, allowing a wider range of learners to access upcoming work on fraction equivalence.

FROM FACTORS AND MULTIPLES TO FRACTION EQUIVALENCE

The work in Unit 2, Fraction Equivalence and Comparison, also builds ideas introduced in grade 3: the size of fractions, the location of fractions on a number line, and equivalent fractions. They use these insights to compare and order fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

Students continue to engage in multiplicative reasoning. They explore doubling and halving—first by repeatedly folding strips of paper in half, which doubles the number of equal parts with each fold, and then by halving the parts in tape diagrams, which creates twice as many parts and smaller unit fractions. Later, they partition segments on tape diagrams and

intervals on number lines into other numbers of equal parts. Such repeated reasoning allows students to more readily relate fractions in which one denominator is a multiple of another denominator and to see fraction equivalence.



IM K-5 Math: Grade 4 Unit 2 Overview

Partitioning a diagram into equal parts becomes increasingly inconvenient as the denominator gets larger. It'd be easy enough to partition a number line into, say, 10 parts, but unwieldy to partition one into 100 parts. This practical limitation motivates a need for generalization, which supports students in relating tenths and hundredths. Understanding of factors and multiples also comes into play as students generalize the process of partitioning a number line to identify equivalent fractions.

Opportunities to strengthen fluency with multiplication are present not only in the core activities but also in the warm-ups, as shown in this sampling of Number Talks from the first two units. The numbers in each sequence encourage students to apply the distributive and associative properties to mentally find the value of each expression.

A Sampling of Number Talks

Unit 1

Lesson 2	Lesson 4	Lesson 7
2×7	10×6	$12 \div 3$
4×7	3×6	$30 \div 3$
3×7	13×6	$60 \div 3$
7×7	12×4	$72 \div 3$

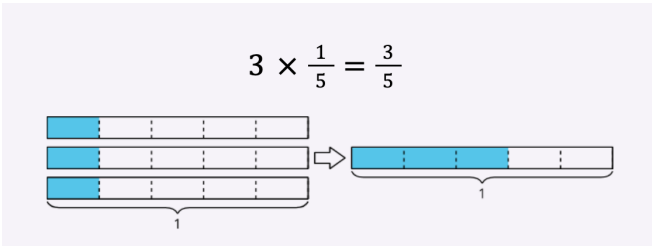
Unit 2

Lesson 5	Lesson 9	Lesson 16
2×12	10×6	5×6
4×12	10×12	5×12
8×12	10×24	6×12
16×12	5×24	11×12

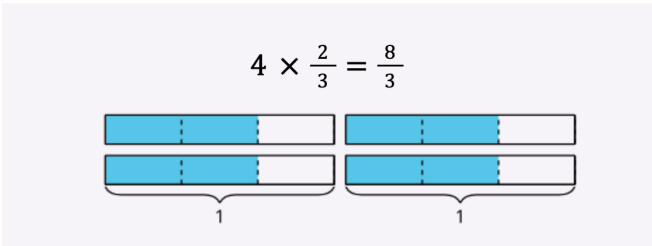
NEW NUMBERS, FAMILIAR OPERATIONS

Having spent time deepening their understanding of the size of fractions, students are now better equipped to see—more abstractly than in grade 3—how fractions can be composed and decomposed, and like whole numbers, be added, subtracted, and multiplied. This work happens in Unit 3, Extending Operations to Fractions.

In grade 3, students saw multiplication in terms of equal groups of whole numbers. Here, they extend that insight to include equal groups of fractional amounts. Through repeated reasoning with diagrams and equations, students see regularity in the product of a whole number and a fraction, namely, that a fraction $a \times \frac{1}{b}$ is $a \times \frac{1}{b}$, and that $n \times a \times \frac{1}{b}$ gives $n \times a \times \frac{1}{b}$.



