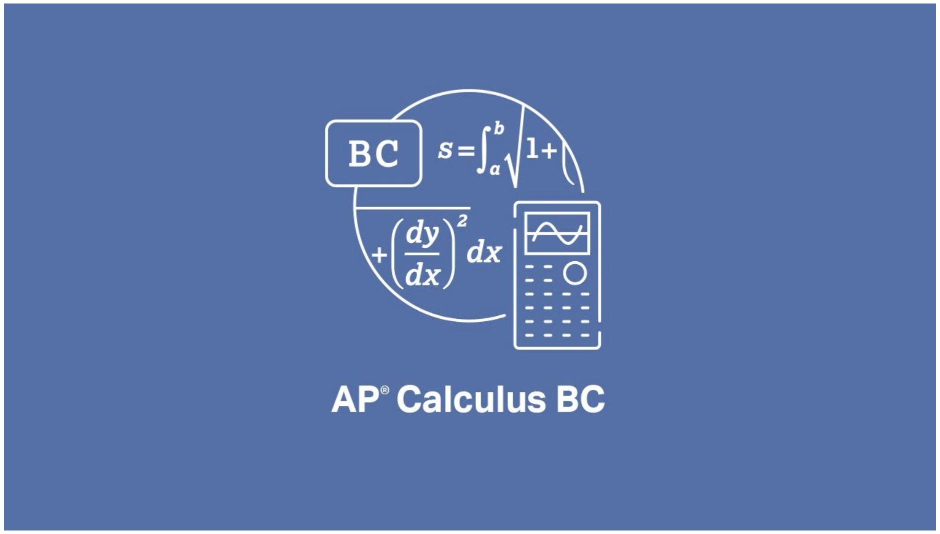
AP CALCULUS BC CURRICULUM



Grade Level(s): 12	**Curriculum Author(s):** Raymond Robillard

Course Description: Students who wish to extend the challenge of AP Calculus AB may enroll in AP Calculus BC. AP Calculus BC encompasses all the material included in AP Calculus AB, but students will also explore advanced integration techniques, improper integrals, sequences and series, tests for convergence, and parametric and polar relationships. Students choosing this option will take the AP Calculus BC exam which assesses knowledge of topics covered on the AP Calculus AB exam as well as those listed above. Students can earn college credit for 2 semesters of Calculus based on their score on the AP exam. Because the additional content will require extra time on the part of the student, enrollment in AP Calculus BC will require the same course seat time as AP Calculus AB with an additional requirement on the part of students to put in time (120+ minutes per week for all of semester 2) in order to gain access to the additional content. Depending on enrollment, this may occur within the school day or after school.

Year At A Glance			
Unit Title	**Overarching Essential Question**	**Overarching Enduring Understanding**	**Vision of A Learner "I Can" Statements**
Advanced Techniques for Integration	How could we reverse differentiation techniques such as the chain rule, the product rule, and the quotient rule?	Recognizing opportunities to apply knowledge of geometry and mathematical rules can simplify integration.	TCC1(9-12); CCE3(9-12); TI3(9-12)
Differential Equations and Arc Lengths	What real world phenomena can we measure using logistic growth models?	Solving differential equations allows us to determine functions and develop models.	TCC4(9-12); CCE3(9-12); TI3(9-12)
Sequences and Series	What can using limits allow us to do?	• The use of limits allows us to show that the areas of unbounded regions may be finite. • Applying limits may allow us to determine the finite sum of infinitely many terms.	TCC1(9-12); TCC3(9-12); CCE3(9-12); TI3(9-12)
Parametric, Vector, and Polar Functions	What are the real-world applications of derivatives, definite integrals, and initial value problems?	• Derivatives allow us to solve real-world problems involving rates of change. • Definite integrals allow us to solve problems involving the accumulation of change in length over an interval. • Solving an initial value problem allows us to determine an expression for the position of a particle moving in the plane. • Recognizing opportunities to apply derivative rules can simplify differentiation.	CCE3(9-12); TI3(9-12); AA3(9-12)

Unit 1 - Advanced Techniques for Integration

Desired Results - Goals, Transfer, Meaning, Acquisition

Established Goals:
Standards for Mathematical Practice:
1.E) Apply appropriate mathematical rules or procedures, with and without technology.
1.C) Identify an appropriate mathematical rule or procedure based on the classification of a given expression(e.g., Use the chain rule to find the derivative of a composite function.)
3.D) Apply an appropriate mathematical definition, theorem, or test.

Vision of A Learner Attributes:

- TCC1(9-12): I can ask purposeful, insightful questions to find a variety of innovative solutions.
- CCE3(9-12): I can show initiative in prompting group discourse and fostering collaboration among others, providing actionable feedback, and working with others to solve problems and/or design products.
- TI3(9-12): I can formulate and investigate probing questions to further my learning.

Understandings: Students will understand that...

- Recognizing opportunities to apply knowledge of geometry and mathematical rules can simplify integration.
- L'Hospital's Rule allows us to determine the limits of some indeterminate forms.

