# High School Mathematics

# Parent Handbook

*For the Integrated Pathway*

Common Core State Standards for Mathematics

For California Public Schools

Grades 9-12

Curriculum & Instruction
Steering Committee

On the cover:
Image from istockphoto.com


Updated September 2013

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# Acknowledgements

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# Introduction

> *"If I had an hour to solve a problem I'd spend 55 minutes thinking about the problem and 5 minutes thinking about solutions."*
>
> Albert Einstein

The challenges our students face in the 21st century global economy are continually changing. In order to be competitive in this global economy, students need to develop the skills to be able to problem-solve creatively.

In recent years, mathematics education in California has focused more on getting an answer than understanding the problem. There are many factors that have contributed to this. The 1997 Mathematics Standards moved California in a good direction with consistent mathematical content being taught at each grade level or high school course. However, the Common Core State Standards for Mathematics are a call to take the next step.

The goal of the Common Core State Standards for Mathematics is for students to be college and career ready upon graduation from high school and to assist students in becoming competitive in a global economy. Therefore, the Common Core State Standards for Mathematics provide not only for rigorous curriculum and instruction, but also conceptual understanding, procedural skill and fluency and the ability to apply mathematics. Students will develop the skills to be able to problem-solve creatively and not be satisfied by just arriving at an answer, thus meeting the challenges of the 21st century.

"These Standards are not intended to be new names for old ways of doing business. They are a call to take the next step. It is time for states to work together to build on lessons learned from two decades of standards based reforms. It is time to recognize that these standards are not just promises to our children, but promises we intend to keep."

*National Governors Association Center for Best Practice and Council of Chief State of School Officers (2010) Common Core State Standards for Mathematics*

The purpose of this handbook is to provide parents and guardians with an introduction to the Common Core State Standards for Mathematics for California public high schools.

# Background Information

"… more time for teachers to teach and students to master concepts."

Jason Zimba, Common Core State Standards for Mathematics Lead Author

These new Standards for Mathematics have been developed to provide students with the knowledge, skills, and understanding in mathematics to be college and career ready when they complete high school. They are internationally benchmarked and assist students in their preparation for enrollment at a public or private university.

Common Core State Standards Initiative Mission Statement:

"The Common Core State Standards provide a consistent, clear understanding of what students are expected to learn, so teachers and parents know what they need to do to help them. The standards are designed to be robust and relevant to the real world, reflecting the knowledge and skills that our young people need for success in college and careers. With American students fully prepared for the future, our communities will be best positioned to compete successfully in the global economy."

These standards also provide focus, coherence, and rigor.

| Focus | Coherence | Rigor |
|---|---|---|
| "Focus implies that instruction should focus deeply on only those concepts that are emphasized in the standards so that students can gain strong foundational conceptual understanding, a high degree of procedural skill and fluency, and the ability to apply the mathematics they know to solve problems inside and outside the mathematics classroom." | "Coherence arises from mathematical connections. Some of the connections in the standards knit topics together at a single grade level. Most connections are vertical, as the standards support a progression of increasing knowledge, skill, and sophistication across the grades." | "Rigor requires that conceptual understanding, procedural skill and fluency, and application be approached with equal intensity." |

The Common Core State Standards for Mathematics include two types of standards:

1. Eight Standards for Mathematical Practice that are the same in each grade level and high school mathematics course.
2. Mathematical Content Standards that are organized into high school courses.

"Together these standards address both 'habits of mind' that students should develop to foster mathematical understanding and expertise and skills and knowledge – what students need to know and be able to do.

The mathematical content standards were built on progressions of topics across a number of grade levels, informed both by research on children's cognitive development and by the logical structure of mathematics."

Adapted from California Common Core State Standards – Mathematics Introduction, page 2

The California State Board of Education adopted these Standards on August 2, 2010, after determining that they were at least as rigorous as the current standards. The Common Core State Standards for Mathematics were the result of a state-led movement by the National Governors Association and the Council of Chief State School Officers. Currently, most states have adopted the Common Core State Standards for Mathematics.

## New Assessments

For the past several years, the California Standards Test (CST) has been used to assess student understanding of the mathematics content standards. These assessments are also referred to as the STAR Program (Standardized Testing and Reporting). In 2014-15, the STAR will be replaced by a new assessment system that is being developed by the Smarter Balanced Assessment Consortium to test student knowledge of the Common Core State Standards for Mathematics. The new assessments will be very different from the CST's in terms of the test structure, the rigor of the mathematical content, and the delivery system. It is planned that by 2016, the entire test will be delivered on-line to each student. At the high school level, the new test will be administered at the end of eleventh grade. In addition to this test, students will still need to pass the California High School Exit Exam (CAHSEE). This handbook will provide some of the sample test items that highlight characteristics of the changes and the rigor of the content that is expected at the end of three years of high school mathematics.

