6TH GRADE

MATH STANDARDS GUIDANCE

WI Math Standards

Domain: Ratios and Proportional Relationships

CLUSTER	STANDARD	EXAMPLES
A. Understand ratio concepts and use ratio reasoning to solve problems. (M)	M.6.RP.A.1: Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities	*For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*
	M.6.RP.A.2: Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship.	For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."
	M.6.RP.A.3: Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number lines, or equations. a. Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. b. Solve unit rate problems including those involving unit pricing and constant speed. c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.	For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?

Domain: The Number System

CLUSTER	STANDARD	EXAMPLES
A. Apply and extend previous understandings of multiplication and division to divide fractions by fractions.	M.6.NS.A.1: Interpret, represent, and compute division of fractions by fractions and solve word problems by using visual fraction models (e.g., tape diagrams, area models, or number lines), equations, and the relationship between multiplication and division.	For example, create a story context for (2/3) ÷ (3/4) such as "How many 3/4 -cup servings are in 2/3 of a cup of yogurt" or "How wide is a rectangular strip of land with length 3/4 mile and area 2/3 square mile?" Explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3.
B. Flexibly and efficiently compute with multi-digit numbers and find common factors and multiples.	M.6.NS.B.2: Flexibly and efficiently divide multi-digit whole numbers using strategies or algorithms based on place value, area models, and the properties of operations.	
	M.6.NS.B.3: Flexibly and efficiently add, subtract, multiply, and divide multi-digit decimals using strategies or algorithms based on place value, visual models, the relationship between operations, and the properties of operations.	
	M.6.NS.B.4: Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor.	For example, express 36 + 8 as 4 (9 + 2).
	M.6.NS.C.5: Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.	

C. Apply and extend previous understandings of numbers to the system of rational numbers. (M)	M.6.NS.C.6: Understand a rational number as a point on the number line. Extend number lines and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., -(-3) = 3, and that 0 is its own opposite. b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. c. Find and position integers and other rational numbers on a horizontal or vertical number line; find and position pairs of integers and other rational numbers on a coordinate plane.	
	M.6.NS.C.7: Understand ordering and absolute value of rational numbers. a. Interpret statements of inequality as statements about the relative position of two numbers on a number line. b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a realworld situation. d. Distinguish comparisons of absolute value from statements about order.	For example, interpret -3 > -7 as a statement that -3 is located to the right of -7 on a number line oriented from left to right. For example, write -3 °C > -7 °C to express the fact that -3°C is warmer than -7 °C. For example, for an account balance of -30 dollars, write |-30| = 30 to describe the size of the debt in dollars. For example, recognize that an account balance less than -30 dollars represents a debt greater than 30 dollars.
	M.6.NS.C.8: Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.	

Domain: The Expressions and Equations

CLUSTER	STANDARD	EXAMPLES
A. Apply and extend previous understandings of arithmetic to algebraic expressions.	M.6.EE.A.1: Write and evaluate numerical expressions involving whole-number exponents.	
	M.6.EE.A.2: Write, read, and evaluate expressions in which letters stand for numbers. a. Write expressions that record operations with numbers and with letters standing for numbers. b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, in the conventional order when there are no parentheses to specify a particular order (order of operations).	For example, express the calculation "Subtract y from 5" as 5 - y. For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms. For example, use the formulas V = s3 and A = 6 s2 to find the volume and surface area of a cube with sides of length s = 1/2.
	M.6.EE.A.3: Apply the properties of operations to generate equivalent expressions.	For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.
	M.6.EE.A.4: Identify when two expressions are equivalent (e.g., when the two expressions name the same number regardless of which value is substituted into them).	For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.

B. Reason about and solve onevariable equations and inequalities.	M.6.EE.B.5: Understand solving an equation or inequality as a process of answering a question: Which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.	
	M.6.EE.B.6: Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number or, depending on the purpose at hand, any number in a specified set.	
	M.6.EE.B.7: Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q, and x are all nonnegative rational numbers.	
	M.6.EE.B.8: Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.	
C. Represent and analyze quantitative relationships between dependent and independent variables. (M)	M.6.EE.C.9: Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.	For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.

Domain: Geometry

CLUSTER	STANDARD	EXAMPLES
A. Solve realworld and mathematical problems involving area, surface area, and volume. (M)	M.6.G.A.1: Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.	
	M.6.G.A.2: Find volumes of right rectangular prisms with fractional edge lengths by using physical or virtual unit cubes. Develop (construct) and apply the formulas $V = l\ w\ h$ and $V = B\ h$ to find volumes of right rectangular prisms in the context of solving real-world and mathematical problems.	
	M.6.G.A.3: Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.	
	M.6.G.A.4: Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving realworld and mathematical problems.	

Domain: Statistics and Probability

CLUSTER	STANDARD	EXAMPLES

A. Develop understanding of statistical variability. (M)	M.6.SP.A.1: Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers.	For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.
	M.6.SP.A.2: Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.	
	M.6.SP.A.3: Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.	
B. Summarize and describe distributions. (M)	M.6.SP.B.4: Display numerical data in plots on a number line, including dot plots, histograms, and box plots.	
	M.6.SP.B.5: Summarize numerical data sets in relation to their context, such as by: a. Reporting the number of observations. b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. c. Describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered and the quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation) were given. d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.	

