

## MATH STANDARDS GUIDANCE

### *WI Math Standards*

*Bridges alignment to WI Math Standards*

Domain: Operations and Algebraic Thinking		
CLUSTER	STANDARD	EXAMPLES
<i>A. Represent and solve problems involving addition and subtraction.</i>	M.2.OA.A.1: Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, ex.; by using drawings and equations with a symbol for the unknown number to represent the problem.	
<i>B. Add and subtract within 20.</i>	M.2.OA.B.2: Flexibly and efficiently add and subtract within 20 using multiple mental strategies which may include counting on; making ten; decomposing a number leading to a ten; using the relationship between addition and subtraction (ex.; knowing that $8 + 4 = 12$ , one knows $12 - 8 = 4$ ); and creating equivalent but easier or known sums (ex.; adding $6 + 7$ by creating the known equivalent $6 + 6 + 1 = 12 + 1 = 13$ ).	
<i>C. Work with equal groups of objects to gain foundations for multiplication.</i>	M.2.OA.C.3: Determine whether a group of objects (up to 20) has an odd or even number of members, ex.; by pairing objects or counting them by twos; write an equation to express an even number as a sum of two equal addends. M.2.OA.C.4: Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.	
Domain: Number and Operations in Base Ten		
CLUSTER	STANDARD	EXAMPLES
<i>A. Understand place value.</i>	M.2.NBT.A.1: Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; ex., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases: a. 100 can be thought of as a bundle of ten tens—called a "hundred". b. The numbers 100,200,300,400,500,600,700,800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones). M.2.NBT.A.2: Count within 1,000; skip-count by fives, tens, and hundreds. M.2.NBT.A.3: Read and write numbers to 1,000 using base-ten numerals, number names, and expanded form. M.2.NBT.A.4: Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, and describe the result of the comparison using words and symbols ( $>$ , $=$ , and $<$ ).	
<i>B. Use place value understanding and properties of operations to add and subtract.</i>	M.2.NBT.B.5: Flexibly and efficiently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. In Grade 2, subtraction with decomposition is an exception and may include drawings or representations. M.2.NBT.B.6: Add up to four two-digit numbers using strategies based on place value and properties of operations. M.2.NBT.B.7: Add and subtract within 1,000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds. M.2.NBT.B.8: Mentally add 10 or 100 to a given number 100 - 900, and mentally subtract 10 or 100 from a given number 100 - 900. M.2.NBT.B.9: Explain why addition and subtraction strategies work, using place value and the properties of operations. These explanations may be supported by drawings or objects.	
Domain: Measurement and Data		
CLUSTER	STANDARD	EXAMPLES
<i>A. Measure and estimate lengths in standard units.</i>	M.2.MD.A.1: Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes. M.2.MD.A.2: Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen. M.2.MD.A.3: Estimate lengths using units of inches, feet, centimeters, and meters. M.2.MD.A.4: Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.	
<i>B. Relate addition and subtraction to length.</i>	M.2.MD.B.5: Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, ex.; by using drawings (such as number lines) and equations with a symbol for the unknown number to represent the problem. M.2.MD.B.6: Represent whole numbers as lengths from 0 on a number line with equally spaced points corresponding to the numbers 0, 1, 2 ... and represent whole-number sums and differences within 100 on a number line.	

C. Work with time and money.	M.2.MD.C.7: Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m. M.2.MD.C.8: Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using \$ and ¢ symbols appropriately.	Example: If you have 2 dimes and 3 pennies, how many cents do you have?
D. Represent and interpret data.	M.2.MD.D.9: 9 Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units. M.2.MD.D.10: Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put together, take-apart, and compare problems using information presented in a bar graph.	

Domain: Geometry		
CLUSTER	STANDARD	EXAMPLES
A. Reason with shapes and their attributes	M.2.G.A.1: Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes. Sizes are compared directly or visually, not compared by measuring. M.2.G.A.2: Partition a rectangle into rows and columns of same-size squares and count to find the total number of them. M.2.G.A.3: Partition circles and rectangles into two, three, or four equal shares, describe and count the shares using the words halves, thirds, and fourths, and use phrases half of, a third of, and a fourth of the whole. Describe the whole as composed of two halves, three thirds, and four fourths. Recognize that equal shares of identical wholes need not have the same shape.	