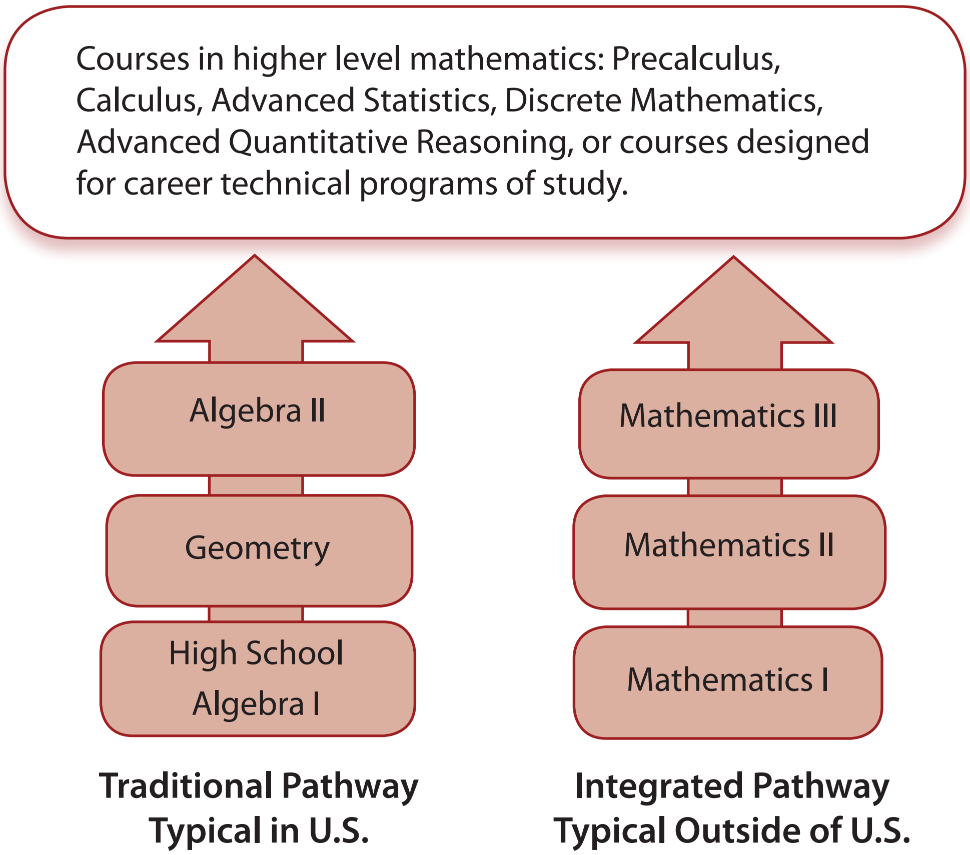
## Standards for Mathematical Practice

The Standards for Mathematical Practice describe behaviors that all students will develop in the Common Core State Standards for Mathematics. These practices rest on important "processes and proficiencies" including problem solving, reasoning and proof, communication, representation, and making connections. These practices will allow students to understand and apply mathematics with confidence.

1. Make sense of problems and persevere in solving them.
   - Find meaning in problems
   - Analyze, predict and plan solution pathways
   - Verify answers
   - Ask themselves the question: "Does this make sense?"
2. Reason abstractly and quantitatively.
   - Make sense of quantities and their relationships in problems
   - Create coherent representations of problems
3. Construct viable arguments and critique the reasoning of others.
   - Understand and use information to construct arguments
   - Make and explore the truth of conjectures
   - Justify conclusions and respond to arguments of others
4. Model with mathematics.
   - Apply mathematics to problems in everyday life
   - Identify quantities in a practical situation
   - Interpret results in the context of the situation and reflect on whether the results make sense
5. Use appropriate tools strategically.
   - Consider the available tools when solving problems
   - Are familiar with tools appropriate for their grade or course (pencil and paper, concrete models, ruler, protractor, calculator, spreadsheet, computer programs, digital content located on a website, and other technological tools)
6. Attend to precision.
   - Communicate precisely to others
   - Use clear definitions, state the meaning of symbols and are careful about specifying units of measure and labeling axes
   - Calculate accurately and efficiently
7. Look for and make use of structure.
   - Discern patterns and structures
   - Can step back for an overview and shift perspective
   - See complicated things as single objects or as being composed of several objects
8. Look for and express regularity in repeated reasoning.
   - When calculations are repeated, look for general methods, patterns and shortcuts
   - Be able to evaluate whether an answer makes sense

# Higher Mathematics Pathways

The Common Core State Standards for Mathematics provides two pathways to organize the standards into courses: the traditional and the integrated pathway. Both pathways require students to accomplish the same mathematical content over a three-year period and include work with making mathematical models. Each school district determines which pathway their high schools will follow. Either pathway will allow students to complete Advanced Placement Calculus with accelerated work.



# Integrated Pathway Course Content

The Integrated Pathway is made up of three courses (Mathematics I, II, and III). The integrated mathematics courses follow the structure began in the K-8 standards of presenting mathematics as a multifaceted, coherent subject, and is the way most other high performing countries present higher mathematics. Each course is comprised of standards selected from the six high school conceptual categories, which were written to encompass the scope of content and skills to be addressed throughout grades 9–12 rather than through any single course.

**By the end of eighth grade**, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. They have defined, evaluated, and compared functions, and used them to model relationships between quantities. Students have worked with radicals and applied the laws of exponents to situations involving integer exponents.

(Note: For information on middle school, see the Common Core State Standards for Mathematics for California Public Schools Middle School Mathematics Parent Handbook.)

