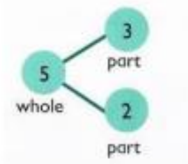
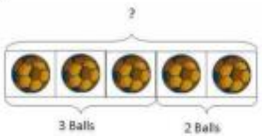
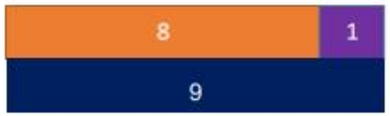
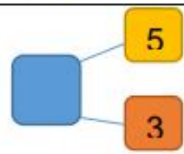
POLICY

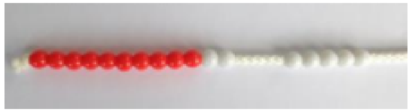
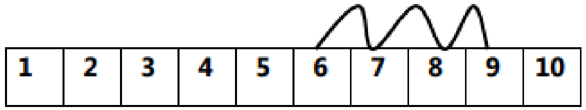
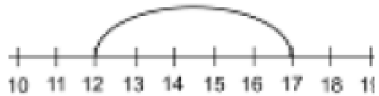
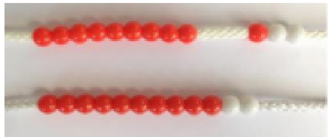
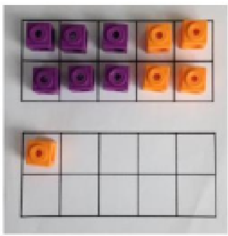
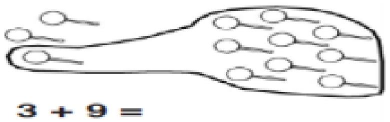
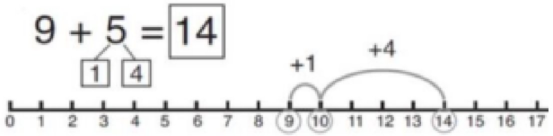
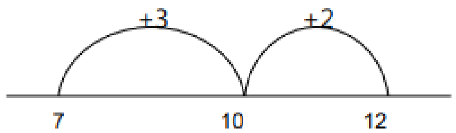
Policy title	Calculation Policy
Written by	Maths Leads Tracy Smith & Clare Baker

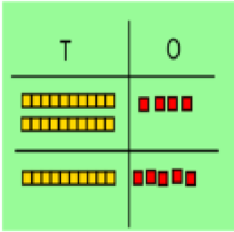
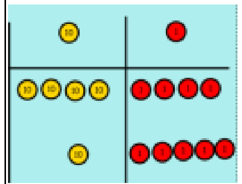
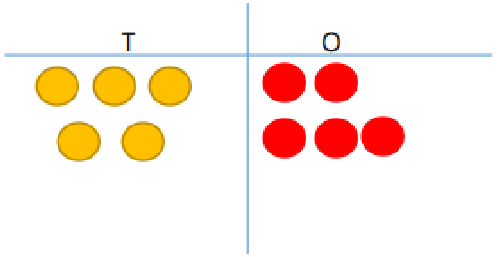
This policy lays out the expectations for both mental and written calculations for the 4 number operations and has been created to support the teaching of a mastery approach to mathematics. This is underpinned by the use of models and images that support conceptual understanding and this policy promotes a range of representations to be used across the primary years. Mathematical understanding is developed through use of representations that are first of all concrete (e.g. Dienes apparatus and place value counters), and then pictorial (e.g. bar models) to then facilitate abstract working (e.g. standard written methods).This policy is a guide through an appropriate progression of representations and if at any point a pupil is struggling with the abstract, they should revert to familiar pictorial and/or concrete materials/representations as appropriate. Although this policy sets out the main methods of mental and written calculations to be taught, it has been appended with a list of recommendations and effective practice teaching ideas aimed at informing and enhancing teaching across all the primary phases. Many of these ideas come from the NCETM's Calculation Guidance document (published October 2015) which is intended to sit alongside a school's calculation policy

