

YEAR AT A GLANCE: *Pre-Calculus* (updated Dec 2022)

	<u>UNIT 1</u>	<u>UNIT 2</u>	<u>UNIT 3</u>	<u>UNIT 4</u>	<u>UNIT 5</u>
Title	Trigonometric Functions	Trigonometric Algebra	Trigonometric Applications	Complex Numbers and Polar Coordinates	Vectors
Unit Length (weeks taught)	4 weeks	4 weeks	3 weeks	3 weeks	3 weeks
Performance Task (e.g., <i>Persuasive Essay, DBQ, Nutritional Analysis, etc.</i>)	Class projects using calculator programs. Unit exam. Daily homework, quizzes, and weekly review assessments. Sinusoidal Modeling Project	Class projects using calculator programs. Unit exam. Daily homework, quizzes, and weekly review assessments. Identities and Equations Partnered Test	Class projects using calculator programs. Unit exam. Daily homework, quizzes, and weekly review assessments. Law of Sines and Cosines Project	Class projects using calculator programs. Unit exam. Daily homework, quizzes, and weekly review assessments. Polar Coordinates Graphing Project Graphing Contest	Class projects using calculator programs. Unit exam. Daily homework, quizzes, and weekly review assessments. Vectors Partnered Test
Enduring Understanding (The big ideas, the “why” we include these ideas)	Students will be able to use trigonometric functions to solve right triangles, find values of trigonometric functions of any angle, and graph trigonometric functions. Students will be able to understand our world is	Students will be able to verify trigonometric identities and solve trigonometric equations.	Students will be able to use the Law of Sines and the Law of Cosines to solve general triangles, find the area of oblique triangles, and solve various trigonometric application problems.	Students will be able to use properties of difference of two squares to find the modulus. Students will be able to relate the modulus visually using vectors. Students will be able to graph complex numbers and identify the magnitude of the complex number, the distance of the complex number from the origin,	Students will recognize that the addition of complex numbers is connected to the addition of vectors. Students will understand that vectors could be used to represent and manipulate data, e.g. to represent payoffs or incidence relationships in a network.

	<p>periodic. The amount of sunlight a city receives on a given day, high and low tides are all real life instances where sinusoids explain and model real life phenomena.</p>			<p>and the direction of the complex number from the origin. Students will be able to express complex numbers in polar coordinate form and in rectangular form.</p>	<p>Students will see that vectors and polar coordinates are useful in solving real-world problems.</p> <p>Students will be able to represent and operate with vectors algebraically in two and three-dimensions, find vector projections, cross products, and dot products of vectors.</p>
<p>Essential Questions (What do we want students to think about)</p>	<p>How is the unit circle used to describe trigonometric functions?</p> <p>How do you graph the basic trigonometric functions on the coordinate plane?</p> <p>How do transformations affect the trigonometric graphs of each function? Such as, how do you determine the period and amplitude of a trigonometric function without looking at the graph of the</p>	<p>What are the relationships between the Pythagorean Identities for Trigonometry?</p> <p>How does the algebraic solution to a trigonometric equation relate to the graphic solution?</p> <p>What is the difference between sine function and the restricted sine function and why is it important when working with the inverse sine function?</p> <p>How is proving or verifying a trigonometric identity different than solving a trigonometric equation?</p>	<p>When is it necessary to use the Law of Sines to solve a triangle?</p> <p>How does trigonometry allow us to calculate distances that can't be measured directly, and to model periodic phenomena?</p>	<p>How is a complex number converted to polar form?</p> <p>How can you graph a complex number in rectangular and polar form?</p> <p>What is the relationship between rectangular and polar form of a complex number?</p> <p>What is the importance of knowing the conjugate of a complex number?</p> <p>Why are functions represented by polar equations?</p>	<p>How is complex number addition connected to vector addition?</p> <p>Why are functions and relations represented by vectors?</p> <p>How is the law of sines learned in Unit 3 connected to the law of sines derived using vectors in 3-space?</p>

	function? What are periodic functions? Why is modeling them so important?				
Common Core Standards	<p>F-TF.A.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.</p> <p>F-TF.A.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.</p> <p>F-TF.A.3 (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$, and $\pi/6$, and use the unit circle to express</p>	<p>F-TF.C.9^[1] (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.</p> <hr/> <p>[1]Students are now responsible for proofs of angle addition and subtraction formulas.</p>	<p>G-SRT.D.9 (+) Derive the formula for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.</p> <p>G-SRT.D.10 (+) Prove the Laws of Sines and Cosines and use them to solve problems.</p> <p>G-SRT.D.11 (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).</p> <p>F-TF.B.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.*</p>	<p>N-CN.A.3 (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.</p> <p>N-CN.B.4 (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.</p> <p>N-CN.B.5 (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation.</p> <p>N-CN.B.6 (+) Calculate the distance between numbers in the complex plane as the modulus of</p>	<p>N-VM.A.1 (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes</p> <p>N-VM.A.2 (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.</p> <p>N-VM.A.3 (+) Solve problems involving velocity and other quantities that can be represented by vectors.</p> <p>N-VM.B.4 (+) Add and subtract vectors</p> <p>a. Add vectors end-to-end, component-wise, and by the parallelogram rule.</p>

the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x , where x is any real number.

