Copy of Math Grade 5 - Module 1

Subject	Grade	Module	Suggested Timeline
Mathematics	5	1	4 weeks

Grade Level Summary

In grade 5, mathematics is about (1) developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to two-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; and (3) developing understanding of volume.

Grade Level Modules

Module 1: Whole Number and Decimal Fraction Place Value to the One-Thousandths

1. Module 2: Multi-Digit Whole Number and Decimal Fraction Operations
2. Module 3: Addition and Subtraction of Fractions
3. Module 4: Multiplication and Division of Fractions and Decimal Fractions
4. Module 5: Addition and Multiplication with Volume and Area
5. Module 6: Graph Points on the Coordinate Plane to Solve Problems

Module Title

Module 1: Whole Number and Decimal Fraction Place Value to the One-Thousandths

Module Overview

In Module 1, whole number patterns with number disks on the place value table are easily generalized to decimal numbers. As students work word problems with measurements in the metric system, where the same patterns occur, they begin to appreciate the value and the meaning of decimals. Fractions of the form 1/10, 1/100, 1/1000 also play a prominent role in the first module and are used to investigate patterns on the place value table.

Module Objectives

At the end of this module, students will be able to independently use their learning to:

- **Demonstrate an understanding that in a multi-digit number, a digit in one place represents 1/10 of what it represents in the place to its left**
- **Explain patterns in the number of zeroes in the product when multiplying a number by powers of 10**
- **Explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10**
- **Use whole number exponents to denote powers of 10**
- **Read and write decimals to thousandths using base 10 numerals, word form, and expanded form**
- **Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols**
- **Round decimals to ones, tenths, hundredths, or thousandths place**

Convert among different sized measurement units within a given measurement system using a provided table of equivalencies.

Standards for Mathematical Practice

MP# 1. Make sense of problems and persevere in solving them
MP# 2. Reason abstractly and quantitatively
MP# 4. Model with mathematics
MP# 5. Use appropriate tools strategically
MP# 6. Attend to precision
MP# 7. Look for and make use of structure (Deductive Reasoning)
MP# 8. Look for and express regularity in repeated reasoning

Focus Standards Addressed in this Module

CC.2.1.5.B.1

Apply place value to show an understanding of operations and rounding as they pertain to whole numbers and decimals.

CC.2.4.5.A.1

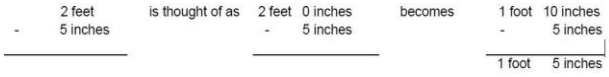
Solve problems using conversions within a given measurement system.

Important Standards Addressed in this Module

Misconceptions	Proper Conceptions
1. **A common misconception**	1. **Reinforcing the concept of**

students have when extending their understanding of whole number place value to decimal place value is thinking that the digits to the right of the decimal point increase in value.

2. A second misconception directly related to comparing whole numbers is the longer the number the greater the number. With whole numbers, a 5-digit number is always greater that a 1-, 2-, 3-, or 4-digit number. However, with decimals a number with one decimal place may be greater than a number with two or three decimal places. For example, 0.5 is greater than 0.12, 0.009 or 0.499.
3. Students fail to convert units in the problem so they are consistent. For example, when subtracting 5 inches from 6 feet. Students obtain an answer of 1 foot.
4. When solving problems that require renaming of units, students fail to use the conversion chart and revert to the base 10 system of renaming. For example, when subtracting 5 inches from 2 feet, students fail to convert 1 foot 12 inches and instead write 1 foot 10 inches. (See example.)



powers of ten is essential for addressing this issue.

2. As we move to the right of the decimal, the digits decrease in value, now becoming a smaller fraction of the whole.
3. Reinforcing the concept of powers of ten is essential for addressing this issue. Rewrite all numbers to include the same number of digits to the right of the decimal point by adding zeros to the number, such as 0.500, 0.120, 0.009 and 0.499. Use a place-value chart to place the numerals for comparison. Rewrite the numbers vertically, lining up the decimal point. Grid paper may be helpful to keep numbers aligned.
4. Use a unit box to organize information. Require students to write the units for each number in the problem. For example,

6 feet

-_____5 inches

4.Provide students with a conversion chart when working with this type of task. It is important for students to realize that methods used to solve whole number problems without a unit of measurement are different than methods used to solve problems involving units of measurement.