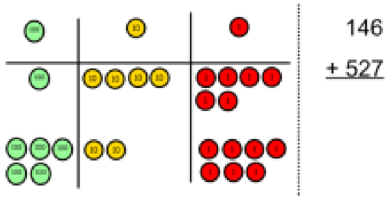
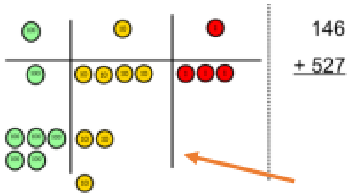
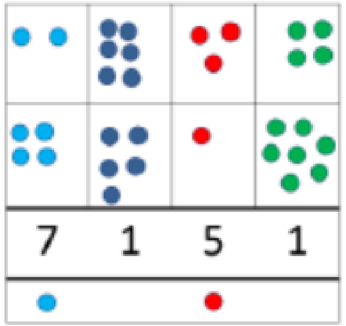
Progression in Calculations

Addition

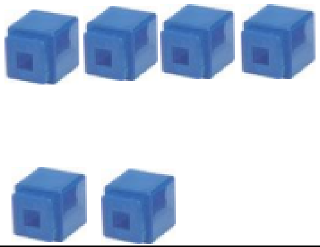
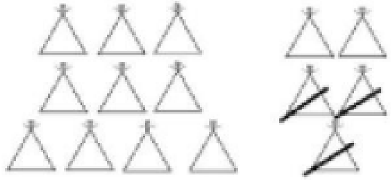
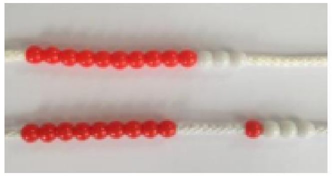
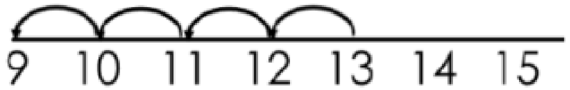
Method	Concrete	Pictorial	Abstract
Stage 1 Counting a set of objects. This can include counting using fingers.	 		 Children relate the number of objects to the numeral.
Stage 2 Combining 2 separate amounts to make 1 whole amount.	  For 4 + 3, count out 4 cubes then 3 more and group them together to see what they have altogether. This can also be represented in a bar. E.g. for 8 + 1: 	   Use pictures to add two numbers together as a group or in a bar. 	 Use the part-part whole diagram as shown above to move into the abstract. 4 + 3 = 7 10 = 6 + 4 Although number sentences are recorded in the concrete and pictorial methods, the abstract method sees the calculation carried out without the use of concrete or pictorial aids.

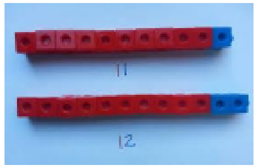
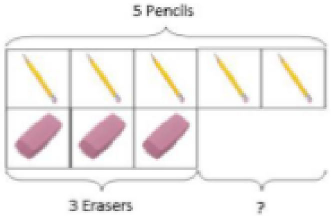
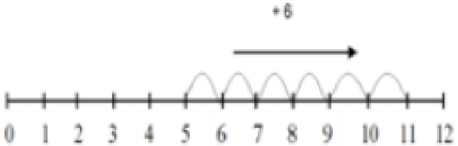
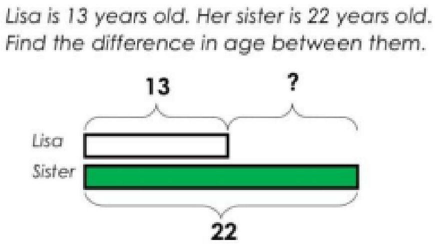
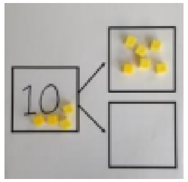
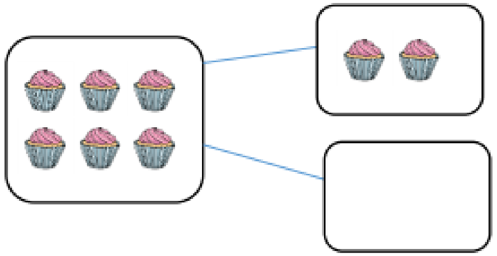
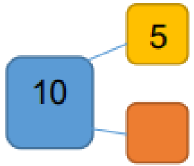
Stage 3 Start at the bigger number and count on	 Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.	Counting on in jumps of 1 using a number line with numbers on it. For 6 + 3 = 9:  This can also be done in bigger jumps or 1 big jump to find the answer. For 12 + 5 = 17: 	5 + 12 = 17 Place the larger number in your head and count on the smaller number to find your answer.
Stage 4 'The Magic 10' Regrouping to make 10 so that the calculation is easier.	Regroup 9 + 3 into 10 + 2 before adding together:   6 + 5 = 11 Start with the bigger number and use the smaller number to make 10.	 Use pictures or a number line. Regroup or partition the smaller number  to make 10 before adding. Children move on to using an 'empty number line'. E.g. 7 + 5 becomes 7 + 3 + 2 	7 + 5 = 7 + 3 + 2 = 12 If I have seven, how many of my 5 do I need to add to make 10. How many more do I still need to add on?