## Mathematics I

### Critical Areas of Instruction

In Mathematics I your child's mathematics experience will focus on:

(1) Extending understanding of numerical manipulation to algebraic manipulation

(2) Synthesizing understanding of function

(3) Deepening and extending understanding of linear relationships

(4) Applying linear models to data that exhibit a linear trend

(5) Establishing criteria for congruence based on rigid motions

(6) Applying the Pythagorean Theorem to the coordinate plane

### Specific Content Examples for the Critical Areas

- Using quantities to model and analyze situations, to interpret expressions, and to create equations to describe situations
- Exploring many examples of functions, including sequences
- Interpreting functions given graphically, numerically, symbolically, and verbally; translating between representations; and understanding the limitations of various representations
- Reasoning with the units in which those quantities are measured when functions describe relationships between quantities
- Extending understanding of integer exponents to consider exponential functions
- Comparing and contrasting linear and exponential functions
- Interpreting arithmetic sequences as linear functions and geometric sequences as exponential functions
- Analyzing and explaining the process of solving an equation, and justifying the process used in solving a system of equations
- Developing fluency writing, interpreting, and translating among various forms of linear equations and inequalities, and using them to solve problems
- Applying related solution techniques and the laws of exponents to the creation and solution of simple exponential equations
- Exploring systems of equations and inequalities, and finding and interpreting solutions

- Using regression techniques to describe approximately linear relationships among quantities
- Using graphical representations and knowledge of the context to make judgments about the appropriateness of linear models
- Looking at residuals to analyze the goodness of fit for linear models
- Establishing triangle congruence criteria, based on analyses of rigid motions and formal constructions
- Solving problems about triangles, quadrilaterals, and other polygons
- Applying reasoning to complete geometric constructions and explain why they work
- Using a rectangular coordinate system to verify geometric relationships, including properties of special triangles and quadrilaterals and slopes of parallel and perpendicular lines

Adapted from California Common Core State Standards – Mathematics
Mathematics I Introduction, pages 86-87

## Mathematics II

### Critical Areas of Instruction

In Mathematics II your child's mathematics experience will focus on:

(1) Extending the laws of exponents to rational exponents

(2) Comparing key characteristics of quadratic functions with those of linear and exponential functions

(3) Creating and solving equations and inequalities involving linear, exponential, and quadratic expressions

(4) Extending work with probability

(5) Establishing criteria for similarity of triangles based on dilations and proportional reasoning

### Specific Content Examples for the Critical Areas

- Extending the laws of exponents to rational exponents and exploring distinctions between rational and irrational numbers by considering decimal representations
- Exploring relationships between number systems: whole numbers, integers, rational numbers, real numbers, and complex numbers
- Comparing the key characteristics of quadratic functions to those of linear and exponential functions
- Selecting from a variety of functions to model phenomena
- Identifying the real solutions of a quadratic equation as the zeros of a related quadratic function
- Expanding experience with functions to include more specialized functions—absolute value, step, and those that are piecewise-defined
- Focusing on the structure of expressions, rewriting expressions to clarify and reveal aspects of the relationship they represent
- Creating and solving equations, inequalities, and systems of equations involving exponential and quadratic expressions
- Computing and interpreting theoretical and experimental probabilities for compound events, and attending to mutually exclusive events, independent events, and conditional probability
- Making use of geometric probability models wherever possible
- Using probability to make informed decisions

- Identifying criteria for similarity of triangles, using similarity to solve problems, and applying similarity in right triangles to understand right triangle trigonometry, with particular attention to special right triangles and the Pythagorean Theorem
- Using knowledge about congruence and similarity to prove theorems involving lines, angles, triangles, and other polygons
- Exploring a variety of formats for writing proofs

Adapted from California Common Core State Standards – Mathematics
Mathematics II Introduction, page 95

## Mathematics III

**Critical Areas of Instruction**

In Mathematics III your child's mathematics experience will focus on four critical areas:

(1) Applying methods from probability and statistics to draw inferences and conclusions from data

(2) Expanding understanding of functions to include polynomial, rational, and radical functions

(3) Expanding right triangle trigonometry to include general triangles

(4) Consolidating functions and geometry to create models and solve contextual problems

**Specific Content Examples for the Critical Areas**

- Identifying different ways of collecting data—including sample surveys—experiments, and simulations and the roles that randomness and careful design play in the conclusions that can be drawn
- Exploring structural similarities between the system of polynomials and the system of integers
- Identifying zeros of polynomials, including complex zeros of quadratic polynomials, and making connections between zeros of polynomials and solutions of polynomial equations
- Examining the Fundamental Theorem of Algebra
- Deriving the Law of Sines and the Law of Cosines in order to find missing measures of general (not necessarily right) triangles
- Developing the notion of radian measure for angles and extend the domain of the trigonometric functions to all real numbers
- Modeling simple periodic phenomena
- Solving exponential equations with logarithms
- Exploring the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of the underlying function
- Identifying appropriate types of functions to model a situation, adjusting parameters to improve the model
- Comparing models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit

Adapted from California Common Core State Standards – Mathematics
Mathematics III Introduction, page 105

# Integrated Pathway Course Content - continued

## Fourth Year Course (PreCalculus)

**Summary of Mathematical Concepts and Skills Being Taught**

This course is highly suggested as preparation before taking a standard Calculus course that would lead to an Advanced Placement Calculus exam.