CLUSTER	STANDARD
Math Practice 1: Make sense of problems and persevere in solving them.	6-8 Mathematically proficient middle school students set out to understand a problem and then look for entry points to its solution. Students identify questions to ask and make observations about the situation by using noticing and attending to aspects of the problem that look familiar. Students make assumptions where needed to make the problem more clearly defined. They analyze problem conditions and goals, translating, for example, verbal descriptions into equations, diagrams, or graphs as part of the process. They consider analogous problems, and try special cases and simpler forms of the original in order to gain insight into its solution. For example, to understand why a 20% discount followed by a 20% markup does not return an item to its original price, they might translate the situation into a tape diagram or a general equation; or they might first consider the situation for an item priced at $100. Mathematically proficient students can explain how alternate representations of problem conditions relate to each other. For example, they can identify connections between the solution to a word problem that uses only arithmetic and a solution that uses variables and algebra; and they can navigate among verbal descriptions, tables, graphs, and equations representing linear relationships to gain insights into the role played by constant rate of change. Mathematically proficient students check their approach, continually asking themselves "Does this approach make sense?" and "Can I solve the problem in a different way?" Students ask themselves these types of questions as a way to preserve through problem solving. While working on a problem, they monitor and evaluate their progress and change course if necessary. Students will reflect and revise their solution as needed. They can understand the approaches of others to solving complex problems and compare approaches.
Math Practice 2: Reason abstractly and quantitatively.	Mathematically proficient middle school students make sense of quantities and relationships in problem situations. For example, they can apply ratio reasoning to convert measurement units and proportional relationships to solve percent problems. They represent problem situations using symbols and then manipulate those symbols in search of a solution (decontextualize). They can, for example, solve problems involving unit rates by representing the situations in equation form. Mathematically proficient students also pause as needed during problem solving to validate the meaning of the symbols involved. In the process, they can look back at the applicable units of measure to clarify or inform solution pathways (contextualize). Students can integrate quantitative information and concepts expressed in text and visual formats. Students can examine the constant and coefficient used in a linear function and express the meaning of those numbers related to a contextual situation. They can work with the function in different representations, such as a graph, keeping in mind the slope and vertical Wisconsin Standards for Mathematics 93 intercept have meaning related to the context. Quantitative reasoning also entails knowing and flexibly using different properties of operations and objects. For example, in middle school, students use properties of operations to generate equivalent expressions and use the number line to understand multiplication and division of rational numbers.

Math Practice 3: Construct viable arguments, and appreciate and critique the reasoning of others.	Mathematically proficient middle school students understand and use assumptions, definitions, and previously established results in constructing verbal and written arguments. They understand the importance of making and exploring the validity of conjectures. They can recognize and appreciate the use of counterexamples. For example, using numerical counterexamples to identify common errors in algebraic manipulation, such as thinking that 5 – 2x is equivalent to 3x. Conversely, given a pair of equivalent algebraic expressions, they can show that the two expressions name the same number regardless of which value is substituted into them by showing which properties of operations can be applied to transform one expression into the other. They can explain and justify their conclusions to others using numerals, symbols, and visuals. They also reason inductively about data, making plausible arguments that take into account the context from which the data arose. For example, they might argue that the great variability of heights in their class is explained by growth spurts, and that the small variability of ages is explained by school admission policies. While communicating their own mathematical ideas is important, middle school students also learn to be open to others' mathematical ideas. They appreciate a different perspective or approach to a problem and learn how to respond to those ideas, respecting the reasoning of others (Gutiérrez 2017, 17-18). Together, students make sense of the mathematics, asking helpful questions such as "How did you get that?" "Why is that true?" and "Does that always work?" that clarify or deepen everyone's understanding. Mathematically proficient students are able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and if there is a flaw in an argument, explain what it is. Students engage in collaborative discussions, drawing on evidence from problem texts and arguments of others, follow conventions for collegial discussions, and qualify their own views in light of evidence presented. They can present their findings and results to a given audience through a variety of formats such as posters, whiteboards, and interactive materials.
Math Practice 4: Model with mathematics.	"In the course of a student's mathematics education, the word 'model' is used in many ways. Several of these, such as manipulatives, demonstration, role modeling, and conceptual models of mathematics, are valuable tools for teaching and learning. However, they are different from the practice of mathematical modeling. Mathematical modeling, both in the workplace and in school, uses mathematics to answer big, messy, reality-based questions" (Bliss and Libertini 2016, 7). Mathematically proficient middle school students formulate their own problems that emerge from natural circumstances as they mathematize the world around them. They can identify the mathematical elements of a situation and generate questions that can be addressed using mathematics (e.g., noticing and wondering). Middle school students can see a complicated problem and understand how that problem contains smaller problems to be solved. They are comfortable making assumptions as they decide "what matters". Mathematically Wisconsin Standards for Mathematics 94 proficient middle students understand that there are multiple solutions to a modeling problem so they are working to find a solution rather than the solution. Students make judgements about what matters and assess the quality of their solution (Bliss and Libertini 2016, 10). They interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving their mathematical modeling approach if it has not served its purpose. As they decontextualize the situation and represent it mathematically, they are also reasoning abstractly (Math Practice 2). In the middle school grades, students encounter mathematical opportunities each and every day at school and at home. Mathematically proficient middle school students might consider how to plan a route to get to school with their friends. Students might then need to make assumptions about travel time and when to leave the house. In the morning they implement their plan and revise it by changing the departure time or including additional friends to the route. As a classroom, students might plan a fundraising event involving selling popcorn after school. In this example, sometimes students will be engaged in only a part of the modeling cycle such as making assumptions about how much to charge or how much popcorn to make (Godbold, Malkevitch, Teague, and van der Kooij 2016, 50). Note: Although physical objects and drawings can be used to model a situation, using these tools absent a contextual situation is not an example of Math Practice 4. For example, drawing an area model to illustrate the distributive property in 4(t + s) = 4t + 4s would not be an example of Math Practice 4. Math Practice 4 is about applying math to a problem in context.
Math Practice 5: Use appropriate tools strategically.	Mathematically proficient middle school students strategically consider the available tools when solving a mathematical problem and while exploring a mathematical relationship. These tools might include pencil and paper, concrete models, a ruler, a protractor, a dynamic graphing tool, a spreadsheet, a statistical package, or dynamic geometry software. Proficient students make sound decisions about when each of these tools might be helpful, recognizing both the insights to be gained and their limitations. For example, they use estimation to check reasonableness; graph functions defined by expressions to picture the way one quantity depends on another; use algebra tiles to see how the properties of operations familiar from the elementary grades continue to apply to algebraic expressions; use an area model to visualize multiplication of rational numbers; use dynamic graphing tools to approximate solutions to systems of equations; use spreadsheets to analyze data sets of realistic size; or use dynamic geometry software to discover properties of parallelograms. Students are also strategic about when not to use tools, such as by simplifying an expression before substituting values into it (Math Practice 7), or rounding the inputs to a calculation and calculating on paper when an approximate answer is enough (Math Practice 6). When making mathematical models, students know that technology can enable them to visualize the results of their assumptions, to explore consequences, and to compare predictions with data (Math Practice 4). Mathematically proficient students are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems.

