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# **Exterior Angles of Triangles**

1. Given parallel lines  $\ell$  and q with transversal p, find the value of x.



$$180 - 122 = 58$$
  
 $58 + 63 + x = 180$   
 $121 + x = 180$   
 $x = 59$ 

The value of x is 59.

## **Interior and Exterior**

2. Identify a location for an exterior angle adjacent to  $\angle F$ .



3. Consider  $\triangle HAT$  in the diagram.



- a. Name the exterior angles of  $\triangle HAT$ .  $\angle CTH$ ,  $\angle OHA$ ,  $\angle GAT$
- b. If the measure of  $\angle HTA$  is 58° and the measure of  $\angle THA$  is 86°, find the measures of the following angles.
  - $m \angle HAT = \underline{36^{\circ}}$  $m \angle GAT = \underline{144^{\circ}}$  $m \angle OHA = \underline{94^{\circ}}$  $m \angle CTH = \underline{122^{\circ}}$

## **Remote Interior Angles**

4. Find the value of *x*.





The value of x is 86.

5. Find the value of *y*.



The value of *y* is 36.

6. Write equations that represent the angle relationships shown in the diagram.



Any exterior angle measure of a triangle is equal to the sum of the remote interior angle measures.

7. Use the relationship between an exterior angle of a triangle and the remote interior angles to find the value of x.



The value of x is 59.

## Find the Angle Measure

For problems 8-13, find the value of *x* in the diagram by using any of the angle relationships you have learned. Label any additional angle measures you use to find the value of *x*.





13. Hint: Extend one segment as a transversal.



76 = x

76 - 34 = 42

42 + 34 = x

The value of x is 76.

The value of x is 81.



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Dylan and Noor are asked to find the measure of  $\angle 1$  in the given triangle.



Dylan finds the measure of  $\angle 1$  this way:

$$156^{\circ} + m \angle 2 = 180^{\circ}$$
$$m \angle 2 = 24^{\circ}$$
$$m \angle 1 + m \angle 2 + 81^{\circ} = 180^{\circ}$$
$$m \angle 1 + 24^{\circ} + 81^{\circ} = 180^{\circ}$$
$$m \angle 1 = 75^{\circ}$$

Noor finds the measure of  $\angle 1$  this way:

$$m \angle 1 + 81^{\circ} = 156^{\circ}$$
$$m \angle 1 = 75^{\circ}$$

Explain whose solution is correct and why.

Both solutions are correct. Dylan uses the interior angle measures of a triangle and the linear pair formed by the exterior angle and the adjacent interior angle to find the measure of  $\ge 1$ .

Noor uses the exterior angle measure of the triangle, which is equal to the sum of the remote interior angle measures, to find the measure of  $\angle 1$ .



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# **Exterior Angles of Triangles**

In this lesson, we

- defined exterior angle and remote interior angles of a triangle.
- determined that the measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles of the triangle.
- solved equations to find angle measures.

## **Examples**

1. Find the measure of  $\angle ACD$ .

 $\angle ACB$  is adjacent to  $\angle ACD$ , so  $\angle ABC$  and  $\angle BAC$  are the

remote interior angles to  $\angle ACD$ .





An exterior angle of a triangle is an angle that forms a linear pair with an interior angle of the triangle. In the diagram,  $\angle 1$  is an exterior angle of the triangle.

Remote interior angles of a triangle are the two interior angles not adjacent to a given exterior angle of the triangle. In the diagram,  $\angle 1$  is an exterior angle, and  $\angle 2$  and  $\angle 3$  are the remote interior angles.

 $\angle ACD$  is an exterior angle of  $\triangle ABC$ .

 $m \angle ABC + m \angle BAC = m \angle ACD$  $96^{\circ} + 28^{\circ} = m \angle ACD$  $124^{\circ} = m \angle ACD$ 

D

*B* 96°

Exterior angle measure

Sum of the remote interior angle measures

2. Find the measure of  $\angle GEF$ .





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2. ∠*ACD* 

For problems 1–6, find the measure of the given angle.

1. ∠*ACD* 





96°

3. ∠*ACB* 



4. ∠*DAB* 



51°

79°



For problems 7-10, find the measure of the given angle. Describe all angle relationships you use to find the unknown angle measure.

