4TH GRADE

MATH STANDARDS GUIDANCE

WI Math Standards

Bridges alignment to WI Math Standards

Domain: Operations and Algebraic Thinking		
CLUSTER	STANDARD	EXAMPLES
A. Use the four operations with whole numbers to solve problems.	M.4.OA.A.1: Interpret a multiplication equation as a multiplicative comparison, e.g., interpret 35 = 5 x 7 as astatement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.	
	M.4.OA.A.2: Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.	
	M.4.OA.A.3: Solve multi-step word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies.	
B. Gain familiarity with factors and multiples.	M.4.OA.B.4: Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite.	
C. Generate and analyze patterns.	M.4.OA.C.5: Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself.	*For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*
D. Multiply and divide within 100.	M.4.OA.D.6: Flexibly and efficiently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 x 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations [e.g., knowing that 7 x 6 can be thought of as 7 groups of 6 so one could think 5 groups of 6 is 30 and 2 more groups of 6 is 12 and 30 + 12 = 42 (informal use of the distributive property)].	

Domain: Numbers and Operations in Base Ten		
CLUSTER	STANDARD	EXAMPLES
A. Generalize place value understanding for multi-digit whole numbers.	M.4.NBT.A.1: Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right.	*For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.*
	M.4.NBT.A.2: Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place and describe the result of the comparison using words and symbols (>, =, and <).	
	M.4.NBT.A.3: Use place value understanding to generate estimates for real-world problem situations, with multidigit whole numbers, using strategies such as mental math, benchmark numbers, compatible numbers, and rounding. Assess the reasonableness of their estimates.	*For example, Is my estimate too low or too high? What degree of precision do I need for this situation?*
B. Use place value understanding and properties of operations to perform multidigit arithmetic.	M.4.NBT.B.4: Flexibly and efficiently add and subtract multi-digit whole numbers using strategies or algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.	
	M.4.NBT.B.5: Multiply a whole number of up to four digits by a one-digit whole number, and multiply two twodigit numbers, using strategies based on place value and the properties of operations. Illustrate an explain the calculation by using equations, rectangular arrays, or area models.	
	M.4.NBT.B.6: 6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, or area models.	

Domain: Number and Operations -- Fractions		
CLUSTER	STANDARD	EXAMPLES

A. Extend understanding of fraction equivalence	M.4.NF.A.1: Understand fraction equivalence. a. Explain why a fraction is equivalent to another fraction by using visual fraction models (e.g., tape diagrams and number lines), with attention to how the number and the size of the parts differ even though the two fractions themselves are the same size. b. Understand and use a general principle to recognize and generate equivalent fractions that name the same amount.	
	M.4.NF.A.2: Compare fractions with different numerators and different denominators while recognizing that comparisons are valid only when the fractions refer to the same whole. Justify the conclusions by using visual fraction models (e.g., tape diagrams and number lines) and by reasoning about the size of the fractions, using benchmark fractions (including whole numbers), or creating common denominators or numerators. Describe the result of the comparison using words and symbols (>, =, and <).	
B. Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.	M.4.NF.B.3: Understand composing and decomposing fractions. a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. b. Decompose a fraction into a sum of unit fractions or multiples of that unit fraction in more than one way, recording each decomposition by an equation. Justify decompositions with explanations, visual fraction models, or equations. For example: 3/8 = 1/8 + 1/8 + 1/8; 3/8 = 1/8 + 2/8; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8. c. Add and subtract fractions, including mixed numbers, with like denominators (e.g., 3/8 + 2/8) and related denominators (e.g., 1/2 +1/4, 1/3 + 1/6) by using visual fraction models (e.g., tape diagrams and number lines), properties of operations, and the relationship between addition and subtraction. d. Solve word problems involving addition and subtraction of fractions with like and related denominators, including mixed numbers, by using visual fraction models and equations to represent the problem. Students are not required to rename fractions in lowest terms nor use least common denominators.	
	M.4.NF.B.4: Apply and extend previous understandings of multiplication to multiply a whole number times a fraction. a. Understand a fraction as a group of unit fractions or as a multiple of a unit fraction. For example, 5/4 can be represented visually as 5 groups of 1/4, as a sum of unit fractions 1/4 + 1/4 + 1/4 + 1/4 + 1/4, or as a multiple of a unit fraction 5 x 1/4. b. Represent a whole number times a non-unit fraction (e.g., 3 x 2/5) using visual fraction models and understand this as combining equal groups of the non-unit fraction (3 groups of 2/5) and as a collection of unit fractions (6 groups of 1/5), recognizing this product as 6/5. c. Solve word problems involving multiplication of a whole number times a fraction by using visual fraction models and equations to represent the problem. Understand a reasonable answer range when multiplying with fractions. For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?	
C. Understand decimal notation for fractions and compare decimal fractions.	M.4.NF.C.5: Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.	*For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.*
	M.4.NF.C.6: Use decimal notation for fractions with denominators 10 or 100, connect decimals to real-world contexts, and represent with visual models (e.g., number line or area model).	*For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line.*
	M.4.NF.C.7: Compare decimals to hundredths by reasoning about their size and using benchmarks. Recognize that comparisons are valid only when the decimals refer to the same whole. Justify the conclusions, by using explanations or visual models (e.g., number line or area model) and describe the result of the comparison using words and symbols (>, =, and <).	
Domain: Measurement and Data		

