11TH GRADE - Advanced Algebra

MATH STANDARDS GUIDANCE

WI Math Standards

Domain: Arithmetic with Polynomials and Rational Expressions (A-APR)

CLUSTER	STANDARD	EXAMPLES
A. Perform arithmetic operations on polynomials.	M.A.APR.A.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	
B. Understand the relationship between zeros and factors of polynomials	M.A.APR.B.2 Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x).	
	M.A.APR.B.3 Identify zeros of polynomials when suitable factorizations are available and use the zeros to construct a rough graph of the function defined by the polynomial.	
C. Use polynomial identities to solve problem	M.A.APR.C.4 Prove polynomial identities and use them to describe numerical relationships.	For example, use (a +20)^2=a^2+40a+400 to mentally or efficiently square numbers in the 20s. (e.g., 222=22+2*40+400=484). Generalize to other double digit numbers. Use a2=(A+b)(a-b)+b^2 and multiples of a*10 to square, e.g., 222=(22+12)(22-12)+122=340+144=484. Recognize the visual representation of (a+2b)^2-a^2=4ab as the area of a frame and find equivalent expressions.
	M.A.APR.C.5 (+) Know and apply the Binomial Theorem for the expansion of (x + y)n in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.	
D. Rewrite rational expressions.	M.A.APR.D.6 Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.	
	M.A.APR.D.7 (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.	

Domain: Creating Equations (A - CED)

CLUSTER	STANDARD	EXAMPLES
A. Create equations that describe numbers or relationships. (M)	M.A.CED.A.1 (F2Y) Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.	
	M.A.CED.A.2 (F2Y) Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	
	M.A.CED.A.3 (F2Y) Represent constraints by equations or inequalities and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.	For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.
	M.A.CED.A.4 (F2Y) Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.	For example, rearrange the formula C = 5/9 (F-32) so you solve for F

Domain: Reasoning with Equations and Inequalities (A-REI)

CLUSTER	STANDARD	EXAMPLES
A. Understand solving equations as a process of reasoning and explain the reasoning.	M.A.REI.A.1 (F2Y) Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	
	M.A.REI.A.2 Solve simple rational and radical equations in one variable and give examples showing how extraneous solutions may arise.	

B. Solve equations and inequalities in one variable.	M.A.REI.B.3 (F2Y) Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	
	M.A.REI.B.4 (F2Y) Solve quadratic equations by inspection (e.g., for x^ 2 = 49), taking square roots, completing the square, the quadratic formula, factoring, and graphing as appropriate to the initial form of the equation. Recognize when	
C. Solve systems of equations.	M.A.REI.C.5 (F2Y) Justify that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.	
	M.A.REI.C.6 (F2Y) Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	
	M.A.REI.C.7 (F2Y) Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.	For example, find the points of intersection between the line y = –3x and the circle x^2 + y^2 = 3.
	M.A.REI.C.8 (+) Represent a system of linear equations as a single matrix equation in a vector variable	
	M.A.REI.C.9 (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).	
D. Represent and solve equations and inequalities graphically.	M.A.REI.D.10 (F2Y) Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).	
	M.A.REI.D.11 (F2Y) Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g (x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations.	
	M.A.REI.D.12 (F2Y) Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality) and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	
Domain: Seeing Structure in Expressions (A-SSE)		
CLUSTER	STANDARD	EXAMPLES
A. Interpret the structure of expressions. (M)	M.A.SSE.A.1 (F2Y) Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. For example, in the expression representing height of a projective, -16t^2+vt+c recognizing there are three terms in the expression, factors within some of the terms, and coefficients. Interpret within the context the meaning of the coefficient -16 as related to gravity, the factor of v as the initial velocity, and the c-term as initial height. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret the expression representing population growth P(1+r)n as the product of P and a factor not depending on P. Interpret the meaning of the P-factor as initial population and the other factor as being related to growth rate and a period of time.	
	M.A.SSE.A.2 (F2Y) Use the structure of an expression to identify ways to rewrite it.	For example, see x4-y4 as (x^2)^2-(y^2)^2, thus recognizing it as a difference of squares that can be factored as (x^2-y^2)(x^2+y^2)
B. Write expressions in equivalent forms to solve problems. (M)	M.A.SSE.B.3 (F2Y) Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. c. Use the properties of exponents to transform expressions for exponential functions.	For example, if the expression 1.15^t represents growth in an investment account at time t (measured in years), it can be rewritten as (1.15^1/12) 12t ≈ 1.012^12t to reveal the approximate equivalent monthly rate of return is 1.2% based on an annual growth rate of 15%.
Domain: Interpreting Functions (F-IF)		
CLUSTER	STANDARD	EXAMPLES
A. Understand the concept of a function and use function notation.	M.F.IF.A.1 (F2Y) Understand that a function from one set, discrete or continuous, (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range.	
	M.F.IF.A.2 (F2Y) Use function notation, evaluate functions. and interpret statements that use function notation in terms of a context.	