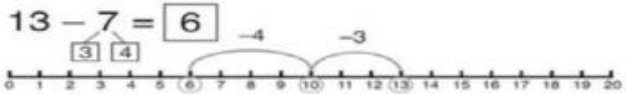
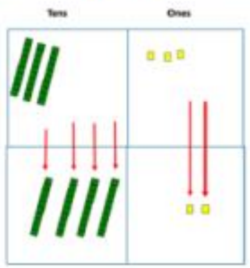
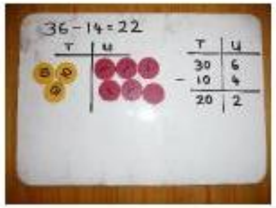
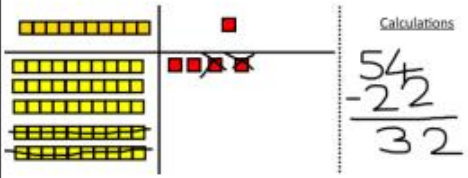
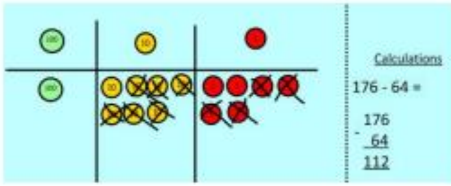
Stage 5 Column addition without regrouping	24 + 15 = 39  Partition the numbers into tens and ones using Dienes blocks. Add together the ones first then add the tens. Finally add the 2 totals together. 44 + 15 = 59  Move onto using place value counters.	After practically using the Dienes blocks and place value counters, children can draw the counters to help them to solve additions. 32 + 23 = 55  	21 + 42 = 21 + 42 Record the calculation vertically adding the column of ones then the column of tens.

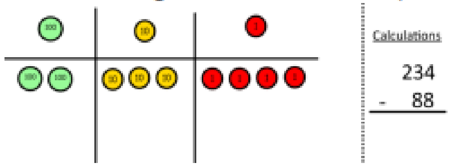
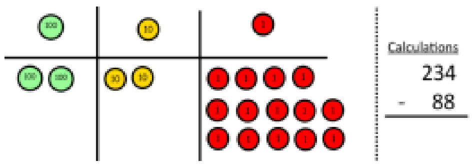
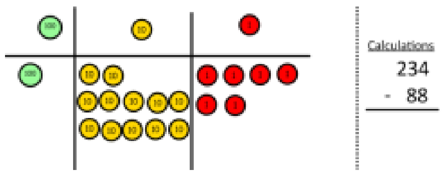
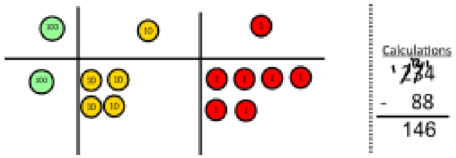
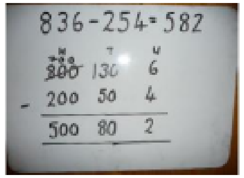
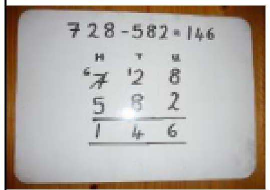
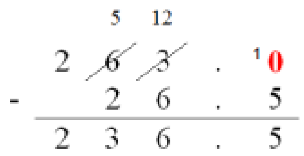
Stage 6 Column addition with regrouping	Make both numbers with place value counters.  In this case, adding the ones gives us 13 which is made up of 10 and 3.  Exchange 10 of these ones for one 10 and add it together with the other tens. Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column if needed. This can also be done with Dienes equipment to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100. As children move on to decimals, money and decimal place value counters can be used to support learning.	Children can draw a pictoral representation of the columns and place value counters to further support their learning and  understanding.	Begin by partitioning the numbers: For 76 + 47 70 + 6 40 + 7 110 + 13 = 123 Move on to clearly show the exchange below the addition: 70 + 6 40 + 7 120 + 3 = 123 10 This then becomes the compact method where numbers aren't partitioned but exchanges still take place: 76 +47 123 11 As the children move on, introduce decimals with and without the same number of decimal places. Money can also be used here. 72.8 + 54.6 127.4 1 1 2 3 . 3 6 1 9 . 0 8 0 5 9 . 7 7 0 + 1 . 3 0 0 9 3 . 5 1 1 2 1 2 **N.B. Exchanged digits need to be recorded below the line when adding.**