Standards for Mathematical Practice	
CLUSTER	STANDARD
Math Practice 1: Make sense of problems and persevere in solving them.	K-5: Mathematically proficient elementary students explain to themselves the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. For example, instead of hunting for "key words" in a word problem, students might use concrete objects or pictures to show the actions of a problem, such as counting out and joining two sets. If students are not at first making sense of a problem or seeing a way to begin, they ask questions about what is happening in the problem that will help them get started. As they work, they continually ask themselves, "Does this make sense?" When they find that their solution pathway does not make sense, they look for another pathway that does. They may consider simpler forms of the original problem; for example, they might replace multi-digit numbers in a word problem with single-digit numbers to better appreciate the quantities in the problem and how they relate.  Mathematically proficient students consider different solution pathways, both their own and those of other students, in order to identify and analyze connections among approaches. They can explain connections among physical models, pictures, diagrams, equations, verbal descriptions, tables, and graphs. Once students have a solution, they often check their answers to problems using a different approach.
Math Practice 2: Reason abstractly and quantitatively.	K-5: Mathematically proficient elementary students make sense of quantities and their relationships in problem situations. They can contextualize quantities and operations by using visual representations or stories. They interpret symbols as having meaning, not just as directions to carry out a procedure. Even as they manipulate the symbols, they can pause as needed to access the meaning of the numbers, the units, and the operations that the symbols represent.  Mathematically proficient students know and flexibly use different properties of operations, numerical relationships, numbers, and geometric objects. They can contextualize an abstract problem by placing it in a context that they can then use to make sense of the mathematical ideas. For example, if a student chooses to evaluate the expression $13 \times 25$ mentally, the student might think of a context to help produce a strategy—for example, by thinking "Thirteen groups of 25 is like having 13 quarters." This prompts a strategy of thinking "I know that 10 quarters is \$2.50 and 3 quarters is \$0.75. \$2.50 and \$0.75 is \$3.25." In this example the student uses a context to think through a strategy for solving the problem, using their knowledge of money and of Wisconsin Standards for Mathematics 25 decomposing one factor based on place value ( $13 = 10 + 3$ ). The student then uses the context to identify the solution to the original problem.  Mathematically proficient students can also make sense of a contextual problem and express the actions or events that are described in the problem using numbers and symbols. If they work with the symbols to solve the problem, they can then interpret their solution in terms of the context. Consider the problem: A teacher wants to bring 10 pumpkins to school to decorate the classroom. Some are big pumpkins and some are small pumpkins. How many of each size pumpkin might the teacher bring to school? When students create the number sentence $4 + 6 = 10$ , they have decontextualized the problem and expressed it with numbers and symbols. When they can explain that the number sentence means, "4 big pumpkins plus 6 small pumpkins equals 10 pumpkins," they demonstrate their ability to recontextualize the numbers and equation back to the word problem.