Prove and apply trigonometric identities.

F-TF.C.8

Prove the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$ and use it to find $\sin\theta$, $\cos\theta$, or $\tan\theta$ given $\sin\theta$, $\cos\theta$, or $\tan\theta$ and the quadrant of the angle.

F-TF.B.6 (+)

Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

F-TF.B.7 (+) Use inverse functions to solve trigonometric equations that

the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

N-CN.A.1 Know there is a complex number such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.

N-CN.A.2 Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

N-CN.C.7 Solve quadratic equations with real coefficients that have complex solutions.

N-CN.C.8 (+) Extend polynomial identities to the complex numbers.

Understand that the magnitude of the sum of two vectors is typically not the sum of the two magnitudes.

b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.

c. Understand vector subtraction $\mathbf{v} - \mathbf{w}$ as $\mathbf{v} + (-\mathbf{w})$ where $-\mathbf{w}$ is the additive inverse of \mathbf{w} with the same magnitude of \mathbf{w} and in the opposite direction.

Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.

N-VM.B.5 (+) Multiply a vector by a scalar.

arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.*

G-SRT.C.6
Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

G-SRT.C.7
Explain and use the relationship between the sine and cosine of complementary angles.

G-SRT.C.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.*

	<u>UNIT 6</u>	<u>UNIT 7</u>	<u>Unit 8</u>	<u>UNIT 9</u>	<u>UNIT 10</u>	<u>UNIT 11</u>
Title	Matrices	Linear Programming	Conic Sections	Polynomial/Rational Functions including Transformations of Functions	Intro to Calculus	Probability
Unit Length <i>(weeks taught)</i>	3 weeks	2 weeks	1 week	6 weeks	4 weeks	2 weeks
Performance Task <i>(e.g., Persuasive Essay, DBQ, Nutritional Analysis, etc.)</i>	Class projects using calculator programs. Unit exam. Daily homework, quizzes, and weekly review assessments. Encoding/Decoding Matrix Project	Class projects using calculator programs. Unit exam. Daily homework, quizzes, and weekly review assessments. Linear Programming Project DESMOS project Linear Programming Partner Test	Class projects using DESMOS to explore the graphs of the various conics and how the equations of the conics impact those graphs.	Class projects using calculator programs. Unit exam. Daily homework, quizzes, and weekly review assessments. 12 Basic Functions activity packet. DESMOS activity exploring the progression of a function as transformations are executed. Polynomial and Rational Functions Partnered Test	Class projects using calculator programs. Unit exam. Daily homework, quizzes, and weekly review assessments.	Class projects using calculator programs. Unit exam. Daily homework, quizzes, and weekly review assessments.
Enduring Understanding (The big ideas, the “why” we include these ideas)	Students will be able to: -find the inverse of a matrix -determine how data can be represented as a matrix	Students will be able to: -connect their understanding of solving systems of inequalities to real-world situations -use problem-solving	Students will be able to: -connect their understanding of graphs of functions and relations to the graphs of the conics -determine the foci,	Students will be able to: -apply transformations to the 12 basic functions -define and divide polynomials.	Students will be able to: - expand a power of a binomials using the Binomial Theorem. - find the coefficient of a given term of a binomial expansion	Students will be able to: -calculate permutations and combinations -use Bernoulli’s theorem to find the probability of an

	<ul style="list-style-type: none"> - determine if the inverse of a matrix exists. -use matrices to solve real-world problems involving system of linear equations - transform linear equations into a single matrix 	<p>and communication skills to maximize/minimize an objective function</p>	<p>directrix, and eccentricity of conics</p>	<ul style="list-style-type: none"> -apply the Remainder and Factor Theorems and make connections between remainders and factors. - determine the maximum number of zeros of a polynomial. -find all rational zeros of a polynomial function. -factor a polynomial completely. -recognize and describe the graphs of various polynomial functions. -identify the properties of general polynomial find the domain of a rational function. -find intercepts, asymptotes, and holes. -describe the end behavior of a function. -write and perform 	<ul style="list-style-type: none"> -find the derivative of a polynomial function using limits and the difference quotient -find the derivative of a polynomial function using derivative rules 	<p>event</p>
--	--	--	--	---	--	--------------