#### 7. ∠*DAB*



### 131°

The exterior angle measure of a triangle is equal to the sum of the remote interior angle measures, so 48 + 83 = 131.



### 126°

The exterior angle measure of a triangle is equal to the sum of the remote interior angle measures, so 100 + 26 = 126.

9. ∠*ACB* 



79°

The exterior angle measure of a triangle is equal to the sum of the remote interior angle measures, so 126 - 47 = 79.

10. ∠*BAD* 





 $\angle BAD$  and  $\angle BAC$  are supplementary, so 180 - 77 = 103.

- 11. Use the diagram and the given information to answer parts (a)–(d).
  - $\overrightarrow{AD}$  and  $\overrightarrow{EI}$  are parallel.
  - $\overrightarrow{JP}$  and  $\overrightarrow{KO}$  are transversals.
  - The measure of  $\angle BCQ$  is 67°.
  - The measure of  $\angle QHI$  is 119°.



a. Find the measure of  $\angle QFH$ .

b. What is the angle relationship between  $\angle BCQ$  and  $\angle QFH$  that verifies the measure of  $\angle QFH$ ?

 $\angle QFH$  and  $\angle BCQ$  are alternate interior angles of parallel lines,  $\overrightarrow{AD}$  and  $\overrightarrow{EI}$ . So  $\angle QFH$  and  $\angle BCQ$  have the same measure.

c. Find the measure of  $\angle FQH$ .

52°

d. What is the relationship between  $\angle FQH$ ,  $\angle QFH$ , and  $\angle QHI$  that verifies the measure of  $\angle FQH$ ?

The measure of the exterior angle of  $\triangle FQH$ ,  $\angle QHI$ , is equal to the sum of the measures of the remote interior angles,  $\angle FQH$  and  $\angle QFH$ .

<sup>67°</sup> 

12. In the diagram,  $\overrightarrow{AB}$  is parallel to  $\overrightarrow{CD}$ . The measure of  $\angle ABE$  is 56°, and the measure of  $\angle EDC$  is 22°.



- a. Find the measure of  $\angle BED$ . Hint: Extend  $\overline{BE}$  so that it intersects  $\overleftrightarrow{CD}$  at a point *F*. 78°
- b. Explain how you found the measure of  $\angle BED$ .

 $\overleftrightarrow{AB}$  is parallel to  $\overleftrightarrow{CD}$ , so  $\angle ABE$  and  $\angle DFE$  both measure 56° because they are congruent alternate interior angles.  $\angle BED$  is an exterior angle of a triangle with  $\angle EFD$  and  $\angle EDF$  as remote interior angles, so the measure of  $\angle BED$  is 78° because 56 + 22 = 78.

13. In the diagram,  $\overleftrightarrow{OP}$  is parallel to  $\overleftrightarrow{LN}$  with transversals  $\overleftrightarrow{JM}$  and  $\overleftrightarrow{KM}$ .



- a. Find the measure of  $\angle JMK$ . 70°
- b. Explain how you found the measure of  $\angle JMK$ .  $\overleftrightarrow{OP}$  and  $\overleftrightarrow{LN}$  are parallel, so  $\angle LMK$  and  $\angle JKM$  both measure 72° because they are congruent alternate interior angles. The interior angle measures of  $\triangle JKM$  sum to 180°, so the measure of  $\angle JMK$  is 70° because 180 - (72 + 38) = 70.

### Remember

For problems 14–17, write an equivalent expression.

 14. 3(x+2) + 7x 15. 5(x+6) + 3x 

 10x + 6 8x + 30 

16. 7(x+2) + 5x17. 8(x+4) + 8x12x + 1416x + 32

18. Figure *ABCDEFG* is congruent to figure *JKLMNPQ*. Describe a sequence of rigid motions that maps figure *ABCDEFG* onto figure *JKLMNPQ*.



A translation along  $\overrightarrow{AJ}$  maps point *A* to point *J*. A 45° counterclockwise rotation around point *J* maps figure *ABCDEFG* onto figure *JKLMNPQ*.

For problems 19 and 20, simplify.

19.  $x^0 \cdot x^5$ 

 $x^5$ 

20.  $(ab^5)^4 (a^2b)^3$  $a^{10}b^{23}$