<p>Math Practice 3: Construct viable arguments and critique the reasoning of others.</p>	<p>K-5: Mathematically proficient elementary students construct verbal and written mathematical arguments that explain the reasoning underlying a strategy, solution, or conjecture. Arguments might use concrete referents such as objects, drawings, diagrams, and actions. Arguments might also rely on definitions, previously established results, properties, or structures. For example, a student might argue that <math>\frac{1}{5} &gt; \frac{1}{9}</math> on the basis that one of 5 equal parts of a whole is larger than one of 9 equal parts of that whole, because with more equal parts, the size of each part must be smaller. Another example is reasoning that two different shapes have equal area because it has already been demonstrated that they are each half of the same rectangle. Students might also use counterexamples to argue that a conjecture is not true—for example, a rhombus is an example that shows that not all quadrilaterals with four equal sides are squares; or, multiplying by 1 shows that a product of two whole numbers is not always greater than each factor.</p> <p>Mathematically proficient students present their arguments in the form of representations, actions on those representations, explanations in words (oral or written), or a combination of these three. Some of their arguments apply to individual problems, but others are about conjectures based on regularities they have noticed across multiple problems (Math Practice 8). As they articulate and justify generalizations, students consider to which mathematical objects (numbers or shapes, for example) their generalizations apply. For example, primary grade students may believe a generalization about the behavior of addition applies to positive whole numbers less than 100 because those are the numbers with which they are currently familiar. As they expand their understanding of the number system, they may reexamine their conjecture for numbers in the hundreds and thousands. Intermediate grade students return to their conjectures and arguments about whole numbers to determine whether they apply to fractions and decimals. Wisconsin Standards for Mathematics 26 While communicating their own mathematical ideas is important, elementary students also learn to be open to others' mathematical ideas. They appreciate a different perspective or approach to a problem and learn how to respond to those ideas, respecting the reasoning of others (Gutiérrez 2017, 17-18). Together, students make sense of the mathematics, asking helpful questions that clarify or deepen everyone's understanding, and reconsider their own arguments in response to the collaboration.</p>
<p>Math Practice 4: Model with mathematics.</p>	<p>K-5: "In the course of a student's mathematics education, the word 'model' is used in many ways. Several of these, such as manipulatives, demonstration, role modeling, and conceptual models of mathematics, are valuable tools for teaching and learning. However, they are different from the practice of mathematical modeling. Mathematical modeling, both in the workplace and in school, uses mathematics to answer big, messy, reality-based questions (Bliss and Libertini 2016, 7). "</p> <p>Mathematically proficient elementary students formulate their own problems that emerge from natural circumstances as they mathematize the world around them. They can identify the mathematical elements of a situation and generate questions that can be addressed using mathematics (e.g., noticing and wondering). Students dig into the context and make assumptions as they decide "what matters." Mathematically proficient elementary students understand that there are multiple solutions to a modeling problem so they are working to find a solution rather than the solution. Students make judgements about what matters and assess the quality of their solution (Bliss and Libertini 2016, 10). They interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving their mathematical modeling approach if it has not served its purpose. As they decontextualize the situation and represent it mathematically, they are also reasoning abstractly (Math Practice 2).</p> <p>In the elementary grades, students encounter mathematical modeling opportunities each and every day at school and at home. Students might consider how the classroom's set of blocks should be shared throughout recess time. Students might then need to make assumptions about how many blocks each student should have as well as the length of time each student should have the blocks. Once a solution is determined, students could be asked to refine their model by posing the question, "What if one of our friends will not be at recess?" Children might also be presented with a bag of apples and simply asked "Is this enough for our class/family?" or consider the question, "Is the carpet in our classroom big enough for our bodies?"</p> <p>Note: Although physical objects and visuals can be used to model a situation, using these tools absent a contextual situation is not an example of Math Practice 4. For example, solving a word problem using counters or a tape diagram would not be modeling with mathematics, instead this is modeling the mathematics. Math Practice 4 is about engaging in solving authentic real-world problems.</p>
<p>Math Practice 5: Use appropriate tools strategically.</p>	<p>K-5: Mathematically proficient elementary students strategically consider the tools that are available when solving a mathematical problem, whether in a real-world or mathematical context. These tools might include physical objects (e.g., manipulatives, pencil and paper, rulers), conceptual tools (e.g., properties of operations, algorithms), drawings or diagrams (e.g., number lines, tally marks, tape diagrams, arrays, tables, graphs), and available technologies (e.g., calculators, online apps).</p> <p>Mathematically proficient students are sufficiently familiar with tools appropriate for their grade and areas of content to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained from their use as well as their limitations. Students choose tools that are relevant and useful to the problem at hand. For example, when determining how to measure length, students may compare the benefits of using non-standard units of measure (e.g., their own hands, paperclips) versus standard units and tools (e.g., an inch or centimeter ruler). As another example, when presented with 1002-3 or 101-98, students subtract strategically, which may involve reasoning, counting, or decomposing rather than using a written algorithm.</p>
<p>Math Practice 6: Attend to precision.</p>	<p>K-5: Mathematically proficient elementary students use precise language to communicate orally and in written form. They come to appreciate, understand, and use mathematical vocabulary not in isolation, but in the context of doing mathematical thinking and problem solving. They may start by using everyday language to express their mathematical ideas and gradually select words with greater clarity and specificity. For example, they may initially use the word "triangle" to refer only to equilateral triangles resting on their bases, but come to understand and use a more precise definition of a triangle as a closed figure with three straight sides. As another example, they may initially explain a solution by saying, "it works" without explaining what "it" means but later clarify their explanation with specific details.</p> <p>In using mathematical representations, students provide appropriate labels to precisely communicate the meaning of their representations (e.g., charts, graphs, and drawings). When making mathematical arguments about a solution, strategy, or conjecture (Math Practice 3), mathematically proficient students learn to craft careful explanations that communicate their reasoning by referring specifically to each important mathematical element, describing the relationships among them, and connecting their words clearly to their representations.</p> <p>Students use mathematical symbols correctly and can describe the meaning of the symbols they use. For example, they use the equal sign consistently and appropriately. They state the meaning of the symbols they choose in relation to the problem at hand. Wisconsin Standards for Mathematics 28 Students use tools and strategies (e.g., measuring tools, estimation) effectively, to maintain a level of precision that is appropriate to the situation. They specify units of measure where needed. Perseverance and attention to detail are mathematical habits of mind; mathematically proficient students check for reasonableness and accuracy by solving a problem a second way, analyzing errors, and learning from them.</p>