Subtraction

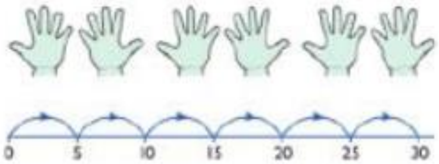
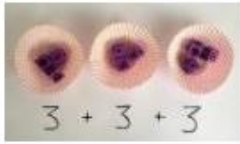
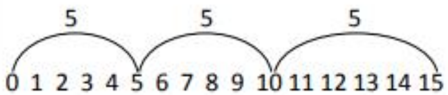
Method	Concrete	Pictorial	Abstract
Stage 1 Taking away ones	Use physical objects, counters, cubes etc. to show how objects can be taken away.  6 – 2 = 4	Cross out drawn objects to show what has been taken away. 4 – 2 = 2   15 – 3 = 12	18 -3= 15 8 – 2 = 6 Although number sentences are recorded in the concrete and pictorial methods children are introduced to them on their own while encouraging them to mentally take away ones.
Stage 2 Counting back	Make the larger number in the subtraction. Move the beads along the bead string and count backwards in ones. 13 – 4  Use counters and move them away from the group counting backwards as they e.ch one is moved away. 	Count back on a number line or number track  Start at the bigger number and count back the smaller number showing the jumps on the number line.  This can progress all the way to counting back using two 2 digit numbers.	For 13 – 4, put 13 in your head and count back 4. What number are you at? Use your fingers to help.

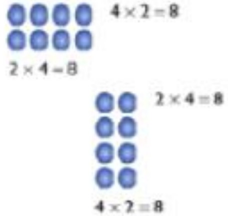
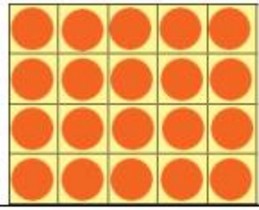
Stage 3 Find the difference	Compare amounts and objects to find the difference.  Use cubes to build towers or make bars to find the difference.  Use basic bar models with items to find the difference.	 11 – 5 = 6 Count on to find the difference. **Comparison Bar Models** Draw bars to find the difference between 2 numbers. 	Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches.
Stage 4 Part Whole Model	 Link to addition- use the part whole model to help explain the inverse between addition and subtraction. If 10 is the whole and 6 is one of the parts. What is the other part? 10 - 6 =	Use a pictorial representation of objects to show the part whole model.  6 – 2 = 4	 Move to using numbers within the part whole model.