"PreCalculus combines the trigonometric, geometric, and algebraic concepts needed to prepare students for the study of Calculus, and strengthens students' conceptual understanding of problems and mathematical reasoning in solving problems. Facility with these topics is especially important for students intending to study calculus, physics, and other sciences, and/or engineering in college. The main topics in the course are complex numbers, rational functions, trigonometric functions and their inverses, inverse functions, vectors and matrices, and parametric and polar curves. Because the standards for this course are mostly (+) standards, students selecting this model PreCalculus course should have met the college and career ready standards of the previous Integrated course Pathways."

Draft California Mathematics Framework
August 2013

PreCalculus Chapter

In all four courses, the Standards for Mathematical Practice complement the content standards so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the high school years.

For a complete list of standards, please refer to the Common Core State Standards for Mathematics document.
http://www.cde.ca.gov/be/st/ss/documents/ccssmathstandardaug2013.pdf

# Accelerated Learning for Students Who Are Ready

Success in Mathematics I is critical for a student to be successful in higher mathematics.

"There are some students who are able to move through mathematics quickly. These students may choose to take high school mathematics beginning in eighth grade or earlier so they can take college-level mathematics in high school."

"Care must be taken to ensure that students master and fully understand all of the important topics in the mathematics curriculum, and that the continuity of the mathematics learning progression is not disrupted. In particular, the Standards for Mathematical Practice ought to continue to be emphasized in these cases."

Common Core State Standards for Mathematics, Appendix A, page 80
www.corestandards.org

In the Common Core State Standards for Mathematics, there are two possible mathematics courses for an eighth grade student to take:

1. Grade 8 Common Core Mathematics
2. Integrated Mathematics 1

# Accelerated Learning for Students Who Are Ready

- continued

The sequence of courses in high school is based on each eighth grade course and the Integrated Pathway:

| Grade 8 | Grade 9 | Grade 10 | Grade 11 | Grade 12 |
|---|---|---|---|---|
| 8th Grade Common Core Standards for Mathematics | Mathematics I | Mathematics II | Mathematics III | PreCalculus |
| Mathematics I | Mathematics II | Mathematics III | PreCalculus | AP Calculus |

**Note:** For a student to reach AP Calculus in grade 12, acceleration must occur.

In recent years, a common method of acceleration was for a student to skip a mathematics class, usually sixth or seventh grade mathematics. This was possible due to the fact that there were many standards from year to year that where repeated in the former California Mathematics Standards. However, the Common Core State Standards for Mathematics requires a new approach to acceleration for a student to reach a high school math course in eighth grade.

The recommendation with Common Core State Standards for Mathematics is to have a thoughtfully designed series of compacted courses. Compacted courses compress the standards of three years of mathematics into two years. For example, one option could be covering seventh grade, eighth grade, and Mathematics I during a student's seventh grade and eighth grade years.

If a student does not accelerate in middle school math, acceleration can also occur at the high school. Just as care should be taken not to rush the decision to accelerate students, care should also be taken to provide more than one opportunity for acceleration. Some students may not have the preparation to enter a "Compacted Pathway" but may still develop an interest in taking advanced mathematics, such as AP Calculus or AP Statistics in their senior year.

Additional opportunities for acceleration may include:

- Allowing students to take two mathematics courses simultaneously
- Allowing students in schools with block scheduling to take a mathematics course in both semesters of the same academic year
- Offering summer courses that are designed to provide the equivalent experience of a full course in all regards, including attention to the Standards for Mathematical Practices
- Creating different compaction ratios, including four years of high school content into three years beginning in ninth grade
- Creating a hybrid Mathematics III-PreCalculus course that allows students to go straight to Calculus

You should consult with your child's teacher or counselor concerning the best acceleration pathway for your student.

# Assessment Examples

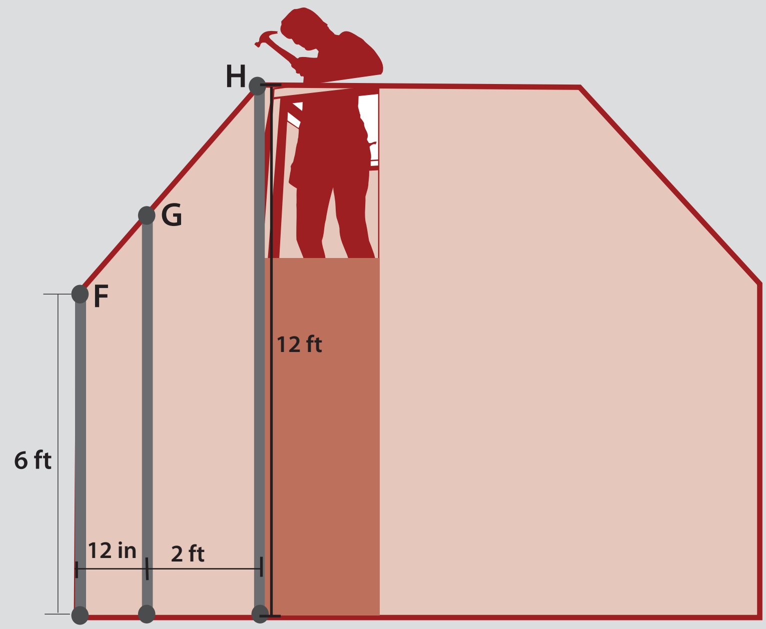
## Smarter Balanced Assessment Consortium Examples

Below are some sample test items (problems) from the Smarter Balanced Assessment Consortium. This is the consortium that is developing new assessments.  (See New Assessments section in this handbook).