Math: Grade 4 Unit 3 Overview

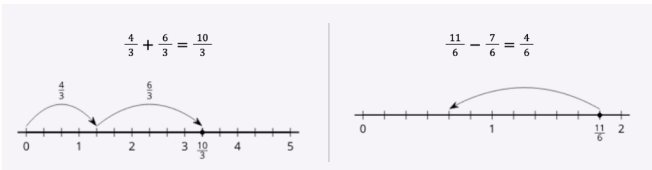


Grade 4 Unit 3 Lesson 4 Synthesis

Everyday contexts are used to help students make sense of non-unit fractions as products and sums. For instance, in the context of using only $\frac{1}{4}$ cup and $\frac{3}{4}$ cup measuring cups to measure the ingredients in a recipe, $\frac{5}{4}$ can be understood as a product ($5 \times \frac{1}{4}$, or using the $\frac{1}{4}$ cup 5 times), a sum of unit fractions with the same denominator ($\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$, or repeatedly adding $\frac{1}{4}$ cup), or a sum of unit and non-unit fractions ($\frac{1}{4} + \frac{1}{4} + \frac{3}{4}$, or adding $\frac{1}{4}$ cup twice and then adding one $\frac{3}{4}$ cup).

IM K–5 Math: Grade 4 Unit 3 Lesson 7 Activity 1

Later, number line diagrams, which in earlier grades supported students in reasoning about addition and subtraction of whole numbers, now support them in adding and subtracting fractions with the same denominator.



IM K–5 Math: Grade 4 Unit 3 Overview

This representation also helps students see that a fraction greater than 1 can be decomposed into a whole number and a fraction, and then be expressed as a mixed number.

Students rely on the understandings developed in Units 1 and 2 to write equivalent fractions in tenths and hundredths, and then find sums of tenths and hundredths. For instance, to find $\frac{3}{10} + \frac{15}{100}$, they reason that $\frac{3}{10}$ is equivalent to $\frac{30}{100}$, so the sum is $\frac{30}{100} + \frac{15}{100}$, which is $\frac{45}{100}$ (**IM K-5 Math: Grade 4 Unit 3 Overview**). The reasoning here paves the way for students to learn about decimal notation in the next unit.

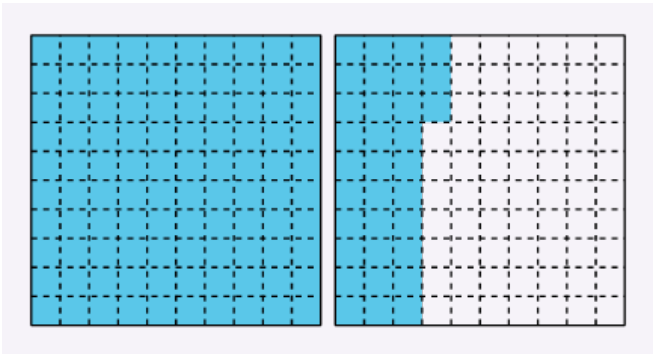
PLACE VALUE, PURPOSEFULLY POSTPONED

In earlier units, students began to develop a mental concept around tenths and hundredths. In Unit 4, they connect decimal fractions to numbers and place value in base ten.

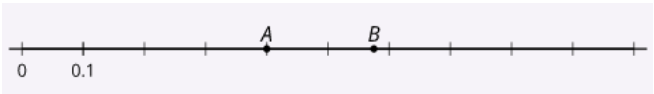
Unit 4, From Hundredths to Hundred-thousands, begins with small numbers. Students learn about decimal notation, reason about the size of tenths and hundredths written as decimals, locate decimals on a number line, and compare and order them. They are prompted to notice the structure of the decimal notation and how it relates to place value.

Each large square represents 1.

What fraction and decimal do the shaded parts of the diagram represent?



IM K-5 Math: Unit 4 Lesson 1 Activity 1



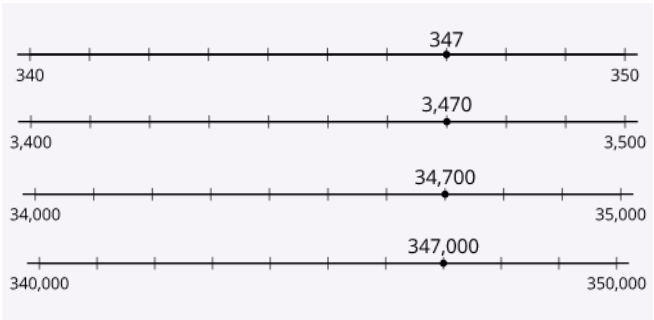
Name the decimal located at point A.
Is the decimal at point A less than or greater than 0.50?
Explain or show your reasoning.

Is the decimal at point B greater or less than 0.06?
Explain your reasoning.
Estimate the decimal at point B.