Essential Questions:

- How could we reverse differentiation techniques such as the chain rule, the product rule, and the quotient rule?
- What structures of integrands allow us to utilize u-substitution, integration by parts, and partial fraction decomposition?
- How could two functions approach infinity or zero at different rates?

Students will know...

- Substitution of variables is a technique for finding antiderivatives.
- For a definite integral, substitution of variables requires corresponding changes to the limits of integration.
- Techniques for finding antiderivatives include rearrangements into equivalent forms, such as long division and completing the square.
- Integration by parts is a technique for finding antiderivatives.
- Some rational functions can be decomposed into sums of ratios of linear, non repeating factors to which basic integration techniques can be applied.

Students will be able to...

- Determine indefinite integrals and evaluate definite integrals using u-substitution.
- Determine indefinite integrals and evaluate definite integrals using integration by parts.
- Determine indefinite integrals and evaluate definite integrals using partial fraction decomposition.
- Determine limits of functions that result in indeterminate forms.

● When the ratio of two functions tends to 0/0 or ∞/∞ in the limit, such forms are said to be indeterminate. ● Limits of the indeterminate forms 0/0 or ∞/∞ may be evaluated using L'Hospital's Rule.	

Key Vocabulary: Definite Integral, Indefinite Integral, U-substitution, Chain Rule, Integration by Parts, Product Rule, Partial Fractions, Partial Fraction Decomposition, Quotient Rule, L'Hospital's Rule, Indeterminate Form

Assessment Evidence

Performance Tasks:	Other Evidence:
● Unit 1 Test-This assessment covers all topics from this unit. Test breakdown is: 70% Integration Techniques(both with definite and indefinite integrals) 30% L'Hospital's Rule	Book Problems: ● U-Substitution (7.2) P. 343, #17,20,21,22,32,39,56,60 ● Integration by Parts (7.3) P. 354 #1,4,5,8,15,17,25,27 ● Partial Fraction Integration (7.5) P. 375 #1,5,11,17 ● L'Hospital's Rule (9.2) P. 458 #17,23,33,39,49,51,65 Previously Released AP FRQ Questions: ● AP 2016 #3, 2015 #5, 2015 #4 Select Problems from AP Classroom Personal Progress Checks from: ● Unit 6 ● Unit 4. *AP Units are structured differently, but all content is the same

Learning Plan

This unit deepens the students' knowledge of integration by detailing techniques that allow for a wider range of functions to be anti-differentiated. Students begin by exploring methods that reverse the processes utilized to differentiation known as the chain rule, the product rule, and the quotient rule. Additional emphasis is placed on recognizing the form of a complex function and viewing it as the sum, product, or composition of other functions.

Students often struggle with the relationship between differentiation and integration. They think that integration is simply differentiation in reverse order. However, to apply the rules of integration correctly, students must think more strategically, taking into consideration how the "pieces" fit together. Students will need explicit guidance for choosing an appropriate antidifferentiation strategy that's based on the underlying patterns in different categories of integrands (e.g., using u-substitution when they recognize that the integrand is a factor of the derivative of a composite function or using integration by parts for an integrand, udv, that is related to a term in the derivative of the product uv).

Students should be careful applying the chain rule, both when differentiating functions defined by integrals and when integrating using u-substitution. Students will need to recognize integrands that are factors of a chain rule derivative and should practice u-substitution until the process is internalized. Students will additionally need to recognize integrands that suggest strategies such as integration by parts or partial fractions and should use mixed practice in preparation for the exam. When using a calculator to evaluate a definite integral in a free-response question, students should present the expression for the definite integral, including endpoints of integration, and an appropriately placed differential.

When evaluating an integral without a calculator, students should present an appropriate antiderivative; they should include a constant of integration with indefinite integrals. As always, students should be careful about parentheses usage and should avoid writing strings of equal signs
equating expressions that are not equal.

Students' understanding of units of measure often reinforces their understanding of contextual applications of differentiation. In problems involving related rates, identifying the independent variable common to related functions may help students to correctly apply the chain rule. When applying differentiation to determine limits of certain indeterminate forms using L'Hospital's rule, students must show that the rule applies.

When utilizing L'Hospital's Rule, emphasize that students must verify that both the numerator and denominator go to 0, or that both approach infinity, as a necessary first step before applying L'Hospital's Rule to determine the limit of an indeterminate form. Students should understand that 0/0 or inf/inf are appropriate labels for indeterminate forms but do not represent values in an equation. Also emphasize that the conclusion of L'Hospital's rule features the ratio of the derivatives of the numerator and denominator, respectively, rather than the derivative of the ratio.

Students will demonstrate the VOL Attributes in this unit through the following common formative assessments:

WHICH TECHNIQUE IS BEST? - In this unit we learn three techniques of integration, but in what situations do you apply which and will there be cases where multiple methods will work? Students will be presented with various integrals and discuss in small groups what aspect of an integrand prompts us to try a specific technique. For example, given $\int_0^1 x^2\sqrt{4x^3 - 2}\,dx$, students may say "I would use u-substitution because a factor of the integrand is the derivative, give or take a constant multiple, or another part of the integrand." Yet another students might say, This is a product of two functions so I want to try using Integration by Parts". Written evidence of this skill is shown through the completion of homework assignments listed above in "Other Evidence." This activity relates to TCC1(9-12): I can ask purposeful, insightful questions to find a variety of innovative solutions.