### Example 1

A construction worker is using wooden beams to reinforce the back wall of a room.

Determine the height, in feet, of the beam that ends at point G.  Explain how you found your answer.




In this problem, students are asked to find the length of the beam that ends at point G in the diagram. This is an example of a real-life problem. To find the answer a student would need to use their understanding of similar triangles, ratio, proportional reasoning, and solving equations. In addition, a student would need to explain in words how they solved the problem. It is possible to arrive at the answer in more than one way.

**Sample Response:**

Two right triangles can be formed by extending a line from point F that is perpendicular to the beam that ends at point H. Now you have a right triangle with a height of 6 and a base of 3 and a smaller right triangle with a height of x and a base of 1.

The larger triangle and the smaller triangle are similar since they are both right triangles and share an angle. The proportion 3:1 = 6:x can be used to find the smaller portion of the beam ending at point G. Solving this proportion gives x = 2.

Smarter Balanced Assessment Consortium
Sample Items and Performance Tasks
http://www.smarterbalanced.org/sample-items-and-performance-tasks/

## Smarter Balanced Assessment Consortium Examples - continued

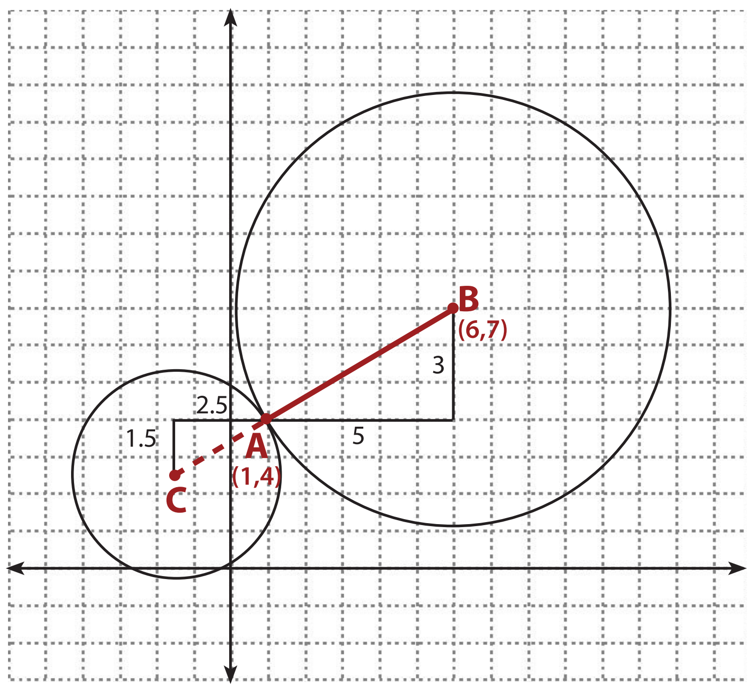
### Example 2

A circle has its center at (6, 7) and goes through the point (1, 4). A second circle is tangent to the first circle at the point (1, 4) and has one-fourth the area.

What are the coordinates for the center of the second circle? Show your work or explain how you found your answer.

**Sample Response:**

The slope between the center of the larger circle and the point (1, 4) is 3/5. Since the area of the smaller circle is one-fourth the area of the larger circle, then the radius of the smaller circle is half of the radius of the larger circle. The slope will be the same, but both distances will be half, so 3/5 becomes 1.5/2.5. So, the coordinates of the center of the smaller circle are (1 – 2.5, 4 – 1.5) = (-1.5, 2.5).




Smarter Balanced Assessment Consortium
Sample Items and Performance Tasks
http://www.smarterbalanced.org/sample-items-and-performance-tasks/

## Smarter Balanced Assessment Consortium Examples - continued

### Example 3

The noise level at a music concert must be no more than 80 decibels (dB) at the edge of the property on which the concert is held.

Melissa uses a decibel meter to test whether the noise level at the edge of the property is no more than 80 dB.

Melissa is standing 10 feet away from the speakers and the noise level is 100 dB.

The edge of the property is 70 feet away from the speakers. Every time the distance between the speakers and Melissa doubles, the noise level decreases by about 6 dB.

Rafael claims that the noise level at the edge of the property is no more than 80 dB since the edge of the property is over 4 times the distance from where Melissa is standing. Explain whether Rafael is or is not correct.

**Sample Response:**

Rafael is not correct because the dB level does not decrease by at least (6)(4) = 24. The decibel level decreases by 6 every time the distance is doubled starting from 10 feet. At 10 feet from the speakers, the volume is 100 dB. At 20 feet, it is 100 – 6 = 94 dB. At 40 feet, it is 94 – 6 = 88 dB. At 80 feet, it is 88 – 6 = 82 dB. Since the property line is 70 feet from the speakers, Rafael is wrong. The volume will be greater than 82 dB.