Concepts	Competencies	Vocabulary
• Decimals • Place Value and Properties of Operations	• Demonstrate an understanding of rounding as it pertains to whole numbers and decimals. • Read, write and compare decimals.	Addend Associative Property of Addition Associative Property of Multiplication

- **Solve problems using simple conversions.**

Base-ten numeral form
Benchmark
Capacity
Centimeter
Commutative Property of Addition
Commutative Property of Multiplication
Compose
Cup
Customary system
Decimal
Decimal point
Decimeter
Decameter
Decompose
Difference
Distributive Property
Elapsed time
Estimate
Expanded form
Exponent
Factor
Fluid ounce
Foot
Gallon
Gram
Greater than
Hundredth
Hundredths
Inch
Inequality
Kilogram
Kilometer
Less than
Liter
Mass
Measurement Systems
Measurement Units
Meter
Metric system
Mile
Milligram
Milliliter
Millimeter
Minuend

Order of Operations
Ounce
Partial product
Pattern
Pint
Place value
Pound
Powers of 10
Product
Quart
Rounding
Sequence
Standard form
Subtrahend
Sum
Tenth
Tenths
Term
Thousandth
Thousandths
Ton
Unit Fractions
Weight
Yard

Assessment(s)

Daily practice sheets

Oral Responses

Daily Math Journal/Open Ended Responses

Pre-Test/Review

End of chapter test

Suggested Strategies to Support Design of Coherent Instruction

Instructional Strategies:

Place Value:

In Grade 5, the concept of place value is extended to include decimal values to thousandths. The strategies for Grades 3 and 4 should be drawn upon and extended for whole numbers and decimal numbers. For example, students should continue to represent, write, and state the value of numbers including

decimal numbers.

This module calls for students to reason about the magnitude of numbers. Students should work with the idea that the tens place is ten times as much as the ones place, and the ones place is $1/10^{th}$ the size of the tens place. Students use base ten blocks, pictures of base ten blocks, and interactive images of base ten blocks to manipulate and investigate these place value relationships.

This standard includes multiplying by multiples of 10 and powers of 10, including 10^2 which is $10 \times 10=100$, and 10^3 which is $10 \times 10 \times 10 =1,000$. Students should have experiences working with connecting the pattern of the number of zeros in the product when you multiply by powers of 10.

Examples:

$$2.5 \times 10^3 = 2.5 \times (10 \times 10 \times 10) = 2.5 \times 1,000 = 2,500$$

Students should reason that the exponent above the 10 indicates how many places the decimal point is moving (not just that the decimal point is moving but that you are multiplying or making the number 10 times greater three times) when you multiply by a power of 10. Since we are multiplying by a power of 10 the decimal point moves to the right.

$$350 \div 103 = 350 \div 1,000 = 0.350 = 0.35$$

$$^{350}/_{10} = 35 \qquad (350 \times {}^{1}/_{10}) \qquad {}^{35}/_{10} = 3.5$$

$$(35 \times {}^{1}/_{10}) \qquad {}^{3.5}/_{10} = 0.35 \qquad (3.5 \times {}^{1}/_{10})$$

This will relate well to subsequent work when operating with fractions. This example shows that when we divide by powers of 10, the exponent above the 10 indicates how many places the decimal point is moving (how many times we are dividing by 10, the number becomes ten times smaller). Since we are dividing by powers of 10, the decimal point moves to the left.

Students need to be provided with opportunities to explore this concept and come to this understanding; this should not just be taught procedurally.

Utilizing number lines for ordering and comparing is a viable method when working with decimals. Placing the numbers on a number line provides students with a concrete representation to help build conceptual understanding.

Number cards, number cubes, spinners and other manipulative can be used to generate decimal numbers. For example, have students roll three number cubes, then create the largest and small number to the thousandths place. Ask students to represent the number with numerals, words, and in expanded form.

Money could be used to decimals. Present contextual situations that require the comparison of the cost of two items to determine the lower or higher priced

item. Students should also be able to identify how many pennies, dimes, dollars and ten dollars, etc., are in a given value. Help students make connections between the number of each type of coin and the value of each coin, and the expanded form of the number. Build on the understanding that it always takes ten of the number to the right to make the number to the left.

<u>Converting Unit Measurements:</u>

This standard calls for students to convert measurements within the same system of measurement in the context of multi-step, real-world problems. Both customary and standard measurement systems are included; students worked with both metric and customary units of length in second grade. In third grade, students work with metric units of mass and liquid volume. In fourth grade, students work with both systems and begin conversions within systems in length, mass and volume.