IM K-5 Math: Unit 4 Lesson 3 Activity 2

After attending to place value in small numbers, students shift their attention to place value in larger numbers. They use base-ten blocks and diagrams to build, read, write, and represent whole numbers beyond 1,000. Working with numbers within 1,000,000 is quite a leap from the incremental increases in the numbers being explored in kindergarten through grade 3 (such as working within 20, 100, or 1,000), but grade 4 students are ready and can generalize more broadly. They observe that ten-thousands are related to thousands in the same way that thousands are related to hundreds, and hundreds are to tens, and tens are to ones—each base-ten unit is 10 times the next smaller unit.

Multiplicative reasoning and the idea of multiples comes into play as students reason about the size of large numbers. Students use their observations of structure to compare, order, and round numbers (by thinking about multiples of 10, 100, 1,000, 10,000, and 100,000 that are the closest to the number).



IM K-5 Math: Grade 4 Unit 4 Section B Overview

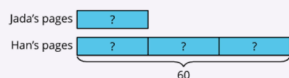
With a deeper understanding of place value, students are better prepared to consolidate their learning about addition and subtraction of whole numbers and to develop fluency as intended within the **K-5 progression of addition and subtraction**.

A CLOSER LOOK AT MULTIPLICATION AND MEASUREMENT

Unit 5, Multiplicative Comparison and Measurement, comes next. Since Unit 4 ends with addition and subtraction of multi-digit numbers, why not go right into multiplication and division of multi-digit numbers? This is because students would benefit from first developing deeper understandings around multiplication and division, measurement, and problem solving before venturing to learn ways to find products and quotients of larger numbers. In other words, the work in Unit 5 gives students better access to the major work of multiplying and dividing multi-digit numbers.

Up to this point, students have understood equal-size groups. Here, students make sense of multiplication as a way to compare quantities. In prior grades, they made additive comparisons, asking “how much more or less?” In multiplicative comparison, the key question is “how many times as much or as many?” Students use diagrams and equations to reason about situations that involve comparing multiplicatively.

Jada read some pages. Han read 60 pages altogether. The diagram shows how their pages compare. How many times as many as Jada's pages did Han read?



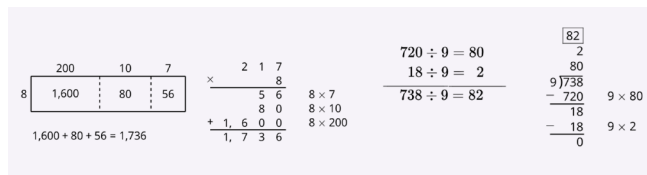
Write a multiplication equation to compare the pages read by Elena and Clare. Use a symbol to represent the unknown.



IM K-5 Math: Grade 4 Unit 5 Overview and Section A Overview

Multiplicative comparison is also helpful in relating units of measurement. For instance, we generally think of 1 yard as 3 times as long as 1 foot, rather than as 2 feet longer than 1 foot. Solving problems that involve measurement conversion and fractional values, though classified as “supporting work” of the grade, also reinforces concepts of fractions and operations that are central in grade 4.

Upon their arrival in Unit 6, Multiplying and Dividing Multi-Digit Numbers, students are more ready for this major work of the grade. They can see the benefits of decomposing factors or dividends by place value, make sense of different representations—diagrams, equations, and algorithms—being used to organize partial products or partial quotients, and make connections between reasoning strategies and representations.



IM K-5 Math: Grade 4 Unit 6 Overview Sections B and C

Put another way, students are better equipped to notice and make use of structure—of base-ten numbers, properties of operations, and the relationship between multiplication and division—to multiply and divide larger numbers effectively and strategically. The progression of the work in Units 1–6 helps to put students on a pathway toward **fluency with these operations as expected in their K-5 journey.**

NEXT STEPS

At first glance, the story of grade 4 may seem to follow a curious path. We hope that a closer look shows a coherent sequence that allows students to make stronger connections across topics and opens the access to grade-level work for a wider range of learners. We also invite your reflections.

- + What other considerations may support the placement of multi-digit operations and place value work in this course?
- + In what other ways might the design decisions improve students' chance of reaching grade-level standards and empower them to know, use, and enjoy mathematics?

PATTI DRAWDY

K-5 Curriculum Specialist. Patti Drawdy began her career as an elementary teacher and now draws upon 33 years of teaching experience in public education at the elementary, middle, high school, and post-secondary levels in her current work as a Curriculum Specialist for IM K-5 Math.