CLUSTER	STANDARD	EXAMPLES
A. Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.	M.4.MD.A.1: Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb., oz.; l, ml; hr., min., sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table.	*For example, know that 1 ft. is 12 times as long as 1 in. Express the length of a 4 ft. snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36).*
	M.4.MD.A.2: Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as a number line that feature a measurement scale.	
	M.4.MD.A.3: Apply the area and perimeter formulas for rectangles in real-world and mathematical problems.	*For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*
B. Represent and interpret data.	M.4.MD.B.4: Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots.	*For example, from a line plot find and interpret the difference in length between the longest and shortes specimens in an insect collection.*
C. Geometric measurement: understand concepts of angle and measure angles.	M.4.MD.C.5: Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint and understand concepts of angle measurement: a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "onedegree angle" and can be used to measure angles. b. An angle that turns through n one-degree angles is said to have an angle measure of n degrees.	
	M.4.MD.C.6: Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.	
	M.4.MD.C.7: Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.	

Domain: Geometry

CLUSTER	STANDARD	EXAMPLES
A. Draw and identify lines and angles, and classify shapes by properties of their lines and angles.	M.4.G.A.1: Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.	
	M.4.G.A.2: Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category and identify right triangles.	
	M.4.G.A.3: Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.	

Standards for Mathematical Practice

CLUSTER	STANDARD

Math Practice 1: Make sense of problems and persevere in solving them.	K-5: Mathematically proficient elementary students explain to themselves the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. For example, instead of hunting for "key words" in a word problem, students might use concrete objects or pictures to show the actions of a problem, such as counting out and joining two sets. If students are not at first making sense of a problem or seeing a way to begin, they ask questions about what is happening in the problem that will help them get started. As they work, they continually ask themselves, "Does this make sense?" When they find that their solution pathway does not make sense, they look for another pathway that does. They may consider simpler forms of the original problem; for example, they might replace multi-digit numbers in a word problem with single-digit numbers to better appreciate the quantities in the problem and how they relate. Mathematically proficient students consider different solution pathways, both their own and those of other students, in order to identify and analyze connections among approaches. They can explain connections among physical models, pictures, diagrams, equations, verbal descriptions, tables, and graphs. Once students have a solution, they often check their answers to problems using a different approach.
Math Practice 2: Reason abstractly and quantitatively.	K-5: Mathematically proficient elementary students make sense of quantities and their relationships in problem situations. They can contextualize quantities and operations by using visual representations or stories. They interpret symbols as having meaning, not just as directions to carry out a procedure. Even as they manipulate the symbols, they can pause as needed to access the meaning of the numbers, the units, and the operations that the symbols represent. Mathematically proficient students know and flexibly use different properties of operations, numerical relationships, numbers, and geometric objects. They can contextualize an abstract problem by placing it in a context that they can then use to make sense of the mathematical ideas. For example, if a student chooses to evaluate the expression 13 x 25 mentally, the student might think of a context to help produce a strategy—for example, by thinking "Thirteen groups of 25 is like having 13 quarters." This prompts a strategy of thinking "I know that 10 quarters is $2.50 and 3 quarters is $0.75. $2.50 and $0.75 is $3.25." In this example the student uses a context to think through a strategy for solving the problem, using their knowledge of money and of Wisconsin Standards for Mathematics 25 decomposing one factor based on place value (13 = 10+3). The student then uses the context to identify the solution to the original problem. Mathematically proficient students can also make sense of a contextual problem and express the actions or events that are described in the problem using numbers and symbols. If they work with the symbols to solve the problem, they can then interpret their solution in terms of the context. Consider the problem: A teacher wants to bring 10 pumpkins to school to decorate the classroom. Some are big pumpkins and some are small pumpkins. How many of each size pumpkin might the teacher bring to school? When students create the number sentence 4+6=10, they have decontextualized the problem and expressed it with numbers and symbols. When they can explain that the number sentence means, "4 big pumpkins plus 6 small pumpkins equals 10 pumpkins," they demonstrate their ability to recontextualize the numbers and equation back to the word problem.