Stage 5 Make 10	14 – 5 =  Make 14 on the ten frame. Take away the four first to make 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9.	 Start at 13. Count back 3 to reach 10. Then count back the remaining 4 so you have taken away 7 altogether. You have reached your answer.	16 – 8= How many do we take off to reach the previous 10? (6) How many do we have left to take off? (2)
Stage 6 Column method without regrouping	75 - 42  Use Dienes blocks to make the bigger number then take the smaller number away. Show how you partition numbers to subtract. Again make the larger number first.  	  Draw the Dienes or place value counters alongside the written calculation to help show working.	Partitioned numbers are written vertically: For 54 – 22 Tens Ones 50 4 - 20 2 30 + 2 = 32 This will lead to a clear written column subtraction: 54 - 22 32

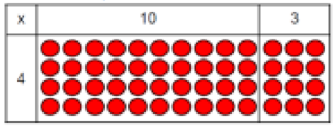
Stage 7 Column method with regrouping	Use Dienes first then move to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges. Make the larger number with the place value counters  Start with the ones. I can't take away 8 ones. I need to exchange a ten for ten ones:  Now I can subtract 8 ones from 14. Next look at the tens. I can't take away 8 tens. I need to exchange a hundred for 10 tens:  Now I can take eight tens from the 12 tens and complete the subtraction.  Show children how the concrete method links to the written method alongside their working. Cross out the numbers when exchanging and show where we write our new amount.		 Children can start their formal written method by partitioning the number into clear place value columns.  Moving forward the children use a more compact method. This will lead to an understanding of subtracting any number including decimals. 

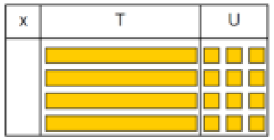
Multiplication

Method	Concrete	Pictorial	Abstract
Stage 1 Counting in multiples	  Count in multiples supported by concrete objects in equal groups.	 Use a number line or pictures to continue support in counting in multiples.	Count out loud in multiples of a number. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30
Stage 2 Repeated addition	   Use different objects to add equal groups.	5 + 5 + 5 = 15   Repeated addition can be shown on a labelled or empty number line. Begin to relate repeated addition to multiplication using 'lots of' e.g. 3 lots of 5 = 15	Write addition sentences to describe objects and pictures.  This then leads to writing related multiplication sentences e.g. 2 x 5 = 10

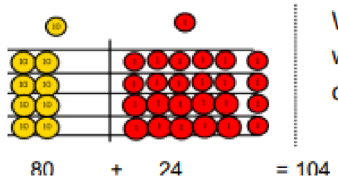
Stage 3 Arrays- showing commutative multiplication	 Create arrays using counters / cubes to show multiplication sentences. 4 x 6 = 24 Begin to look at arrays in different orientations to make the link between, for example, 5 x 3 = 15 and 3 x 5 = 15 (commutativity) 	Draw arrays in different rotations to find **commutative** multiplication sentences.  Link arrays to area of rectangles. 	Use an array to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15

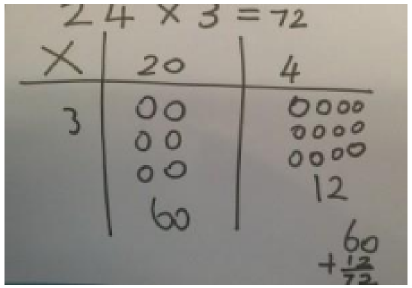
<table>
<tr>
<td>Stage 4
Grid Method</td>
<td>Show the link with arrays to first introduce the grid method.

4 x 13

4 rows of 10
4 rows of 3

Move on to using Dienes to move towards a more compact method.

4 rows of 13

Move on to place value counters to show how we are finding groups of a number.

4 x 26

We are multiplying by 4 so we need 4 rows with each containing 26.</td>
<td>Children can represent the work they have done with place value counters in a way that they understand.

They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.
</td>
<td>Start with multiplying 2-digit by 1-digit numbers showing the addition alongside the grid.