WHO WINS THE "TUG OF WAR"? - L'hospital's rule can be considered the "Tug of War" of Calculus in that two changing quantities are competing to determine the end behavior of a function. Whole group discussion to introduce this concept will include classic examples that produce indeterminate forms of 0/0 or ∞/∞, building up to more complex examples like 1^{∞}. Students will respond to such prompts as "if the numerator getting infinitely close to infinity is pushing the value of the fraction to infinity, and the denominator getting infinitely large makes the value of the fraction approach 0, which one wins?" Give appropriate wait time for the students' curiosity to take over and come to such questions/conclusions as related to the rates at which they approach 0 or infinity, and the connection of these rates to the derivative to help them discover the premise of L'Hospital's Rule. As part of the discussion, students will define L'Hospital's Rule, first informally, then using appropriate language and symbols. Written evidence of this skill is shown through the completion of homework assignments listed above in "Other Evidence." This activity relates to TI3(9-12): I can formulate and investigate probing questions to further my learning.

SELF DIRECTED REVIEW SESSION - Students take initiative as they determine the use of review time to prepare for assessments. Students are provided with a topic list and a set of review problems, however, they are given the freedom to do as much or as little of it as needed, go in whatever order makes most sense to them, request additional problems from me as needed on specific topics, and can move around the room freely utilizing any board space, manipulatives, classroom resources, etc. They are encouraged to work in groups seeking out partners that want to work on the same skill. I wander the class and coach students only when requested and only if they have first sought guidance/feedback from their peers. This activity relates to CCE3(9-12): I can show initiative in prompting group discourse and fostering collaboration among others, providing actionable feedback, and working with others to solve problems and/or design products.

Teacher Resources:
AP Classroom
Topic Outline
Folder of All Released MC and FRQ Questions

Unit 2 - Differential Equations and Arc Lengths	
Desired Results - Goals, Transfer, Meaning, Acquisition	
Established Goals: Standards for Mathematical Practice: 2.C) Identify a re-expression of mathematical information presented in a given representation. 3.G) Confirm that solutions are accurate and appropriate. 4.D) Use appropriate graphing techniques. 1.E) Apply appropriate mathematical rules or procedures, with and without technology. 3.F) Explain the meaning of mathematical solutions in context. 3.D) Apply an appropriate mathematical definition, theorem, or test.	
Vision of A Learner Attributes: ● TCC4(9-12): I can integrate my learning to adapt to experiences in the classroom, career and life. ● CCE3(9-12): I can show initiative in prompting group discourse and fostering collaboration among others, providing actionable feedback, and working with others to solve problems and/or design products. ● TI3(9-12): I can formulate and investigate probing questions to further my learning.	
Understandings: Students will understand that... ● Solving differential equations allows us to determine functions and develop models. ● Definite integrals allow us to solve problems involving the accumulation of change in length over an interval.	**Essential Questions:** ● How can we visually represent a derivative of a function without knowing the function itself? ● How can we calculate a function value without knowing the function itself? ● What real world phenomena can we measure using logistic growth models? ● How can we measure the length of a curve?
Students will know... ● Differential equations relate a function of an independent variable and the function's derivatives. ● Derivatives can be used to verify that a function is a solution to a given differential equation. ● There may be infinitely many general solutions to a differential equation.	**Students will be able to...** ● Interpret verbal statements of problems as differential equations involving a derivative expression. ● Verify solutions to differential equations. ● Estimate solutions to differential equations. ● Determine general solutions to differential equations. ● Determine particular solutions to differential equations.

● A slope field is a graphical representation of a differential equation on a finite set of points in the plane. ● Slope fields provide information about the behavior of solutions to first-order differential equations. ● Solutions to differential equations are functions or families of functions. ● Euler's method provides a procedure for approximating a solution to a differential equation or a point on a solution curve. ● Some differential equations can be solved by separation of variables. ● Antidifferentiation can be used to find general solutions to differential equations. ● A general solution may describe infinitely many solutions to a differential equation. There is only one particular solution passing through a given point. ● Solutions to differential equations may be subject to domain restrictions. ● Specific applications of finding general and particular solutions to differential equations include motion along a line and exponential growth and decay. ● The model for exponential growth and decay that arises from the statement "The rate of change of a quantity is proportional to the size of the quantity" is dy/dt = .ky with initial condition y = y0 when t = 0 and has solutions of the form y=y0*e^(kt). ● The model for logistic growth that arises from the statement "The rate of change of a quantity is jointly proportional to the size of the quantity and the difference between the quantity and the carrying capacity" is dy/dt = ky(a − y). ● The logistic differential equation and initial conditions can be interpreted without solving the differential equation. ● The limiting value (carrying capacity) of a logistic differential equation as the independent variable approaches infinity can be determined using the logistic growth model and initial conditions. ● The value of the dependent variable in a logistic differential equation at the point when it is changing fastest can be	● Interpret the meaning of a differential equation and its variables in context. ● Interpret the meaning of the logistic growth model in context. ● Determine the length of a curve in the plane defined by a function, using a definite integral.

determined using the logistic growth model and initial conditions. ● The length of a planar curve defined by a function can be calculated using a definite integral.	

