

# ADVANCED QUANTITATIVE REASONING

## Graded Course of Study

**Conceptual Category:** Number and Quantity

**Ohio Learning Standards Domain:** Quantities

**Clusters**

- Reason quantitatively and use units to solve problems

**Ohio Statewide Remediation Free Standards Category:** Number and Operations

**Categories**

- Structure of the Number System
- Operations

**Ohio Statewide Remediation Free Standards Category:** Algebra

**Categories**

- Graphing
- Functions and Applications

**Ohio Statewide Remediation Free Standards Category:** Geometry

**Categories**

- Measurement

**Conceptual Category:** Algebra and Functions

**Ohio Learning Standards Domain:** Creating Equations

**Clusters**

- Create equations that describe numbers or relationships

**Ohio Learning Standards Domain:** Interpreting Functions

**Clusters**

- Interpret functions that arise in applications in terms of the context

**Ohio Learning Standards Domain:** Interpreting Categorical and Quantitative Data

**Clusters**

- Summarize, represent, and interpret data on two categorical and quantitative variables
- Interpret linear models

**Conceptual Category: Algebra and Functions (continued)****Ohio Learning Standards Domain: Linear, Quadratic, and Exponential Models****Clusters**

- Construct and compare linear, quadratic, and exponential models, and solve problems

**Ohio Learning Standards Domain: Quantities****Clusters**

- Reason quantitatively and use units to solve problems

**Ohio Learning Standards Domain: Interpreting Functions****Clusters**

- Understand the concept of a function, and use function notation
- Analyze functions using different representations

**Ohio Learning Standards Domain: Building Functions****Clusters**

- Build a function that models a relationship between two quantities

**Conceptual Category: Probability & Statistics****Ohio Learning Standards Domain: Interpreting Categorical and Quantitative Data****Clusters**

- Summarize, represent, and interpret data on a single count or measurement variable

**Ohio Learning Standards Domain: Quantities****Clusters**

- Reason quantitatively and use units to solve problems

**Ohio Learning Standards Domain: Making Inferences and Justifying Conclusions****Clusters**

- Understand and evaluate random processes underlying statistical experiments
- Make inferences and justify conclusions from sample surveys, experiments, and observational studies

**Conceptual Category:** Modeling with Algebra and Functions**Ohio Learning Standards Domain:** Quantities**Clusters**

- Reason quantitatively and use units to solve problems

**Ohio Learning Standards Domain:** Seeing Structure in Expressions**Clusters**

- Write expressions in equivalent forms to solve problems

**Ohio Learning Standards Domain:** Creating Equations**Clusters**

- Create equations that describe numbers or relationships

**Ohio Learning Standards Domain:** Linear, Quadratic, and exponential Models**Clusters**

- Construct and compare linear, quadratic, and exponential models, and solve problems

**Ohio Learning Standards Domain:** Interpreting Functions**Clusters**

- Interpret functions that arise in applications in terms of the context

**Mathematical Practices**

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

**Theme 1****Ohio Learning Standards 2017****QUANTITIES**

N.Q

Reason quantitatively and use units to solve problems.

N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

**CREATING EQUATIONS**

A.CED

Create equations that describe numbers or relationships.

A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★

- a. Focus on applying linear and simple exponential expressions.
- b. Focus on applying simple quadratic expressions.
- c. Extend to include more complicated function situations with the option to graph with technology.

A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.★

- a. While functions will often be linear, exponential, or quadratic, the types of problems should draw from more complicated situations.

**INTERPRETING FUNCTIONS**

F.IF

Interpret functions that arise in applications in terms of the context.

F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include the following: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.★

- a. Focus on linear and exponential functions.
- b. Focus on linear, quadratic, and exponential functions.

F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★

**INTERPRETING CATEGORICAL AND QUANTITATIVE DATA**

S.ID

Summarize, represent, and interpret data on two categorical and quantitative variables.

S.ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.★

- a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions, or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.
- b. Informally assess the fit of a function by discussing residuals.
- c. Fit a linear function for a scatterplot that suggests a linear association.

Interpret linear models.

S.ID.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.★

S.ID.9 Distinguish between correlation and causation.★

**Uniform Statewide Standards for Remediation-Free Status 2018**

Number and Operations

Structure of the Number System

A. Understand and convert between different representations of numbers (decimal, percent, fraction, scientific notation, radicals...).

Operations

D. Compute and explain the solutions to problems involving ratio, proportion, percent, scientific notation, square roots, and numbers with integer and rational exponents;

Algebra

Graphing

C. Read a graph to interpret solutions to an equation and identify and interpret characteristics such as intercepts, extrema, and rates of change.

E. . Interpret transformations of functions from both a graphical and algebraic perspective.

Geometry

Measurement

E. Solve problems involving measurement, including problems requiring a choice of scale and unit

**Theme 2****Ohio Learning Standards 2017****CREATING EQUATIONS**

A.CED

Create equations that describe numbers or relationships.

A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★

c. Extend to include more complicated function situations with the option to graph with technology.