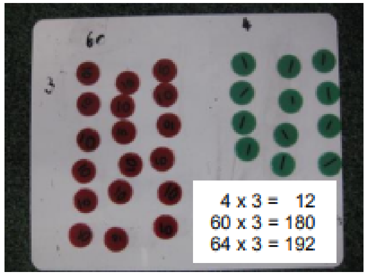
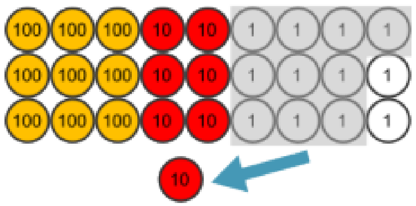
<table>
<tr><th>×</th><th>30</th><th>5</th></tr>
<tr><td>7</td><td>210</td><td>35</td></tr>
</table>

210 + 35 = 245

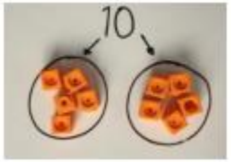
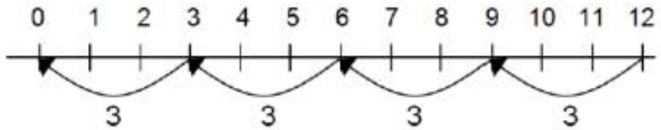
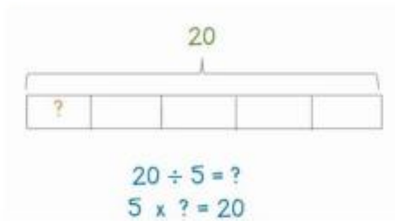
Moving forward, multiply 2, 3 and 4-digit numbers showing the different rows within the grid method.

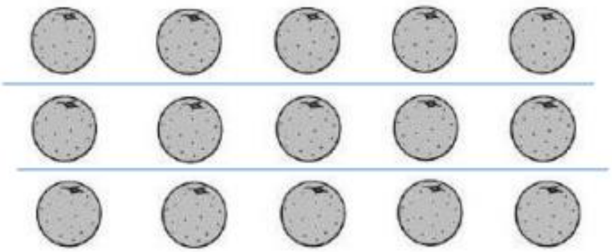
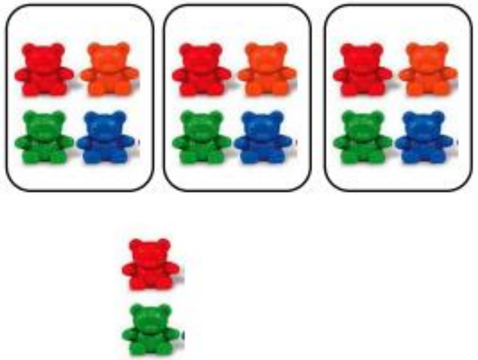
13 x 28
<table>
<tr><th>X</th><th>20</th><th>8</th></tr>
<tr><td>10</td><td>200</td><td>80</td></tr>
<tr><td>3</td><td>60</td><td>24</td></tr>
</table>

280
+ 84
364
1</td>
</tr>
</table>

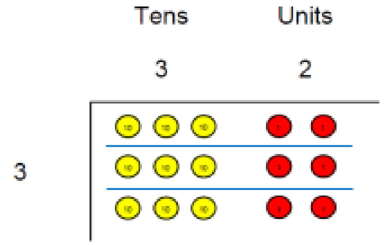
Stage 5 Column multiplication	Children can continue to be supported by place value counters for carrying out column multiplication. They can partition and record each calculation vertically.  It is important to get into the habit of multiply the ones first and note down their answer followed by the tens which they note below. The idea of exchanging will support them in moving on to a more compact method: 3 x 324  	As with stage 4, children can represent the work they have done with place value counters in a way that they understand. They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking.	As with the grid method, numbers of more than one digit are partitioned but this time the calculation is recorded vertically. To support them, children need to write out what they are solving next to their answer. For 38 x 7 38 X 7 56 8 x 7 210 30 x 7 266 Remind the children about the importance of lining up their numbers clearly in columns. This then moves to the more compact method of short multiplication: For 38 x 7 38 X 7 266 5 For 56 x 27 56 x 27 392 56 x 7 1120 56 x 20 1512 1 Start by multiplying the ones digit, recording the last digit of the answer in the answer line but exchanging any tens and putting them under the tens column to be added on after multiplying the tens digit. Again, the last digit in the answer is recorded in the answer line and any hundred are exchanged, this time to the hundreds column, and so on.