Math Practice 6: Attend to precision.	Mathematically proficient middle school students communicate precisely to others both verbally and in writing. They present claims and findings, emphasizing salient points in a focused, coherent manner with relevant evidence, sound and valid reasoning, well-chosen details, and precise language. They use clear definitions in discussion with others and in their own reasoning and determine the meaning of symbols, terms, and phrases as used in specific mathematical contexts. For example, they can use the definition of a rational number to explain why $\sqrt{2}$ is irrational. Middle school students and describe congruence and similarity in terms of transformations in the plane. They decide which parts of a problem need to be defined by a variable, state the meaning of the symbols, consistently and appropriately, such as independent and dependent variables of a linear equation. They are careful about specifying units of measure, and label axes to display the correct correspondence between quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate to the context. For example, they accurately apply scientific notation to large numbers and use measures of center to describe data sets. In statistics and probability, students must attend to precision in the manner they write their statistical questions, the manner in which they collect their data, and the process they use to develop a simulation. Mathematically proficient middle school students care that an answer is right or reasonable; they attend to precision when they check their work; they solve the problem another way; they make revisions where appropriate.
Math Practice 7: Look for and make use of structure.	Mathematically proficient middle school students look closely to discern a pattern or structure. They might use the structure of the number line to demonstrate that the distance between two rational numbers is the absolute value of their difference, ascertain the relationship between slopes and solution sets of systems of linear equations, and see that the equation $3x = 2y$ represents a proportional relationship with a unit rate of $3/2 = 1.5$. They might recognize how the Pythagorean Theorem is used to find distances between points in the coordinate plane and identify right triangles that can be used to find the length of a diagonal in a rectangular prism. They also can step back for an overview and shift perspective, as in finding a representation of consecutive numbers that shows all sums of three consecutive whole numbers are divisible by six. They can see complicated things as single objects, such as seeing two successive reflections across parallel lines as a translation along a line perpendicular to the parallel lines or understanding 1.05a as an original value, a, plus 5% of that value, 0.05a. They can evaluate numeric expressions without combining each term in the order they are given.
Math Practice 8: Look for and express regularity in repeated reasoning.	Mathematically proficient middle school students notice if calculations are repeated, and look for both general methods and general and efficient methods. Working with tables of equivalent ratios, they might deduce the corresponding multiplicative relationships and make generalizations about the relationship to rates. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity with which interior angle sums increase with the number of sides in a polygon might lead them to the general formula for the interior angle sum of an n-gon. As they work to Wisconsin Standards for Mathematics 96 solve a problem, mathematically proficient students maintain oversight of the process while attending to the details. They continually evaluate the reasonableness of their results throughout all stages of the process.