Key Vocabulary: Slope Field, Differential Equation, General Solution, Particular Solution, Initial Condition, Initial Value Problem, Euler's Method, Separable Equation, Newton's Law of Heating and Cooling, Half-life, Logistic Growth, Logistic Curve, Carrying Capacity, Logistic Differential Equation, Arc Length, Smooth Curve,

Assessment Evidence

Performance Tasks: ● Unit 2 Test-This assessment covers all topics from this unit. Test breakdown is: 20% Slope FIelds 20% Euler's Method 40% Initial Value Logistic Models 20% Arc Length	**Other Evidence:** Book Problems: ● Slope Fields and Euler's Method (7.1) P.335 #29-40,57 ● Logistic Curves (7.5) P.375 #33,34 ● Lengths of Curves (8.4) P..424 #1,4,9,11,15 Previously Released AP MC Questions: ● BC MC 2003(Various Questions) Previously Released AP FRQ Questions: ● AP 2014 #3, 2016 #4, 2014 #4 Select Problems from AP Classroom Personal Progress Checks from: ● Unit 7 ● Unit 8. *AP Units are structure differently, but all content is the same

Learning Plan

In this unit, students will set up and solve separable differential equations. Slope fields can be used to represent solution curves to a differential equation and build understanding that there are infinitely many general solutions to a differential equation, varying only by a constant of integration. Students can locate a unique solution relevant to a particular situation, provided they can locate a point on the solution curve. By writing and solving differential equations leading to models for exponential growth and decay and logistic growth, students build understanding of topics introduced in Algebra II and other courses. Furthermore, understanding that the length problems studied in this unit are limiting cases of Riemann sums of segment lengths saves students from memorizing a long list of seemingly unrelated formulas and develops meaningful understanding of integration.

In this unit, students will translate mathematical information from one representation to another by matching equations and slope fields, rewriting verbal statements as differential equations, and sketching slope fields that match their symbolic representations. Provide students with explicit guidance on how to select an appropriate graphing technique. As students practice Euler's method, encourage them to transfer skills using tangent line approximations, rather than simply memorizing an algorithm. This unit also involves geometric applications of integration to find arc length. Relating graphical representations to symbolic representations, such as Riemann sums and definite integrals, develops these skills and helps students to master the content.

Because the problems in this unit model real world scenarios, help students to develop proficiency in transferring the mathematical procedures they've learned in "x's and y's" to equivalent environments with variable names other than x, y, and t. Using differentiation to confirm that solutions to differential equations are accurate and appropriate also helps students to develop an understanding of what it means to say that an equation is a solution to a differential equation.

Students should practice setting up and solving contextual questions involving separable differential equations until the solution strategy becomes routine: separate variables, antidifferentiate both sides of the equation and add a constant of integration, use initial conditions to determine the constant of integration, and rearrange the resulting expression to complete the solution. Failure to separate variables or omitting the constant of integration severely limits the number of points a student can earn on the AP Exam. A common error in antidifferentiation is to assume that all differential equations involving fractions have logarithmic solutions, presumably because some do.

Students should learn to recognize the forms of differential equations resulting in exponential and logistic models. These may be used or interpreted without performing the derivation. Students should also be reminded that differential equations give us information about the derivative and may be used directly to find information about a slope or rate of change.

Students will demonstrate the VOL Attributes in this unit through the following common formative assessments:

ROLLER COASTER ACTIVITY - How fast are you actually going on a roller coaster? In this activity students calculate the average speed of a roller coaster. The activity begins by asking students to consider that average speed can be calculated as total distance traveled divided by total time elapsed. The activity provides the amount of time from start to finish, however, it requires students to calculate the length of the track to find the total distance traveled. This is done by sketching the graph of the track as a polynomial by overlaying the picture of the track against a coordinate grid with a scale, taking data points, using our calculator to get a polynomial regression, then using the resulting function with this units method for finding arc length. In all, this activity provides a method for calculating speed using many mathematical concepts with a heavy emphasis on the Mathematical Practice of Modeling. This activity relates to TCC4(9-12): I can integrate my learning to adapt to experiences in the classroom, career and life.

LOGISTIC GROWTH MODEL FOLLOW UP - Logistic Growth Models are often one of the most intimidating concepts in this course, but it is also one that, once students begin to make connections between the model, graph, and the real world phenomena that it measures, leaves students with as many, if not more questions, than they started with. This is because it connects so many mathematical concepts from asymptotes, initial values, rate of change, end behavior, and thus it creates so many "aha moments". This activity is the followup to the lessons on this concept. It asks students to make next steps by considering what real world scenarios can be represented with a Logistic Growth Model, and for those that don't perfectly fit the model, what adjustments need to be made in order to create an appropriate model. Some examples that students have applied their learning to in this activity have included Covid-19 spread, changes of value of collectibles over time, and the temperature change of a beverage, warming to room temperature. This activity relates to TI3(9-12): I can formulate and investigate probing questions to further my learning.