				<p>arithmetic operations on complex numbers.</p> <p>-find the number of zeros of a polynomial.</p> <p>-give the complete factorization of polynomial expressions. functions</p>		
<p>Essential Questions (What do we want students to think about)</p>	<p>What is the procedure that is used to verify two matrices are inverses of each other?</p> <p>How can you use a formula to find the inverses of 2x2 matrices?</p> <p>How can you use inverse matrices to solve systems of linear equations?</p>	<p>How can you use systems of inequalities in two variables to model and solve real-life problems?</p> <p>How can you use linear programming to model and solve real-life problems?</p>	<p>How do the graphs of conics relate to real world phenomena?</p>	<p>What is the procedure that is used to find real zeros of a polynomial function?</p>	<p>How can you use the Binomial Theorem to expand binomials?</p> <p>How does the derivative of a function relate to its graph?</p>	<p>What is important about permutation, combination, tree diagrams and other methods of counting?</p>
<p>Common Core Standards</p>	<p>N-VM.C.8 (+) Add, subtract, and multiply matrices of appropriate dimensions.</p> <p>N-VM.C.10^[1] (+) Understand</p>	<p>A-CED.A.1 Create equations and inequalities in one variable and use them to solve problems. <i>Include equations arising from linear and quadratic functions, and simple rational</i></p>	<p>G-GPE.A.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle</p>	<p>A-SSE.B.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.*</p>	<p>** Beyond the scope of the current Common core Standards.</p>	<p>S-CP.B.8 (+) Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A) \times P(B A) = P(B) \times P(A B)$, and interpret the answer in terms of the model.</p>

	<p>that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of and in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.</p> <p>N-VM.C.11 (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.</p> <p>N-VM.C.12 (+) Work with matrices as transformations of the plane, and interpret the absolute value of the determinant in</p>	<p><i>and exponential functions.</i></p> <p>A-CED.A.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.</p> <p>A-CED.A.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. <i>For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.</i></p>	<p>given by an equation.</p> <p>G-GPE.A.2 Derive the equation of a parabola given a focus and directrix.</p> <p>G-GPE.A.3 (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.</p>	<p>a. Factor a quadratic expression to reveal the zeros of the function it defines.</p> <p>b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.</p> <p>N-CN.C.9 (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.</p> <p>A-APR.C.5 (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.[1]</p> <p>A-APR.D.7 (+) Understand that rational expressions form a system analogous to the rational numbers,</p>		<p>S-CP.B.9 (+) Use permutations and combinations to compute probabilities of compound events and solve problems.</p> <p>S-MD.B.6 (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).</p> <p>S-MD.B.7 (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).</p> <p>S-CP.A.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).</p> <p>S-CP.A.2</p>
--	--	--	---	--	--	--

	<p>terms of area.</p> <p>A-REI.B.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.</p> <p>A-REI.C.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.</p> <p>N-VM.C.6 (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.</p> <p>N-VM.C.7 (+) Multiply matrices by scalars to produce new</p>			<p>closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.</p> <p>F-IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*</p> <p>d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.</p> <hr/> <p>[1]The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.</p>		<p>Understand that two events and are independent if the probability of and occurring together is the product of their probabilities, and use this characterization to determine if they are independent.</p> <p>S-CPA.3 Understand the conditional probability of A given B as $P(A \text{ and } B)/P(B)$, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.</p> <p>S-CPA.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the</p>
--	--	--	--	--	--	---

	<p>matrices, e.g., as when all of the payoffs in a game are doubled.</p> <p>N-VM.C.8 (+) Add, subtract, and multiply matrices of appropriate dimensions.</p> <p>N-VM.C.9 (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.</p> <p>N-VM.C.10 (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of and in the real numbers. The determinant of a</p>					<p>two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. <i>For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.</i></p> <p>S-CPA.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. <i>For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if</i></p>
--	---	--	--	--	--	---

square matrix is nonzero if and only if the matrix has a multiplicative inverse.

N-VM.C.11 (+)
Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.

A-REI.C.8 (+)
Represent a system of linear equations as a single matrix equation in a vector variable.

A-REI.C.9 (+)
Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension or greater).

you have lung cancer.

S-CP.B.6 Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.

S-CP.B.7 Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.

[1]N.VM and G.CO standards are included in the context of defining transformations of the plane rigorously using complex numbers and matrices and linking rotations and reflections to multiplication by complex number and/or by matrices to show how geometry software and video games work.