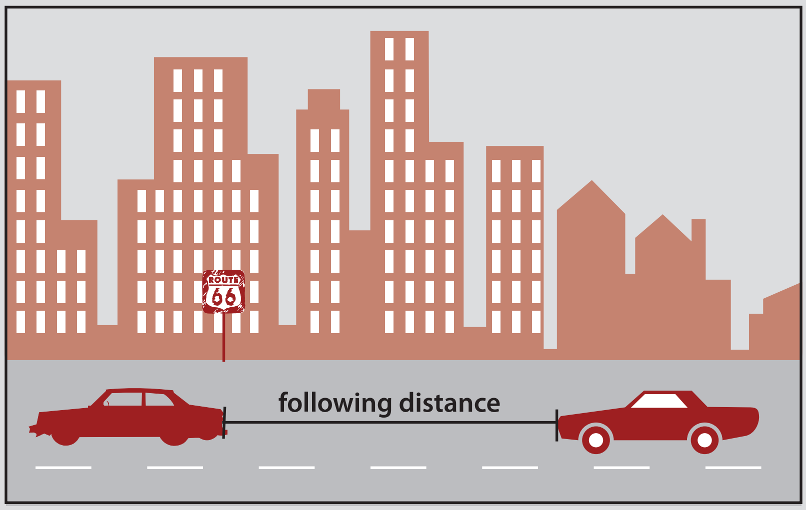
Smarter Balanced Assessment Consortium
Sample Items and Performance Tasks
http://www.smarterbalanced.org/sample-items-and-performance-tasks/

## Smarter Balanced Assessment Consortium Examples - continued

### Example 4

The "two-second rule" is used by a driver who wants to maintain a safe following distance at any speed. A driver must count two seconds from when the car in front of him or her passes a fixed point, such as a tree, until the driver passes the same fixed point. Drivers use this rule to determine the minimum distance to follow a car travelling at the same speed. A diagram representing this distance is shown.

As the speed of the cars increases, the minimum following distance also increases. Explain how the "two-second rule" leads to a greater minimum following distance as the speed of the cars increases. As part of your explanation, include the minimum following distances, in feet, for cars traveling at 30 miles per hour and 60 mile per hour.




**Sample Response:**

The minimum following distance is determined by the formula $d = rt$, where d is the minimum following distance, $r$ is the rate (or speed), and $t$ is the time. The "two-second rule" says that the time needed between cars traveling at the same speed remains constant at 2 seconds, so as the speed of the cars increases by a certain factor, then the minimum following distance must increase by the same factor. Since the speed of the cars is measured in miles per hour, and the "two-second rule" measures time in seconds, the formula shown below can be used to determine the minimum following distance, in feet.

$$d = r \bullet \frac{5280}{1} \bullet \frac{1}{3600} \bullet 2$$

For cars traveling at 30 miles per hour, the minimum following distance is 88 feet. For cars traveling at 60 miles per hour, the minimum following distance is 176 feet.

Smarter Balanced Assessment Consortium
Sample Items and Performance Tasks
http://www.smarterbalanced.org/sample-items-and-performance-tasks/

## Additional Examples to Demonstrate Understanding of Mathematical Content and Skills

### Algebra and Function Examples

Create expressions and equations in two or more variables to represent relationships between quantities. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

- A container of ice cream is taken from the freezer and sits in a room for *t* minutes. Its temperature in degrees Fahrenheit is $a - b \cdot 2^{-t} + b$, where a and b are positive constants. Write this expression in a form that shows that the temperature is always
  1. Less than $a + b$
  2. Greater than $a$

**Response:**

The form $a + b - b \cdot 2^{-t}$ for the temperature shows that it is $a + b$ minus a positive number, so always less than $a + b$. The form $a + b\,(1 - 2^{-t})$ reveals that the temperature is always greater than *a*, because it is *a* plus *a* positive number.

**CCSS-M Progressions, High School Algebra**

Build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

- Build a function that models the temperature of a cup of tea over a period of time, if the ambient room temperature is 70° and a cup of tea is made with boiling water at a temperature of $212^0$.

**Response:**

The ambient room temperature can be represented as the constant function $f(t) = 70$. The exponentially decaying function $g(t) = 142e^{-kt}$ is used to represent the decaying difference between the temperature of the tea and the temperature of the room. Therefore the function that models this situation is $T(t) = 70 + 142e^{-kt}$.

**California Draft Mathematics Framework, Algebra II**

## Additional Examples to Demonstrate Understanding of Mathematical Content and Skills - continued

### Algebra and Function Examples - continued

Fit a linear function for a scatter plot that suggests a linear association.

Find the points of intersection between the line and the circle, given the equations of both.

- Find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 90$.