SELF DIRECTED REVIEW SESSION - Students take initiative as they determine the use of review time to prepare for assessments. Students are provided with a topic list and a set of review problems, however, they are given the freedom to do as much or as little of it as needed, go in whatever order makes most sense to them, request additional problems from me as needed on specific topics, and can move around the room freely utilizing any board space, manipulatives, classroom resources, etc. They are encouraged to work in groups seeking out partners that want to work on the same skill. I wander the class and coach students only when requested and only if they have first sought guidance/feedback from their peers. This activity relates to CCE3(9-12): I can show initiative in prompting group discourse and fostering collaboration among others, providing actionable feedback, and working with others to solve problems and/or design products.

Teacher Resources:
AP Classroom
Topic Outline
Folder of All Released MC and FRQ Questions

Unit 3 - Sequences and Series

Desired Results - Goals, Transfer, Meaning, Acquisition

Established Goals:
Standards for Mathematical Practice:
3.D) Apply an appropriate mathematical definition, theorem, or test.
1.E) Apply appropriate mathematical rules or procedures, with and without technology.
2.C) Identify a re-expression of mathematical information presented in a given representation.

Vision of A Learner Attributes:

- TCC1(9-12): I can ask purposeful, insightful questions to find a variety of innovative solutions.
- TCC3(9-12): I can integrate relevant information to produce multiple valid solutions.
- CCE3(9-12): I can show initiative in prompting group discourse and fostering collaboration among others, providing actionable feedback, and working with others to solve problems and/or design products.
- TI3(9-12): I can formulate and investigate probing questions to further my learning.

Understandings: Students will understand that... • The use of limits allows us to show that the areas of unbounded regions may be finite. • Applying limits may allow us to determine the finite sum of infinitely many terms. • Power series allow us to represent associated functions on an appropriate interval.	**Essential Questions:** • How do we calculate the area of an infinitely extended area? • In what ways can we determine the divergence or convergence of an infinite series? • How can we represent all functions as infinite polynomials? • How precisely can we estimate function values with Taylor polynomials, and what determines this level of precision?
Students will know… • An improper integral is an integral that has one or both limits infinite or has an integrand that is unbounded in the interval of integration. • Improper integrals can be determined using limits of definite integrals. • The use of limits allows us to show that the areas of unbounded	**Students will be able to...** • Evaluate an improper integral or determine that the integral diverges. • Determine whether a series converges or diverges. • Approximate the sum of a series. • Represent a function at a point as a Taylor polynomial. • Approximate function values using a Taylor polynomial.

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regions may be finite.

The nth partial sum is defined as the sum of the first n terms of a series.
A geometric series is a series with a constant ratio between successive terms.
The nth term test, integral test, comparison test, limit comparison test, alternating series test. ratio test, and root test are all methods to determine whether a series converges or diverges.
In addition to geometric series, common series of numbers include the harmonic series, the alternating harmonic series, and p-series.
A series may be absolutely convergent, conditionally convergent, or divergent.
If a series converges absolutely, then it converges.
If a series converges absolutely, then any series obtained from it by regrouping or rearranging the terms has the same value.
If an alternating series converges by the alternating series test, then the alternating series error bound can be used to bound how far a partial sum is from the value of the infinite series.
The coefficient of the nth degree term in a Taylor polynomial for a function f centered at x = a is f(n) (a) / n!
In many cases, as the degree of a Taylor polynomial increases, the nth degree polynomial will approach the original function over some interval.
Taylor polynomials for a function f centered at x = a can be used to approximate function values of f near x = a.
The Lagrange error bound can be used to determine a maximum interval for the error of a Taylor polynomial approximation to a function.
In some situations, the alternating series error bound can be used to bound the error of a Taylor polynomial approximation to the value of a function.
If a power series converges, it either converges at a single point or has an interval of convergence.
The ratio test can be used to determine the radius of convergence of a power series.
The radius of convergence of a power series can be used to

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Determine the error bound associated with a Taylor polynomial approximation.
Determine the radius of convergence and interval of convergence for a power series.
Represent a function as a Taylor series or a Maclaurin Series
Interpret Taylor series and Maclaurin series.
Represent a given function as a power series.

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identify an open interval on which the series converges, but it is necessary to test both endpoints of the interval to determine the interval of convergence. • If a power series has a positive radius of convergence, then the power series is the Taylor series of the function to which it converges over the open interval. • The radius of convergence of a power series obtained by term-by-term differentiation or term-by-term integration is the same as the radius of convergence of the original power series. • A Taylor polynomial for f (x) is a partial sum of the Taylor series for f (x). • The Maclaurin series for 1/(1-x) is a geometric series. • The Maclaurin series for sin x, cos x, and e^x provides the foundation for constructing the Maclaurin series for other functions. • Using a known series, a power series for a given function can be derived using operations such as term-by-term differentiation or term-by-term integration, and by various methods (e.g., algebraic processes, substitutions, or using properties of geometric series).	