Math Practice 3: Construct viable arguments, and appreciate and critique the reasoning of others.	K-5: Mathematically proficient elementary students construct verbal and written mathematical arguments that explain the reasoning underlying a strategy, solution, or conjecture. Arguments might use concrete referents such as objects, drawings, diagrams, and actions. Arguments might also rely on definitions, previously established results, properties, or structures. For example, a student might argue that 1/5 >1/9 on the basis that one of 5 equal parts of a whole is larger than one of 9 equal parts of that whole, because with more equal parts, the size of each part must be smaller. Another example is reasoning that two different shapes have equal area because it has already been demonstrated that they are each half of the same rectangle. Students might also use counterexamples to argue that a conjecture is not true—for example, a rhombus is an example that shows that not all quadrilaterals with four equal sides are squares; or, multiplying by 1 shows that a product of two whole numbers is not always greater than each factor. Mathematically proficient students present their arguments in the form of representations, actions on those representations, explanations in words (oral or written), or a combination of these three. Some of their arguments apply to individual problems, but others are about conjectures based on regularities they have noticed across multiple problems (Math Practice 8). As they articulate and justify generalizations, students consider to which mathematical objects (numbers or shapes, for example) their generalizations apply. For example, primary grade students may believe a generalization about the behavior of addition applies to positive whole numbers less than 100 because those are the numbers with which they are currently familiar. As they expand their understanding of the number system, they may reexamine their conjecture for numbers in the hundreds and thousands. Intermediate grade students return to their conjectures and arguments about whole numbers to determine whether they apply to fractions and decimals. Wisconsin Standards for Mathematics 26 While communicating their own mathematical ideas is important, elementary students also learn to be open to others' mathematical ideas. They appreciate a different perspective or approach to a problem and learn how to respond to those ideas, respecting the reasoning of others (Gutiérrez 2017, 17-18). Together, students make sense of the mathematics, asking helpful questions that clarify or deepen everyone's understanding, and reconsider their own arguments in response to the collaboration.
Math Practice 4: Model with mathematics.	K-5: "In the course of a student's mathematics education, the word 'model' is used in many ways. Several of these, such as manipulatives, demonstration, role modeling, and conceptual models of mathematics, are valuable tools for teaching and learning. However, they are different from the practice of mathematical modeling. Mathematical modeling, both in the workplace and in school, uses mathematics to answer big, messy, reality-based questions (Bliss and Libertini 2016, 7). " Mathematically proficient elementary students formulate their own problems that emerge from natural circumstances as they mathematize the world around them. They can identify the mathematical elements of a situation and generate questions that can be addressed using mathematics (e.g., noticing and wondering). Students dig into the context and make assumptions as they decide "what matters." Mathematically proficient elementary students understand that there are multiple solutions to a modeling problem so they are working to find a solution rather than the solution. Students make judgements about what matters and assess the quality of their solution (Bliss and Libertini 2016, 10). They interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving their mathematical modeling approach if it has not served its purpose. As they decontextualize the situation and represent it mathematically, they are also reasoning abstractly (Math Practice 2). In the elementary grades, students encounter mathematical modeling opportunities each and every day at school and at home. Students might consider how the classroom's set of blocks should be shared throughout recess time. Students might then need to make assumptions about how many blocks each student should have as well as the length of time each student should have the blocks. Once a solution is determined, students could be asked to refine their model by posing the question, "What if one of our friends will not be at recess?" Children might also be presented with a bag of apples and simply asked "Is this enough for our class/family?" or consider the question, "Is the carpet in our classroom big enough for our bodies?" Note: Although physical objects and visuals can be used to model a situation, using these tools absent a contextual situation is not an example of Math Practice 4. For example, solving a word problem using counters or a tape diagram would not be modeling with mathematics, instead this is modeling the mathematics. Math Practice 4 is about engaging in solving authentic real-world problems.