YENCHE TIOANDA

3-12 Curriculum Specialist. Yenche is passionate about learning and helping others learn. Her adventures in mathematics education began in Philadelphia, where she taught high school students as a member of Teach For America and later served as a founding teacher of Science Leadership Academy.



THE STORY OF Grade 5

■ Sarah Caban

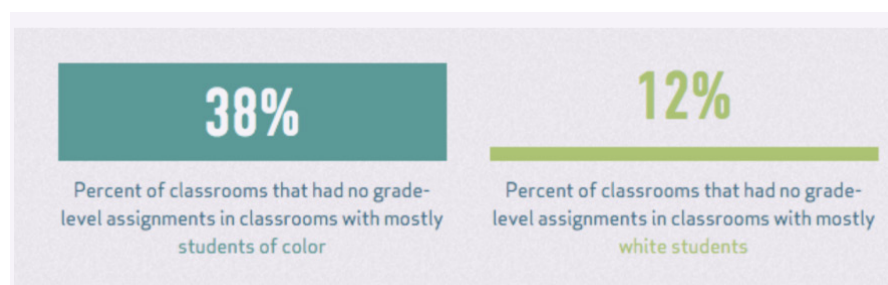
K–5 Professional Learning Specialist



From the start of the year, we want students to know they are capable of engaging in grade-level mathematics.

In the **Opportunity Myth** (2018), data shows that there is an opportunity gap for historically marginalized students—often students of color—between the grade-level expectations laid out in standards and students’ opportunities to engage with this content in their math classes.

Oftentimes, when students are perceived as being not “ready” for content, they are often limited to below-grade-level work. This practice disproportionately occurs in classrooms where the majority of students are students of color.



from **The Opportunity Myth** (2018)

If we want to close the opportunity gap, we have to do many things differently than we have done in the past, one of which is re-examine the sequence of the content we put in front of students. The IM K–5 mathematical story of grade 5 was intentionally written so all students have access to grade-level content.





Many grade 5 math curricula start the year with units on place value concepts and whole number multiplication and division algorithms. When considering whether to start the year this way, we wondered:

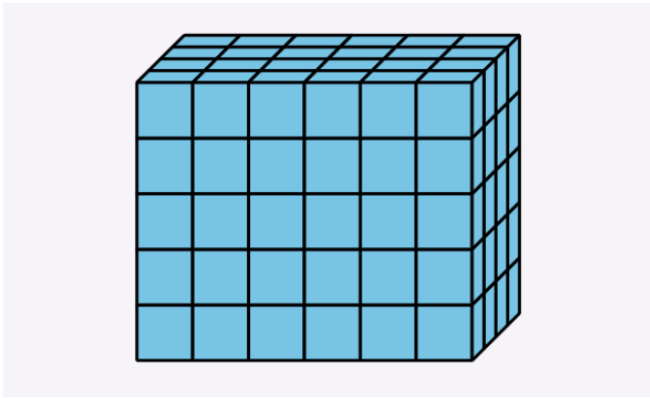
- + What message does the mathematical content in the early units of the year send to students about what it means to “do math” and what is valued in math class?
- + What assumptions might educators make about students based on their successes or challenges with multiplication and division algorithms at the beginning of the year?
- + How might these assumptions impact students’ access to grade-level content throughout the course of the year?

EXTEND AN INVITATION

From the start of the year, we want to send the message that mathematics is an opportunity to be curious, collaborative, and playful. To do this, we begin

with the concept of volume. Because this concept is introduced for the first time in grade 5, students do not have any previous negative experiences or connotations. Students are invited to be curious and creative, and teachers have opportunities to notice and build on what students do know and can do. It also diminishes, if not eliminates, the assumptions both teachers and students might make about what it looks like to be “good at math.”

In early lessons, students have many opportunities to build with unit cubes and use mathematically informal words and phrases to explain the layered structure of rectangular prisms. Teachers listen for and display the words students use—such as “slices,” “groups,” or “layers,”—to describe how they counted the cubes to determine the volume. Students connect their less formal language to multiplication expressions that represent the volume of rectangular prisms before they are introduced to more formal math vocabulary and procedures.