Key Vocabulary: Sequence, Term, Nth Term, Finite, Infinite, Explicitly-defines, Recursively-defined, Arithmetic, Geometric, Common Difference, Common Ratio, Limit, Converge, Diverge, Squeeze Theorem, Improper Integral, Comparison Test, P-Integral, Series, Partial Sums, Interval of Convergence, Radius of Convergence, Power Series, Center, Taylor Polynomial, Taylor Series, Maclaurin Polynomial, Maclaurin Series, Taylor's Theorem, Taylor's Formula, Alternating Series Error Bound, Truncation Error, Lagrange Error Bound, Remainder Bounding Theorem, Nth Term Test, Integral Test, Comparison Test, Limit Comparison Test, Alternating Series Test. Ratio Test, Root Test, Absolute Convergence, Conditional Convergence, P-Series, Harmonic Series,

Assessment Evidence

Performance Tasks:	Other Evidence:
• Unit 3 Test-This assessment covers all topics from this unit. Test breakdown is: 20% Improper Integrals 30% Taylor and Maclaurin Series 30% Intervals of Convergence 20% Tests for Convergence	Book Problems: • Sequences (9.1) P. 449 #1,4,7,10,13,16,25,29,31,34,37,41 • Improper Integrals (9.4) P. 475 #1,3,12,20,26,31,41 • Infinite Series (10.1) P. 489 #1,2,7,10-12,15,19,20 • Infinite Series (Continued) P. 489 #21,24,27,31,54,59 • Taylor Series (10.2) P. 500 #1,4-6,11,14,21

	• Taylor's Theorem(10.3) P. 509 #13,15,18,19,21,23 • Radius of Convergence (10.4) P. 521 #3,5,7,10,17,19,23,26 • Testing Convergence at Endpoints (10.5) P. 534 #1,5,9,12,18,19,25,39,42 Previously Released AP MC Questions: • BC MC 2008(Various Questions) Previously Released AP FRQ Questions: • AP 2017 #5, 2017 #4, 2017 #1, 2017 #3, 2018 #3, 2018 #4, 2015 #1, 2013 #6, 2018 #1, 2018 #6, 2015 #6, 2014 #6, 2017 #6, 2016 #6, 2015 #3, 2016 #1 Select Problems from AP Classroom Personal Progress Checks from: • Unit 10 • Unit 6 *AP Units are structure differently, but all content is the same

Learning Plan

In this unit, students need to understand that a sum of infinitely many terms may converge to a finite value. They can develop intuition by exploring graphs, tables, and symbolic expressions for series that converge and diverge and for Taylor polynomials. Students should build connections to past learning, such as how evaluating improper integrals relates to the integral test or how using limiting cases of power series to represent continuous functions relates to differentiation and integration. Students who rely solely on memorizing a long list of tests and procedures typically find little success achieving a lasting conceptual understanding of series.

In this unit, students will need to develop proficiency with complex series notation and the ability to communicate their reasoning. Emphasize appropriate use of notation, precision of language, and establishing conditions for using a particular test. Remind students that a sound justification relies upon both mathematical evidence and reasons why that evidence supports the conclusion.

Additionally, students will need to practice determining which application is appropriate for different scenarios (for example, using the definitions of harmonic or p-series to classify certain infinite series) and then applying associated procedures accurately. Students will also need to practice using Taylor polynomials to approximate the value of a function, choosing and implementing an appropriate method to bound the error involved in the approximation, and effectively communicating supporting work.

Connecting representations is an important skill to develop in this unit. For example, students will need to identify infinite power series to represent functions presented symbolically or move between graphic and symbolic representations of an interval of convergence.

Students are more likely to demonstrate an incomplete understanding of series or to struggle with communicating their understanding of it compared to other topics. Continue to model and expect correct notation and language to present solutions, explain reasoning, and justify conclusions. For example, using the ratio test to find a radius of convergence, or operating on a known series to create another series, requires proficient, well-presented algebra. Applying a convergence test requires explicit verification that all necessary conditions are met. Determining that a given number is an error bound requires calculating an appropriate value and communicating that the value is less than the given number.

Intentional focus on the recurrent theme of using limiting cases to move from discrete approximations to analytic calculations and determinations is one way to help students to finish the year with a strong performance on the AP Exam and to come away with an enduring,meaningful understanding of Calculus.

Students will demonstrate the VOL Attributes in this unit through the following common formative assessments:

TESTS FOR CONVERGENCE FLOW CHART - In this unit we learn lots of different tests for convergence. Depending on the series, some tests work, others don't, and some may be inconclusive. In some cases, multiple approaches will work, but some are much more efficient depending on the structure of the series. In this activity, students will create a flowchart that can be used to determine the best approach to take to verify whether a Series converges or diverges. Examples might be "if the series has a factorial, utilize the root test" or, "if you see an alternating factor, consider the alternating series test." This flowchart can then be used by the student moving forward. This activity relates to TCC1(9-12): I can ask purposeful, insightful questions to find a variety of innovative solutions and TI3(9-12): I can formulate and investigate probing questions to further my learning.