	M.F.IF.A.3 (F2Y) Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.	For example, in an arithmetic sequence, $f(x) = f(x-1) + C$ or in a geometric sequence, $f(x) = f(x-1) * C$, where C is a constant.
B. Interpret functions that arise in applications in terms of context. (M)	M.F.IF.B.4 (F2Y) For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.	
	M.F.IF.B.5 Relate the domain of a function to its graph and find an appropriate domain (discrete or continuous) in the context of the given problem.	
	M.F.IF.B.6 Calculate and interpret the average rate of change of a linear or nonlinear function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.	
C. Analyze functions using different representations. (M)	M.F.IF.C.7 M.F.IF.C.7a (F2Y) Graph functions expressed symbolically and show key features of the graph using an efficient method. a. Graph linear and quadratic functions and show intercepts, maxima, and minima, and exponential functions, showing intercepts and end behavior. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. c. Graph polynomial functions, identifying zeros when suitable factorizations are available and showing end behavior. d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available and showing end behavior. e. Graph logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.	
	M.F.IF.C.8 (F2Y) Write a function defined by an expression in equivalent forms to reveal and explain different properties of the function. a. Use an efficient process to rewrite $f(x) = ax2+bx+c$ as $f(x) = a(x-h)2 +k$ or $f(x)=a(x-p)(x-q)$ to determine the characteristics of the function and interpret these in terms of a context. b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions, where t is in years, such as y = (1.01)12t is approximately y = (1.127)t, where t is in years, meaning it is a 1% growth rate each month and a 12.7% growth rate each year. Identify percent rate of change in functions, where t is in years, such as y = (1.2)(t/10) is approximately y = (1.018)t, meaning it is a 20% growth rate each decade and a 1.8% growth rate each year.	
	M.F.IF.C.9 (F2Y) Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).	For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

Domain: Building Functions (F-BF)

CLUSTER	STANDARD	EXAMPLES
A. Build a function that models a relationship between two quantities. (M)	M.F.BF.A1 Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. c. Work with composition of functions using tables, graphs, and symbols. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.	For example: The temperature of a cup of coffee can be modeled by combining together a function representing difference in temperature and the actual room temperature, which results in an exponential model. An average cost function can be created by dividing the cost of purchasing n items by the number of n items purchased, which results in a rational function.
B. Build new functions from existing functions.	M.F.BF.B.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) using transformations for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.	
	M.F.BF.B.4 Identify and create inverse functions, using tables, graphs, and symbolic methods to solve for the other variable. For example: Each car in a state is assigned a unique license plate number, and each license plate number is assigned to a unique car; thus there is an inverse relationship. Rearrange the formula C=59(F32) so you solve for F. You examine a table of values and realize the inputs and outputs are invertible. Two graphs are symmetrical about the line y = x.	
	M.F.BF.B5 Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.	