How do the expressions $5 \times 245 \times 24$ and $6 \times 206 \times 20$ represent the volume of the rectangular prism? Explain or show your reasoning.

Over time, students connect their informal strategies for finding the volume of rectangular prisms to generalized formulas. While students are developing conceptual understanding of volume, the quantities used in the problems were deliberately chosen to be “friendlier” numbers. This allows students to strengthen their fluency with multiplication in advance of the work on the standard algorithm for multiplication, which is introduced in Grade 5 Unit 4. After studying the standard algorithm, students revisit the concept of volume, solving problems that involve larger multi-digit numbers as side length measures. This progression allows students time to develop big, conceptual ideas and reinforce fluency work from earlier grades before demanding procedural fluency in new grade 5 content. It also decreases the likelihood that students will be perceived as not ready for new grade-level topics because of their proficiency with computation. We believe these choices will increase student access to grade-level content.

PUT FRACTIONS AT THE FOREFRONT

Throughout the year, we want to maximize students’ opportunities to access grade-level content. In Units 2 and 3, we introduce grade-level content while providing ongoing opportunities to reinforce prior grade-level understandings and fluencies, all within the work of fractions. While many curricula organize whole number computation before fractions, this sequence has implications for extending an opportunity gap. If students are perceived as not being able to use whole number multiplication and division algorithms

correctly, they may be confined to remediation cycles and denied the opportunity to engage with grade-level fraction concepts and procedures. We know that understanding fraction operations is a major indicator of success with topics in high school mathematics so we want to ensure that each student has the opportunity to make sense of grade-level fraction understandings and operations early in the year (Siegler et. al., 2012, p. 695).

CONTINUE TO BUILD ON PRIOR GRADE-LEVEL CONTENT

In Units 2 and 3, students revisit the meaning of fractions, multiplication, and division. They connect what they know about these topics to new learning.

Students begin Unit 2 by interpreting a fraction as division of the numerator by the denominator. Through this work, they continually deepen their conceptual understanding of fractions and division. In the example below, students connect what they know about unit fractions and division.

1. Complete the table.

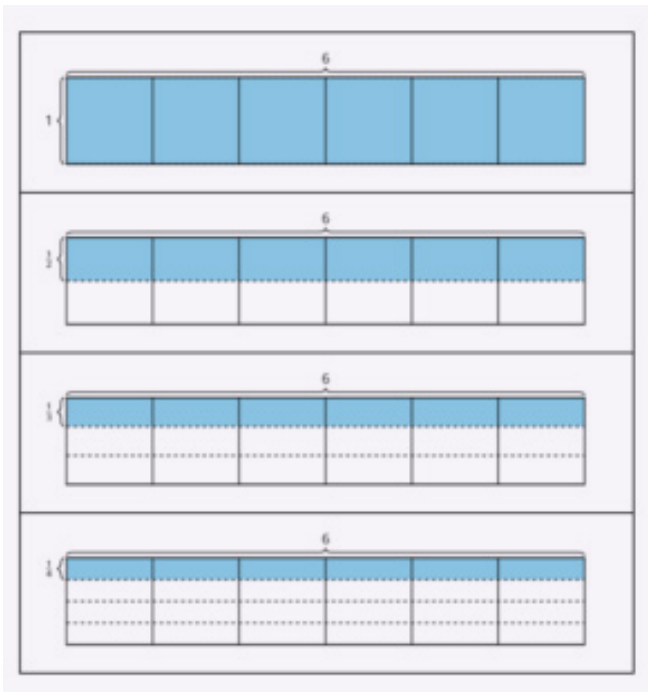
sandwiches	number of people sharing a sandwich	amount of sandwich for each person	division expression
1	2	?	?
1	3	?	?
1	4	?	?
1	8	?	?

2. Choose one row of the table and draw a diagram to show your reasoning for that row.

3. What patterns do you notice?

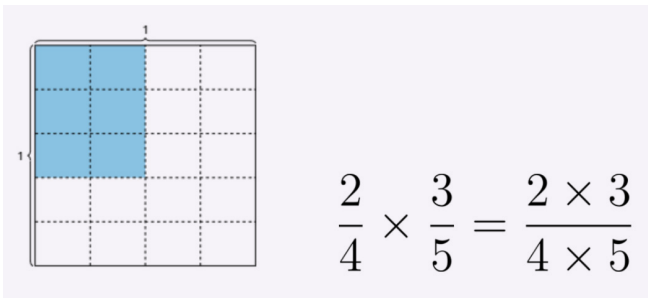
As the unit progresses, students extend their understanding of multiplication as they multiply a whole number by a fraction. In the example below, students leverage what they know about whole number multiplication to solve problems involving multiplication of a whole number and a unit fraction.