Division

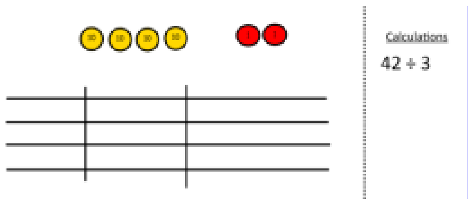
Method	Concrete	Pictorial	Abstract
Stage 1 Sharing objects equally	 I have 10 cubes, can you share them equally in 2 groups?  	Children use pictures or shapes to share quantities.  8 ÷ 2 = 4	Share 9 buns between three people. 9 ÷ 3 = 3
Stage 2 Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.  There are 10 sweets. How many people can have 2 sweets each?	Use a number line to show jumps in groups. The number of jumps equals the number of groups.  Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.  	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group?

Stage 3 Division within arrays	 Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	 Draw an array and use lines to split the array into groups to make multiplication and division sentences.	Find the inverse of multiplication and division sentences by creating four linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7
Stage 4 Division with a remainder	14 ÷ 3 = Divide objects into groups or share equally and see how much is left over. 	Draw dots and group them to divide an amount and clearly show a remainder.  Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder. 13 ÷ 4 = 3 r1  As knowledge of place value improves, children can begin to jump in multiples of 10: 92 ÷ 3 = 30 r2 	Children use knowledge of times table facts to quickly calculate divisions involving remainders. For example: 27 ÷ 5 = 5 r2 Go on to combining knowledge of times tables with place value to calculate more difficult divisions. For example: 137 ÷ 4 = 34 r1

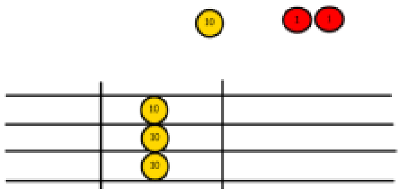
Stage 5
Short division



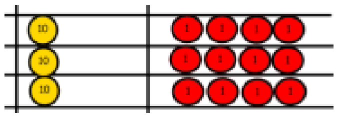
Use place value counters to divide using the bus stop method alongside



42 ÷ 3=
Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.

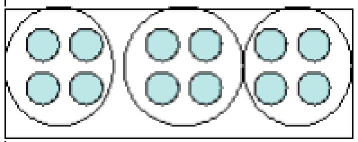


We exchange this ten for ten ones and then share the ones equally among the groups.



We look at how much is in 1 group so the answer is 14.

Children can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.



Encourage them to move towards counting in multiples to divide more efficiently.

Begin with divisions that divide equally with no remainder.

  1 8
4) 7 ^{3}2

Move onto divisions with a remainder.

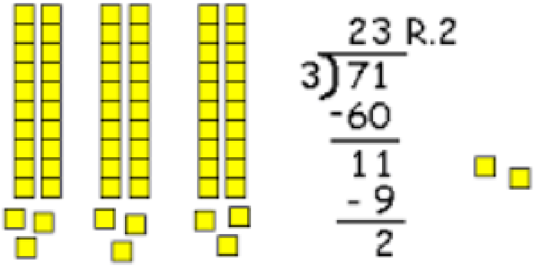
  1 9 r3
4) 7 ^{3}9

This can also be recorded as a fraction: 19 ¾

Finally move into decimal places to divide the total accurately.

  1 9 . 7 5
4) 7 ^{3}9.^{3}0 ^{2} 0

  1 4 . 6
35) 5 1 ^{16}1.^{21}0

Stage 6 Long division	 Using dienes or place value counters, we start with 7 tens and 1 one, to be divided into 3 groups. We can put 2 tens in each group, so we write a 2 in the tens column. In all, we've put 6 tens into the groups (3 x 2 tens), so we write 6 tens (60) below. We are left with 11 (1 ten and 1 one). We will need to exchange the ten for 10 ones so we can put 3 ones in each group (using 9 ones in all), and we will have a remainder of 2.		432 ÷ 15 becomes 2 8 r 12 1 5 | 4 3 2 3 0 0 1 3 2 1 2 0 1 2 Answer: 28 remainder 12 432 ÷ 15 becomes 2 8 1 5 | 4 3 2 3 0 0 15×20 1 3 2 1 2 0 15×8 1 2 Answer: $28\frac{4}{5}$ $\frac{12}{15} = \frac{4}{5}$ 432 ÷ 15 becomes 2 8 · 8 1 5 | 4 3 2 · 0 3 0 ↓ 1 3 2 1 2 0 ↓ 1 2 0 1 2 0 0 Answer: 28·8