Domain: Linear, Quadratic, and Exponential Models (F-LE)

CLUSTER	STANDARD	EXAMPLES
A. Construct and compare linear, quadratic, and exponential models and solve problems. (M)	M.F.LE.A.1 (F2Y) Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.	
	M.F.LE.A.2 (F2Y) Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	
	M.F.LE.A.3 (F2Y) Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.	
	M.F.LE.A.4 For exponential models, express as a logarithm the solution to abcct= d where a, c, and d are numbers and the base b is 2,10, or e; evaluate the logarithm using technology.	

CLUSTER	STANDARD	EXAMPLES
B. Interpret expressions for functions in terms of the situation they model.	M.F.LE.B.5 (F2Y) Interpret the parameters in a linear or exponential function in terms of a context.	

Domain: Trigonometric Functions (F-TF)

CLUSTER	STANDARD	EXAMPLES
A. Extend the domain of the trigonometric functions of the unit circle.	M.F.TF.A.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.	
	M.F.TF.A.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.	
	M.F.TF.A.3 (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosine, and tangent for π–x, π+x, and 2π–x in terms of their values for x, where x is any real number.	
	M.F.TF.A.4 (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.	
B. Model periodic phenomena with trigonometric functions. (M)	M.F.TF.B.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.	
	M.F.TF.B.6 (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.	
	M.F.TF.B.7 (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.	
C. Prove and apply trigonometri c identities.	M.F.TF.C.8 Prove the Pythagorean identity sin^2(θ) + cos^2 (θ) = 1 and use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle.	
	(+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.	

Domain: The Complex Number System (N-CN)

CLUSTER	STANDARD	EXAMPLES
A. Perform arithmetic operations with complex numbers.	M.N.CN.A.1 Know there is a complex number i such that , and every complex number has the form a + bi with a and b real. Understand why complex numbers exist.	
	M.N.CN.A.2 (+) Use the relation and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	
	M.N.CN.A.3 (+) Find the conjugate of a complex number; use conjugates to find moduli (absolute values) and quotients of complex numbers.	
B. Represent complex numbers and their operations on the complex plane	M.N.CN.B.4 (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers) and explain why the rectangular and polar forms of a given complex number represent the same number.	
	M.N.CN.B.5 (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation.	For example, because has modulus 2 and argument 120˚.

	M.N.CN.B.6 (+) Calculate the distance between numbers in the complex plane as the modulus of the difference and the midpoint of a segment as the average of the numbers at its endpoints.	
C. Use complex numbers in polynomial identities and equations.	M.N.CN.C.7 Solve quadratic equations with real coefficients that have complex solutions. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b.	
	M.N.CN.C.8 (+) Extend polynomial identities to the complex numbers. For example, rewrite x 2+ 4 as (x + 2i)(x - 2i).	
	M.N.CN.C.9 (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.	

Domain: The Real Number System (N-RN)

CLUSTER	STANDARD	EXAMPLES
A. Extend the properties of exponents to rational exponents.	M.N.RN.A.1 (F2Y) Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents.	
	M.N.RN.A.2 (F2Y) Rewrite expressions involving radicals and rational exponents using the properties of exponents.	
B. Use properties of rational and irrational numbers.	M.N.RN.B.3 Explain why the sum or product of two rational numbers is rational, that the sum of a rational number and an irrational number is irrational, and that the product of a nonzero rational number and an irrational number is irrational.	

Domain: Interpreting Categorical and Quantitative Data (S-ID)

CLUSTER	STANDARD	EXAMPLES
B. Summarize, represent, and interpret data on two categorical and quantitative variables. (M)	M.SP.ID.B.6 (F2Y) Represent data on two quantitative variables on a scatter plot and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize appropriate families of functions to model. b. Informally assess the fit of a function by plotting and analyzing residuals. c. Fit a linear function for a scatter plot that suggests a linear association.	

CLUSTER	STANDARD	EXAMPLES
1	Make sense of problems and persevere in solving them.	
2	Reason abstractly and quantitatively.	
3	Construct viable arguments, and appreciate and critique the reasoning of others.	
4	Model with mathematics.	
5	Use appropriate tools strategically.	

6	Attend to precision.	
7	Look for and make use of structure.	
8	Look for and express regularity in repeated reasoning.	