Mathematical Proficiency

Our analyses of the mathematics to be learned, our reading of the research in cognitive psychology and mathematics education, our experience as learners and teachers of mathematics, and our judgment as to the mathematical knowledge, understanding, and skill people need today have led us to adopt a composite, comprehensive view of successful mathematics learning. Recognizing that no term captures completely all aspects of expertise, competence, knowledge, and facility in mathematics we have chosen mathematical proficiency to capture what we think it means for anyone to learn mathematics successfully. Mathematical proficiency has five strands:

- **Conceptual Understanding:** Comprehension of mathematics concepts, operations, and relationships. For example, what division means, why an algorithm works. The students must build the relationships and connections themselves.

- **Procedural Fluency:** Skill in carrying out procedures flexibly, accurately, efficiently, and appropriately.

- **Strategic Competency:** Non-routine problem solving – you are not sure what you are going to have to do… the ability to formulate, represent and solve problems.

- **Adaptive Reasoning:** Capacity for logical thought, reflection, explanations, and justification. This is difficult to engender to our students. It’s what makes tinkers, conjecturers, wonderers, pattern finders – people who can live with getting stuck, individuals who can articulate what they are doing even in times of disequilibrium.

- **Productive Disposition:** Habitual inclination to see math as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s one efficacy.

The most important observation we make about these five strands is that they are interwoven and interdependent. This observation has implications for how students acquire mathematical proficiency, how teachers develop that proficiency in their students, and how teachers are educated to achieve that goal.