Find the area of the shaded region. Explain or show your reasoning.



In Unit 3, students continue to use area diagrams to make sense of fraction multiplication. They build on the new understanding of multiplying a whole number by a fraction while applying prior-grade understanding of area. They recognize that, when multiplying fractions, the unit is the size of the piece being tiled within the unit square. They count those unit tiles by multiplying the number of tiles in each shaded row by the number of shaded rows.

For example, when asked to determine the area of the shaded region below, students know that the unit square is partitioned into a 4×5 array, so the size of each tile is $\frac{1}{20}$. They then multiply 2×3 to find how many tiles are shaded and determine that the area of the shaded region is $\frac{6}{20}$.



Students later apply these understandings to multiply any two fractions.

Students are offered another opportunity to revisit the major work of prior grades in the context of the major work of grade 5 when they are introduced to division of a unit fraction by a whole number and a whole number by a unit fraction. As shown in the example below, the sequence we chose offers students the opportunity to be curious and creative while deepening and extending their understanding of the meaning of division.

1. Solve.

$$36 \div 3$$

$$12 \div 3$$

$$9 \div 3$$

$$6 \div 3$$

$$3 \div 3$$

$$1 \div 3$$

2. What patterns do you notice?

3. Why is the quotient getting smaller?

4. What do you know about this expression: $\frac{1}{3} \div 3$?

5. Draw a diagram to show what $\frac{1}{3} \div 3$ might look like.

Sequencing fraction multiplication and division before whole number algorithms invites teachers to notice and celebrate what students do know about multiplication and division, and minimizes the chances teachers and students will make assumptions about what students don't know about fraction operations based on their multi-digit computation.

MAKE ALGORITHMS ACCESSIBLE

By the time students encounter whole number multiplication and division algorithms in Unit 4, they have had significant experience building the necessary fluencies and understandings to be successful. Teachers have had significant opportunities to observe what each student knows and how they apply these understandings.

In telling the story of grade 5, we chose to begin with an invitation to engage in grade-level mathematics, introduce fractions early in the year, and postpone multi-digit multiplication and division algorithms until later in the year. We think these choices will:

- + send the message that each student is creative, curious, and capable of doing grade-level math in the beginning of the year
- + support teachers to notice and build on what students know and can do
- + increase opportunities for students to access grade-level content

We hope these choices lead to a world where more historically marginalized students have the opportunity to know, use, and enjoy mathematics.

NEXT STEPS

IM's [Grade 5, Unit 1](#) is available at the [IM Community Hub](#). Try it out! Tell us how you and your students found opportunities to be curious, collaborative, and playful.

For more about how the story begins for other grades in IM K–5 Math, read Kristin Gray's blog post, [First Impressions: The First Units in IM K–5 Math](#).



SARAH CABAN

K–5 Professional Learning Specialist. Sarah believes that students learn math by doing math together, and sees great value in listening to students' ideas. She started her teaching career on an island off the coast of Maine, teaching in a K–8 one-room schoolhouse, then she spent four years teaching grades 4 and 6 as a K–12 math coach and coordinator. At Illustrative Mathematics, Sarah was the Grade 5 lead writer for IM K–5 Math v.1 and has facilitated professional learning for IM K–5 Math and IM 6–8 Math.



About Us

Illustrative Mathematics is a nonprofit organization dedicated to creating a world where all learners know, use, and enjoy mathematics. We provide free and accessible high-quality instructional materials and a full suite of professional learning solutions, supported by an incredible bench of experienced IM Certified facilitators. IM K–12 Math is a problem-based core curriculum built on the principle that all students are capable learners of grade-level mathematics.

Founded in 2011 at the University of Arizona, Illustrative Mathematics is guided and inspired by educators who work in spirited teams to do extraordinary things. The result is a comprehensive suite of math curricula and professional learning offerings designed to encourage engaging and affirming math instruction and support the development of positive mathematical identities. Learn more at [**IllustrativeMathematics.org**](https://IllustrativeMathematics.org).