**Response:**

$x^2 + (-3x)^2 = 90$ ← Substituting (-3*x*) for *y* in the equation

$x^2 + 9x^2 = 90$

$10x^2 = 90$

$x^2=9$ ← Substituting two values for *y* in the equation

$x = 3$ and $x = -3$

$y = -3x$

$y = (-3)(3)$ and $y = (-3)(-3)$

$y = -9$ $y = 9$

The points of intersection are (3, −9) and (−3, 9)

## Additional Examples to Demonstrate Understanding of Mathematical Content and Skills - continued

### Algebra and Function Examples - continued

Interpret functions that arise in applications in terms of the context. Key features to be included in interpretations: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

- The below table provides some U.S. Population data from 1982 to 1988:

U.S. Population 1982-1988

| Year | Population (in thousands) | Change in Population (in thousands) |
|---|---|---|
| 1982 | 231,664 | --- |
| 1983 | 233,792 | 233,792 - 231,664 = 2,128 |
| 1984 | 235,825 | 2,033 |
| 1985 | 237,924 | 2,099 |
| 1986 | 240,133 | 2,209 |
| 1987 | 242,289 | 2,156 |
| 1988 | 244,499 | 2,210 |

**Notice**: The change in population from 1982 to 1983 is 2,128,000, which is recorded in thousands in the first row of the 3rd column. The other changes are computed similarly. All population numbers in the table are recorded in thousands.

Source: http://www.census.gov/popest/archives/1990s/popclockest.txt

a. If we were to model the relationship between the U.S. population and the year, would a linear function be appropriate? Explain why or why not.

b. Mike decides to use a linear function to model the relationship. He chooses 2,139, the average of the values in the 3rd column, for the slope. What meaning does this value have in the context of this model?

c. Use Mike's model to predict the U.S. population in 1992.

**Response:**

a. The table shows that over one-year periods, the population increases by approximately the same amount (just a little over 2 million per year). Hence a linear function is appropriate to model the relationship between the population and the year over this short time interval.

b. The slope of the linear function, which will model this relationship, will measure the change in population per change in time. Its units will be millions-of-people per year in this problem. A value of 2139 thousand = 2,139,000 for the slope would mean that the population is growing by approximately 2,139,000 people per year.

c. The population in 1988 was 244,499,000. Mike's choice for the slope from problem (b) indicates a population growth of about 4 • (2,139,000) = 8,556,000 people between 1988 and 1992. Therefore Mike's model predicts the 1992 population to be approximately 253,055,000 people. Note the actual population of the United States in 1992 was about 255 million.

**Illustrative Mathematics, High School Functions Standards**

## Additional Examples to Demonstrate Understanding of Mathematical Content and Skills - continued

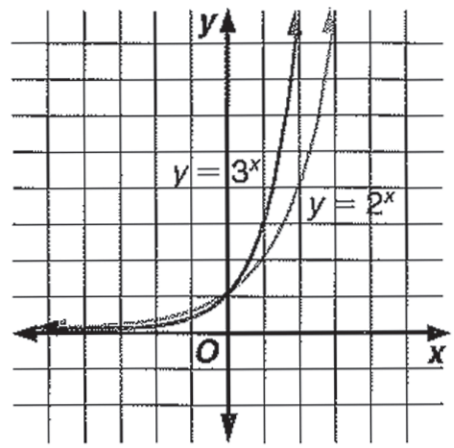
### Algebra and Function Examples - continued

- Discuss the behavior of the graphs of $y = 2^x$ and $y = 3^x$. Compare the values of $2^x$ and $3^x$ on the intervals $-10 \le x < 0$ and $0 < x \le 10$.

**Response:**

Both graphs have *y*-intercepts at (0, 1). In the interval $-10 \le x < 0$, $2^x > 3^x$ and both graphs approach the x-axis as *x* approaches -10.

In the interval $0 < x \le 10$, $3^x > 2^x$.




Translate between the geometric description and the equation for a conic section.

- Using the geometric description of a circle, derive the equation for a circle.

**Response:**

A circle consists of all points (x, y) that are at a distance r > 0 from a fixed center (h, k). For any point lying on the circle, $\sqrt{(x - h)^2 + (y - k)^2} = r$ so that $(x - h)^2 + (y - k)^2 = r^2$ determines the circle.

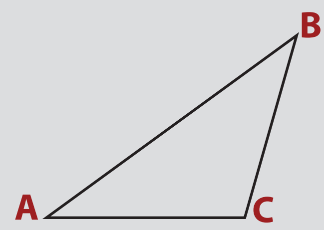
# Assessment Examples - continued

## Additional Examples to Demonstrate Understanding of Mathematical Content and Skills - continued

### Proof Examples

Prove that the sum of the interior angles in a triangle is 180º.

- Given: $\triangle ABC$. To prove: $m\angle A + m\angle B + m\angle C = 180$.



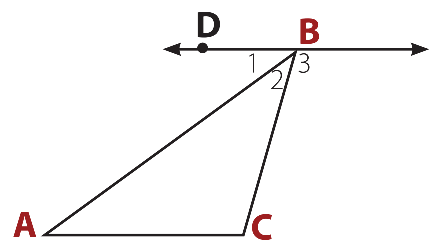

**Response:**

Draw $\overleftrightarrow{BD}$ with $\overleftrightarrow{BD} \,//\, \overleftrightarrow{AC}$. (Through a point not on a line, there is exactly one line parallel to the given line.) By angle addition,

$m\angle 1 + m\angle 2 + m\angle 3 = 180$.