Appendix

Listed below are a range of recommendations and teaching ideas aimed at informing and enhancing the teaching of primary mathematics:

1. Developing children's understanding of the = symbol

The = symbol is an assertion of equivalence. If we write 3 + 4 = 6 + 1 then we are saying that what is on the left of the = symbol is equivalent to what is on the right of the symbol. But many children interpret = as always being an instruction to work out the value of a calculation. This is as a result of always seeing it used as follows:

3 + 4 =

5 × 7 =

16 – 9 =

If children only think of = as meaning "Work out the answer to this calculation" then they are likely to get confused by empty box questions such as:

3 + ☐ = 8 and are are very likely to struggle with even simple algebraic equations, such as: 3y = 18. This can be overcome by doing the following:

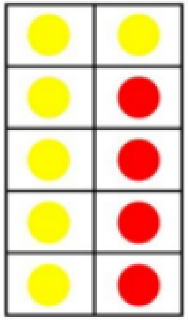
- Vary the position of the = symbol e.g. 24 = 4 x 6
- Include lots of empty box problems e.g. 12 - ☐ = 4; ☐ x 6 = 24
- Teach inequality alongside equality e.g. 5 + 9 ☐ 3 x 5 (< > or =?)

2. Recognising the actual value of ones, tens, hundreds etc. in a number

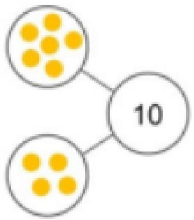
Many children are able to recognise the value of each digit in a number like 347 but find it harder to explain, for example, how many tens there are in 347. Once they are able to recognise that there are 34 tens (rather than 4 tens), it makes it much easier to be able to carry out a calculation such as 347 + 30 as they are adding 3 tens to the 34 tens. Traditionally, children often struggle when tackling a calculation involving crossing over a hundred e.g. 293 + 10 but using this method takes much of the difficulty away as they only need to add 1 ten to the 29 tens to give 30 tens and an answer of 303. It is equally effective when subtracting e.g. for 112 – 20, we subtract 2 tens from the 11 tens to leave is with 9 tens and an answer of 92.

3. Reasoning about mathematical relationships

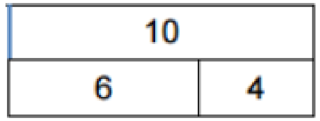
Children need to be exposed to images and structures that help them to make links between inverse operations from an early age



6 + 4 = 10
4 + 6 = 10
10 – 4 = 6
10 – 6 = 4



6 + 4 = 10
4 + 6 = 10
10 – 4 = 6
10 – 6 = 4



6 + 4 = 10
4 + 6 = 10
10 – 4 = 6
10 – 6 = 4

Part Whole Model

Bar Model

Opportunities should be taken wherever possible to demonstrate how children can use what they already know to work out a related fact e.g.:

.- if 6 + 4 = 10, then 6 tens + 4 tens = 10 tens i.e. 60 + 40 = 100

- If you know 3 + 5, you can use this to work out 23 + 5

4. Developing children's fluency with basic number facts

Fluent computational skills are dependent on accurate and rapid recall of basic number bonds to 20 and times-tables facts. Research has shown that spending a short time every day on these basic facts quickly leads to improved fluency.

5. Developing fluency in mental calculations (The Magic 10)

Although the Magic 10 already has a place in this calculation policy, it is worth emphasising the importance of this approach. Children who learn to 'make 10' to create an easier calculation are able to develop mental fluency and an ability to look for patterns. Using knowledge of number bonds that make 10, they can see that 9 + 6 = 9 + 1 + 5 = 10 + 5 = 15