<p>Math Practice 7: Look for and make use of structure.</p>	<p>Mathematically proficient elementary students use structures such as place value, the properties of operations, and attributes of shapes to solve problems. In many cases, they have identified and described these structures through repeated reasoning (Math Practice 8). When students use an algorithm to solve <math>53-17</math> in order to fully understand how to decompose the tens and ones, they must understand that 53 can be seen as 4 tens and 13 ones, not just 5 tens and 3 ones.</p> <p>When younger students recognize that adding 1 results in the next counting number, they are identifying the basic structure of whole numbers. When older students calculate <math>16 \times 9</math>, they might apply the structure of place value and the distributive property to find the product: <math>16 \times 9 = (10 + 6) \times 9 = (10 \times 9) + (6 \times 9)</math>. To determine the volume of a <math>3 \times 4 \times 5</math> rectangular prism, students might see the structure of the prism as five layers of <math>3 \times 4</math> arrays of cubes.</p> <p>Students in elementary grades look for and make use of structure when they view expressions as objects to observe and interpret. For example, students might observe that <math>120 - 41</math> must be one less than <math>120 - 40</math> because "if you subtract one more, the result will be one less" (Math Practice 8). Students can interpret the expression <math>5 \times 3 + 6 \times 3</math> as "five groups of three and six more groups of three" or notice there are a total of 11 groups of 3.</p> <p>A word problem that involves distributing 29 marbles among 4 vases could lead (Math Practice 4) to an equation model <math>(29 - 1) \div 4 = 7</math>, where the expression on the left-hand side not only has the value 7 but also suggests, based on its structure, a process of discarding 1 marble and dividing the rest of the marbles equally into 4 groups of 7.</p>
<p>Math Practice 8: Look for and express regularity in repeated reasoning.</p>	<p>Mathematically proficient elementary students look for and identify regularities as they solve multiple related problems. Students make and test conjectures, reason about and express these regularities as generalizations about structures and relationships, and then use those generalizations to solve problems (Math Practice 7).</p> <p>For example, younger students might notice that when tossing two-color counters to find combinations of a given number, over time students will notice a pattern (commutative property of addition). For example, when tossing six 2-sided counters, they may get 2 red, 4 yellow and 4 red, 2 yellow and when tossing 4 counters, they get 1 red, 3 yellow and 3 red, 1 yellow. Wisconsin Standards for Mathematics 29 In the elementary grades students can recognize and use patterns to help them become flexible with addition. For example, given the number string below, students may recognize they can take one away from the 5 and add it to the first number to make a multiple of ten. They also may notice a pattern related to the first digit increasing by 10, therefore the answer increases by 10.</p> <p>9+5 19+5 29+5 39+5</p> <p>When drawing and representing fractions, students might notice a consistent relationship between the numerator and denominator of fractions that equal one half (e.g., that the numerator is half the denominator and the denominator is two times the numerator). They can generalize from these repeated examples that all fractions equal to one half show this relationship.</p> <p>As students practice articulating their observations both verbally and in writing, they learn to communicate with greater precision (Math Practice 6). As they explain why these generalizations must be true, they construct, critique, and compare arguments (Math Practice 3).</p>