Since // lines $\Longrightarrow$ AIA $\cong$, $m\angle 1 = m\angle A$ and $m\angle 3 = m\angle C$. So by substitution,

$m\angle A + m\angle B + m\angle C = 180$.

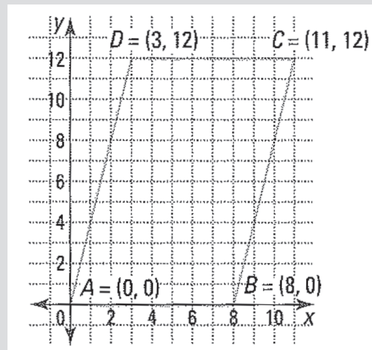


## Additional Examples to Demonstrate Understanding of Mathematical Content and Skills - continued

### Proof Examples - continued

Prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle.

- Quadrilateral *ABCD* has vertices *A* = (0, 0), *B* = (8, 0), *C* = (11, 12), and *D* = (3, 12) as shown at the right. Prove or disprove that *ABCD* is a parallelogram.




**Response:**

*ABCD* is a parallelogram if both pairs of opposite sides are parallel. Calculate the slopes of the sides of *ABCD*.

Justifications

definition of slope

Conclusions

1. slope of $\overline{AD} = \frac{12-0}{3-0} = 4$

   slope of $\overline{BC} = \frac{12-0}{11-8} = 4$

   slope of $\overline{DC} = \frac{12-12}{11-3} = \frac{0}{8} = 0$

   slope of $\overline{AB} = \frac{0-0}{8-0} = \frac{0}{8} = 0$

2. $\overline{AD} \parallel \overline{BC}$, $\overline{DC} \parallel \overline{AB}$

3. ABCD is a parallelogram.

Parallel lines have the same slope

definition of parallelogram

## Additional Examples to Demonstrate Understanding of Mathematical Content and Skills - continued

### Probability and Statistics Example

Use probability to evaluate outcomes of decisions.

- A seven-year-old boy has a favorite treat, Super Fruity Fruit Snax. These "Fruit Snax" come in pouches of 10 snack pieces per pouch, and the pouches are generally sold by the box, with each box containing 4 pouches.

  The snack pieces come in 5 different fruit flavors, and usually each pouch contains at least one piece from each of the 5 flavors. The website of the company that manufactures the product says that equal numbers of each of the 5 fruit flavors are produced and that pouches are filled in such a way that each piece added to a pouch is equally likely to be any one of the five flavors.

  Of all the 5 fruit flavors, the seven-year-old boy likes mango the best. One day, he was very disappointed when he opened a pouch and there were no (zero) mango flavored pieces in the pouch. His mother (a statistician) assured him that this was no big deal and just happens by chance sometimes.

a. If the information on the company's website is correct,

   i. What proportion of the population of snack pieces is mango flavored?

   ii. On average, how many mango flavored pieces should the boy expect in a pouch of 10 snack pieces?

   iii. What is the chance that a pouch of 10 would have no mango flavored pieces? Was the mother's statement reasonable? Explain. (Hint: if none of the 10 independently selected pieces are mango, then all 10 pieces are "not mango.")

b. The family then finds out that there were in fact no mango flavored pieces in any of the 4 pouches in the box they purchased. Again, if the information on the company's website is correct,

   i. What is the chance that an entire box of 4 pouches would have no mango-flavored pieces? (Hint: How is this related to your answer to question (iii) in part (a)?)

   ii. Based on your answer and based on the fact that this event of an entire box with "no mangoes" happened to this family, would you be concerned about the company's claims, or would you say that such an event is not surprising given the company's claims? Explain.

## Additional Examples to Demonstrate Understanding of Mathematical Content and Skills - continued

### Probability and Statistics Example - continued

**Response:**

a.

i. If each of the 5 flavors occurs equally in the population, the proportion of mango flavored pieces is 1/5 or .20.

ii. If the proportion of mango flavored pieces in the extremely large population is 1/5 or .20, then a random sample of 10 pieces (a pouch) should have mango flavored pieces on average.

iii. The chance of selecting a single non-mango flavored piece is 4/5 or .80. If the selection of each piece is independent, then the chance of getting 10 non-mango flavored pieces is or about .1074. So yes, the mother's statement that a single pouch might not have any mango flavored pieces seems reasonable, happening in slightly more than 1 out of 10 pouches.

b.

i. An entire box would consist of 40 snack pieces. As above, the chance of selecting a single non-mango flavored piece is 4/5 or .80. If the selection of each piece is independent, then the chance of getting 40 non-mango flavored pieces is or about .0001. Also, a student could use the answer determined in 1c above for the probability of a single "mango-less" pouch (.1074) and raise that value to the fourth power to represent 4 pouches. $(.1074)^4$ is about .0001.

ii. Most students would say that such an event is too rare to occur by chance (1 in 10,000 boxes) if the company's claims are true. Therefore, the company's claims may need to be called into question. However, some students who are familiar with sweepstakes and offers from children's cereals, snacks, etc. may say that a 1 in 10,000 chance is not that unusual and is more likely than winning a grand prize, etc. The important thing is that the student's decision should be explained carefully and with reference to the answer to Question b.i.

**Illustrative Mathematics**
**High School Statistics and Probability Standards**