INVENTING NEW FUNCTIONS ACTIVITY - This activity builds on student knowledge of Taylor and Maclaurin Series for common functions such as sin x, cos x, e^x, and 1/(1-x). Students are shown that from known functions we can build new functions through substitution, multiplication, differentiation, and integration. Then, they are given an "open playground" to build and construct as many new unique, different, and/or complex functions as they can. Students will do so on white board space around the room so that, after a certain period of time, students can walk around the room and comment and provide feedback on each other's creations. For example, given the knowledge that $e^{x} = 1 + x + x^{2}/2! + x^{3}/3! + ...$ students may show an expansion for $e^{x^{2}} = 1 + x^{2}/1! + x^{4}/2! + x^{6}/3! + ...$. This activity relates to TCC3(9-12): I can integrate relevant information to produce multiple valid solutions.

SELF DIRECTED REVIEW SESSION - Students take initiative as they determine the use of review time to prepare for assessments. Students are provided with a topic list and a set of review problems, however, they are given the freedom to do as much or as little of it as needed, go in whatever order makes most sense to them, request additional problems from me as needed on specific topics, and can move around the room freely utilizing any board space, manipulatives, classroom resources, etc. They are encouraged to work in groups seeking out partners that want to work on the same skill. I wander the class and coach students only when requested and only if they have first sought guidance/feedback from their peers. This activity relates to CCE3(9-12): I can show initiative in prompting group discourse and fostering collaboration among others, providing actionable feedback, and working with others to solve problems and/or design products.

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Unit 4 - Parametric, Vector, and Polar Functions	
Desired Results - Goals, Transfer, Meaning, Acquisition	
Established Goals: Standards for Mathematical Practice: 2.D) Identify how mathematical characteristics or properties of functions are related to different representations. 1.E) Apply appropriate mathematical rules or procedures, with and without technology. 1.D) Identify an appropriate mathematical rule or procedure based on the relationship between concepts (e.g., rate of change and accumulation) or processes (e.g., differentiation and ints inverse process, anti-differentiation) to solve problems. 3.D) Apply an appropriate mathematical definition, theorem, or test.	
Vision of A Learner Attributes: ● CCE3(9-12): I can show initiative in prompting group discourse and fostering collaboration among others, providing actionable feedback, and working with others to solve problems and/or design products. ● TI3(9-12): I can formulate and investigate probing questions to further my learning. ● AA3(9-12): I can adjust my expectations and behaviors to succeed in a changing and unpredictable environment.	
Understandings: Students will understand that... ● Derivatives allow us to solve real-world problems involving rates of change. ● Definite integrals allow us to solve problems involving the accumulation of change in length over an interval. ● Solving an initial value problem allows us to determine an expression for the position of a particle moving in the plane. ● Recognizing opportunities to apply derivative rules can simplify differentiation.	**Essential Questions:** ● How can a third variable, time, be integrated into a relationship between two variables? ● What can vectors allow us to measure and model? ● In what other ways can we define all possible locations on a 2-dimensionally measured plane?
Students will know… ● Methods for calculating derivatives of real-valued functions can be extended to parametric functions. ● For a curve defined parametrically, the value of dy/dx at a point on the curve is the slope of the line tangent to the curve at that point. dy/dx, the slope of the line tangent to a curve defined	**Students will be able to...** ● Calculate derivatives of parametric functions. ● Determine the length of a curve in the plane defined by parametric functions, using a definite integral. ● Calculate derivatives of vector-valued functions.

using parametric equations, can be determined by dividing dy/dt by dx/dt , provided dx/dt does not equal zero. ● The length of a parametrically defined curve can be calculated using a definite integral. ● Methods for calculating derivatives of real-valued functions can be extended to vector-valued functions ● Methods for calculating integrals of real-valued functions can be extended to parametric or vector-valued functions. ● Derivatives can be used to determine velocity, speed, and acceleration for a particle moving along a curve in the plane defined using parametric or vector-valued functions. ● For a particle in planar motion over an interval of time, the definite integral of the velocity vector represents the particle's displacement (net change in position) over the interval of time, from which we might determine its position. The definite integral of speed represents the particle's total distance traveled over the interval of time. ● Methods for calculating derivatives of real-valued functions can be extended to functions in polar coordinates. ● For a curve given by a polar equation $r = f(\theta)$,derivatives of r, x, and y with respect to θ, and first and second derivatives of y with respect to x can provide information about the curve. ● The concept of calculating areas in rectangular coordinates can be extended to polar coordinates. ● Areas of regions bounded by polar curves can be calculated with definite integrals.	● Determine a particular solution given a rate vector and initial conditions. ● Determine values for positions and rates of change in problems involving planar motion. ● Calculate derivatives of functions written in polar coordinates. ● Calculate areas of regions defined by polar curves using definite integrals.