Math Practice 5: Use appropriate tools strategically.	K-5: Mathematically proficient elementary students strategically consider the tools that are available when solving a mathematical problem, whether in a real-world or mathematical context. These tools might include physical objects (e.g., manipulatives, pencil and paper, rulers), conceptual tools (e.g., properties of operations, algorithms), drawings or diagrams (e.g., number lines, tally marks, tape diagrams, arrays, tables, graphs), and available technologies (e.g., calculators, online apps). Mathematically proficient students are sufficiently familiar with tools appropriate for their grade and areas of content to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained from their use as well as their limitations. Students choose tools that are relevant and useful to the problem at hand. For example, when determining how to measure length, students may compare the benefits of using non-standard units of measure (e.g., their own hands, paperclips) versus standard units and tools (e.g., an inch or centimeter ruler). As another example, when presented with 1002-3 or 101-98, students subtract strategically, which may involve reasoning, counting, or decomposing rather than using a written algorithm.
Math Practice 6: Attend to precision.	K-5: Mathematically proficient elementary students use precise language to communicate orally and in written form. They come to appreciate, understand, and use mathematical vocabulary not in isolation, but in the context of doing mathematical thinking and problem solving. They may start by using everyday language to express their mathematical ideas and gradually select words with greater clarity and specificity. For example, they may initially use the word "triangle" to refer only to equilateral triangles resting on their bases, but come to understand and use a more precise definition of a triangle as a closed figure with three straight sides. As another example, they may initially explain a solution by saying, "it works" without explaining what "it" means but later clarify their explanation with specific details. In using mathematical representations, students provide appropriate labels to precisely communicate the meaning of their representations (e.g., charts, graphs, and drawings). When making mathematical arguments about a solution, strategy, or conjecture (Math Practice 3), mathematically proficient students learn to craft careful explanations that communicate their reasoning by referring specifically to each important mathematical element, describing the relationships among them, and connecting their words clearly to their representations. Students use mathematical symbols correctly and can describe the meaning of the symbols they use. For example, they use the equal sign consistently and appropriately. They state the meaning of the symbols they choose in relation to the problem at hand. Wisconsin Standards for Mathematics 28 Students use tools and strategies (e.g., measuring tools, estimation) effectively, to maintain a level of precision that is appropriate to the situation. They specify units of measure where needed. Perseverance and attention to detail are mathematical habits of mind; mathematically proficient students check for reasonableness and accuracy by solving a problem a second way, analyzing errors, and learning from them.
Math Practice 7: Look for and make use of structure.	Mathematically proficient elementary students use structures such as place value, the properties of operations, and attributes of shapes to solve problems. In many cases, they have identified and described these structures through repeated reasoning (Math Practice 8). When students use an algorithm to solve 53-17 in order to fully understand how to decompose the tens and ones, they must understand that 53 can be seen as 4 tens and 13 ones, not just 5 tens and 3 ones. When younger students recognize that adding 1 results in the next counting number, they are identifying the basic structure of whole numbers. When older students calculate 16 x 9, they might apply the structure of place value and the distributive property to find the product: 16 x 9 = (10 + 6) x 9 = (10 x 9) + (6 x 9). To determine the volume of a 3 x 4 x 5 rectangular prism, students might see the structure of the prism as five layers of 3 x 4 arrays of cubes. Students in elementary grades look for and make use of structure when they view expressions as objects to observe and interpret. For example, students might observe that 120 – 41 must be one less than 120 – 40 because "if you subtract one more, the result will be one less" (Math Practice 8). Students can interpret the expression 5 x 3 + 6 x 3 as "five groups of three and six more groups of three" or notice there are a total of 11 groups of 3. A word problem that involves distributing 29 marbles among 4 vases could lead (Math Practice 4) to an equation model (29 – 1) ÷ 4 = 7, where the expression on the left-hand side not only has the value 7 but also suggests, based on its structure, a process of discarding 1 marble and dividing the rest of the marbles equally into 4 groups of 7.
Math Practice 8: Look for and express regularity in repeated reasoning.	Mathematically proficient elementary students look for and identify regularities as they solve multiple related problems. Students make and test conjectures, reason about and express these regularities as generalizations about structures and relationships, and then use those generalizations to solve problems (Math Practice 7). For example, younger students might notice that when tossing two-color counters to find combinations of a given number, over time students will notice a pattern (commutative property of addition). For example, when tossing six 2-sided counters, they may get 2 red, 4 yellow and 4 red, 2 yellow and when tossing 4 counters, they get 1 red, 3 yellow and 3 red, 1 yellow. Wisconsin Standards for Mathematics 29 In the elementary grades students can recognize and use patterns to help them become flexible with addition. For example, given the number string below, students may recognize they can take one away from the 5 and add it to the first number to make a multiple of ten. They also may notice a pattern related to the first digit increasing by 10, therefore the answer increases by 10. 9+5 19+5 29+5 39+5 When drawing and representing fractions, students might notice a consistent relationship between the numerator and denominator of fractions that equal one half (e.g., that the numerator is half the denominator and the denominator is two times the numerator). They can generalize from these repeated examples that all fractions equal to one half show this relationship. As students practice articulating their observations both verbally and in writing, they learn to communicate with greater precision (Math Practice 6). As they explain why these generalizations must be true, they construct, critique, and compare arguments (Math Practice 3).