A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.★

a. While functions will often be linear, exponential, or quadratic, the types of problems should draw from more complicated situations.

**INTERPRETING FUNCTIONS**

F.IF

Understand the concept of a function, and use function notation.

F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by  $f(0) = f(1) = 1$ ,  $f(n + 1) = f(n) + f(n - 1)$  for  $n \geq 1$ .

Analyze functions using different representations.

F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

a. Focus on linear and exponential functions.

b. Focus on linear, quadratic, and exponential functions.

**BUILDING FUNCTIONS F.BF**

Build a function that models a relationship between two quantities.

F.BF.1 Write a function that describes a relationship between two quantities.★

(+) c. Compose functions. For example, if  $T(y)$  is the temperature in the atmosphere as a function of height, and  $h(t)$  is the height of a weather balloon as a function of time, then  $T(h(t))$  is the temperature at the location of the weather balloon as a function of time.

F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★

**LINEAR, QUADRATIC, AND EXPONENTIAL MODELS**

F.LE

Construct and compare linear, quadratic, and exponential models, and solve problems.

F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.★

a. Show that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.

- b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
- c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).★

## QUANTITIES

N.Q

Reason quantitatively and use units to solve problems.

N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. ★

N.Q.2 Define appropriate quantities for the purpose of descriptive modeling.★

N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.★

**Theme 3****Ohio Learning Standards 2017****QUANTITIES**

N.Q

Reason quantitatively and use units to solve problems.

N.Q.2 Define appropriate quantities for the purpose of descriptive modeling. ★

N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. ★

**INTERPRETING CATEGORICAL AND QUANTITATIVE DATA**

S.ID

Summarize, represent, and interpret data on a single count or measurement variable.

S.ID.1 Represent data with plots on the real number line (dot plots, histograms, and box plots) in the context of real-world applications using the GAISE model. ★

S.ID.2 In the context of real-world applications by using the GAISE model, use statistics appropriate to the shape of the data distribution to compare center (median and mean) and spread (mean absolute deviation, interquartile range, and standard deviation) of two or more different data sets. ★

S.ID.3 In the context of real-world applications by using the GAISE model, interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). ★

Summarize, represent, and interpret data on two categorical and quantitative variables.

S.ID.5 Summarize categorical data for two categories in two-way frequency tables.

Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. ★

**MAKING INFERENCES AND JUSTIFYING CONCLUSIONS**

S.IC

Understand and evaluate random processes underlying statistical experiments.

S.IC.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population. ★

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

S.IC.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. ★

S.IC.4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. ★

S.IC.6 Evaluate reports based on data. ★



**Theme 4****Ohio Learning Standards 2017****QUANTITIES**

N.Q

Reason quantitatively and use units to solve problems.

N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. ★

N.Q.2 Define appropriate quantities for the purpose of descriptive modeling. ★

**SEEING STRUCTURE IN EXPRESSIONS**

A.SSE

Write expressions in equivalent forms to solve problems.

A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. ★

- c. Use the properties of exponents to transform expressions for exponential functions. For example,  $8^t$  can be written as  $2^{3t}$ .

**CREATING EQUATIONS**

A.CED

Create equations that describe numbers or relationships.

A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. ★

- a. Focus on applying linear and simple exponential expressions.
- b. Focus on applying simple quadratic expressions.
- c. Extend to include more complicated function situations with the option to graph with technology.

A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods. ★

- a. While functions will often be linear, exponential, or quadratic, the types of problems should draw from more complicated situations.

A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. ★

- a. Focus on formulas in which the variable of interest is linear or square. For example, rearrange Ohm's law  $V = IR$  to highlight resistance  $R$ , or rearrange the formula for the area of a circle  $A = (\pi)r^2$  to highlight radius  $r$ .
- b. Focus on formulas in which the variable of interest is linear. For example, rearrange Ohm's law  $V = IR$  to highlight resistance  $R$ .
- c. Focus on formulas in which the variable of interest is linear or square. For example, rearrange the formula for the area of a circle  $A = (\pi)r^2$  to highlight radius  $r$ .
- d. While functions will often be linear, exponential, or quadratic, the types of problems should draw from more complicated situations.

## INTERPRETING FUNCTIONS

F.IF

Interpret functions that arise in applications in terms of the context.

F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★

## BUILDING FUNCTIONS

F.BF

Build a function that models a relationship between two quantities.

F.BF.1 Write a function that describes a relationship between two quantities. ★

a. Determine an explicit expression, a recursive process, or steps for calculation from context.

i. Focus on linear and exponential functions.

ii. Focus on situations that exhibit quadratic or exponential relationships.

b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

(+) c. Compose functions. For example, if  $T(y)$  is the temperature in the atmosphere as a function of height, and  $h(t)$  is the height of a weather balloon as a function of time, then  $T(h(t))$  is the temperature at the location of the weather balloon as a function of time.

## LINEAR, QUADRATIC, AND EXPONENTIAL MODELS

F.LE

Construct and compare linear, quadratic, and exponential models, and solve problems.

F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). ★