Key Vocabulary: Parametric-defined Function, Arc Length, Cycloid, Vector, Position Vector, Velocity Vector, Acceleration Vector, Zero Vector, Components, Standard Representation, Magnitude, Direction Angle, Initial Point, Terminal Point, Equivalent Vectors, Scalars, Vector Addition, Scalar Multiplication, Unit Vector, Tail-to-head Representation, Parallelogram Representation, Displacement, Distance Traveled, Polar-defined Function, Polar Equation, Pole, Initial Ray, Directed Distance, Limacon, Lemniscate, Cardioid

Assessment Evidence	
Performance Tasks: ● Unit 4 Test-This assessment covers all topics from this unit. Test breakdown is:	**Other Evidence:** Book Problems: ● Parametric Functions (11.1) P. 547

35% Parametric Functions 15% Vector Functions 50% Polar Functions	#1,5,7,10,16,23,25,28,29,31,43 ● Vectors in the Plane (11.2) P. 558 #1-5,8,11,16,17,20,21,29,39 ● Polar Functions (11.3) P. 571 #2,4,5,8,10,12,18,21,25,29 ● Polar Functions (Continued) (11.3) P. 571 #39,41,43,45,51,56,57 Previously Released AP MC Questions: ● BC MC 2012(Various Questions) Previously Released AP FRQ Questions: ● AP 2018 #2, 2016 #2, 2015 #2, 2014 #1, 2016 #5, 2014 #5, 2018 #5, 2017 #2, 2012 #1, 2013 #2, 2012 #3, 2013 #4 Select Problems from AP Classroom Personal Progress Checks from: ● Unit 9 *AP Units are structure differently, but all content is the same

Learning Plan

In this unit, students will build on their understanding of straight-line motion to solve problems in which particles are moving along curves in the plane. Students will define parametric equations and vector-valued functions to describe planar motion and apply calculus to solve motion problems. Students will learn that polar equations are a special case of parametric equations and will apply calculus to analyze graphs and determine lengths and areas. This unit should be treated as an opportunity to reinforce past learning and transfer knowledge and skills to new situations, rather than as a new list of facts or strategies to memorize.

As students transition to parametric and vector-valued functions, they'll need to practice previously learned concepts and skills to reinforce the new procedures and representations they're learning in Unit 9. As with particle motion on a line, students learning to handle motion in the plane will need
to practice interpreting which procedure is needed for different scenarios (differentiation or integration) and solving for speed, velocity, distance traveled, or initial position.

Reinforce the importance of precise notation, particularly regarding the variable of differentiation, as well as correct application of the chain rule. Leibniz notation helps students to remember how to find the derivative of y with respect to x for coordinates defined using the parameter t: dy/dx = dy/dt / dx/dt = dy/dt * dt/dx provided dx/dt =/= 0. Since dy/dx is in terms of t, students must be particularly careful when determining d2y/dx2. Similarly, using definite integrals to represent lengths and areas defined by polar curves is based on the same principles as calculating lengths and areas defined by the graphs of more familiar functions (i.e., the limit of a Riemann sum). Students will need to practice with trigonometric identities, radian measures and formulas for arc length and area of a sector to reinforce practice with associated calculus topics.

While students need more experience shifting mindsets from rectangular to polar coordinate systems, errors in arithmetic, algebra, trigonometry, and procedures such as the chain rule are often even more problematic. Provide opportunities for students to reinforce familiar skills and concepts as they practice new techniques in preparation for the AP Exam. As with analysis of graphs, sign charts can be useful tools for identifying answers to questions about the direction of motion or whether speed is increasing or decreasing, for example. To earn points for justification, however, students must connect their work to a relevant definition or theorem, as in the Scoring Guidelines for 2017 AB5. Continue to emphasize accounting for initial values, as in past units, as well as precise communication and notational fluency. Paying attention to subscripts in problems involving more than one particle is essential to clear communication.

Students will demonstrate the VOL Attributes in this unit through the following common formative assessments:
HOW CAN WE REPRESENT IT? - This activity is a class discussion to start the unit. It is a challenging one for students and requires a good amount of wait time to do with fidelity since students do struggle at first. Begin by asking the question: “How else can you uniquely identify any location on a coordinate grid other than with horizontal (x) and vertical (y) coordinates?” This is meant to get students thinking about the big ideas of representing location parametrically, in terms of vectors, or with polar coordinates. Common student responses tend to lean, at first, toward segmented representations of the standard coordinate algorithm, but often students eventually come back to what they explored with the use of bearings in PreCalculus, that we can instead use a direction and a distance, which is the basis of vector and polar functions. Ask students to consider measuring the path of a train if they need help weeding out the possibility of defining x and y coordinates in terms of time. This activity relates to TI3(9-12): I can formulate and investigate probing questions to further my learning and AA3(9-12): I can adjust my expectations and behaviors to succeed in a changing and unpredictable environment.

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