To convert from one unit to another unit in the standard and metric system, the relationship between the units must be known. In order for students to have a better understanding of the relationships between units, they need to use measuring tools in class. The number of units must relate to the size of the unit.

Example 1: **100 cm = 1 meter**

Example 2: **12 inches = 1 foot and 3 feet = 1 yard.**

When converting in the metric system, have students extend their prior knowledge of the base-ten system as they multiply or divide by powers of ten. Teaching conversions should focus on the relationship of the measurements, not merely rote memorization. The questions ask the student to find out the size of each of the subsets. Students are not expected to know e.g. that there are 5280 feet in a mile. If this is to be used as an assessment task, the conversion factors should be given to the students. However, in a teaching situation it is worth having them realize that they need that information rather than giving it to them upfront; having students identify what information they need to have to solve the problem and knowing where to go to find it.

Once students have an understanding of the relationships between units and how to do conversions, they are ready to solve multi-step problems that require conversions within the same system. Allow students to discuss methods used in solving the problems. Begin with problems that allow for renaming the units to represent the solution before using problems that require renaming to find the solution.

Note: This section is not designed to provide a day-to-day lesson plan view; rather, it is offered as a guide to inform lesson planning.

Differentiation

Interdisciplinary Connections

Additional Resources

Textbook: 5th Grade Houghton Mifflin Math

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Copy of Math Grade 5 - Module 2

Subject	Grade	Module	Suggested Timeline
Mathematics	5	2	6 weeks

Grade Level Summary

In grade 5, mathematics is about (1) developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to two-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; and (3) developing understanding of volume.

Grade Level Modules

Module 1: Whole Number and Decimal Fraction Place Value to the One-Thousandths

Module 2: Multi-Digit Whole Number and Decimal Fraction Operations

Module 3: Addition and Subtraction of Fractions

Module 4: Multiplication and Division of Fractions and Decimal Fractions

Module 5: Addition and Multiplication with Volume and Area

Module 6: Graph Points on the Coordinate Plane to Solve Problems

Module Title

Module 2: Multi-Digit Whole Number and Decimal Fraction Operations

Module Overview

Module 2 starts by giving students a chance to sharpen their skills in multiplying and dividing (decimal) numbers by 1-digit whole numbers. Now they are ready to generalize the 1-digit algorithms to the multi-digit whole number versions (multi-digit decimal multiplication such as 4.1. 3.4 and division such as 4.5 ÷ 1.5 are studied in Module 4). For multiplication, students must grapple with and fully understand the distributive property (one of the key reasons for teaching the multi-digit algorithm). While the multi-digit multiplication algorithm is a straightforward generalization of the one-digit multiplication algorithm, the division algorithm with two-digit divisor requires far more care to teach because

students have to also learn estimation strategies, error correction strategies, and the idea of successive approximation (all of which are central concepts in math, science, and engineering).

Module Objectives

At the end of this module, students will be able to independently use thier learning to:

- **Multiply multi-digit whole numbers, not to exceed three digits by three digits**
- **Find whole number quotients of whole numbers with up to four digit dividends and two digit divisors**
- **Add, subtract, multiply, and divide decimals to hundredths (no divisors with decimals)**
- **Use multiple grouping symbols (parentheses, brackets, or braces) in numerical expressions, and evaluate expressions containing these symbols**
- **Write simple expressions that model calculations with numbers**
- **Interpret numerical expressions without evaluating them**

Standards for Mathematical Practice

MP# 1. Make sense of problems and persevere in solving them
MP# 2. Reason abstractly and quantitatively
MP# 4. Model with mathematics
MP# 5. Use appropriate tools strategically
MP# 6. Attend to precision
MP# 7. Look for and make use of structure (Deductive Reasoning)
MP# 8. Look for and express regularity in repeated reasoning

Focus Standards Addressed in this Module

CC.2.1.5.B.2

Extend an understanding of operations with whole numbers to perform operations including decimals.

CC.2.2.5.A.1

Interpret and evaluate numerical expressions using order of operations.

Important Standards Addressed in this Module

Misconceptions	Proper Conceptions
1. Students may believe the order in which a problem with mixed	1. Allow students to use calculators to determine the value of the

operations is written is the order to solve the problem.

2. Students might compute the sum or difference of decimals by lining up the right-hand digits as they would whole number. For example, in computing the sum of 15.34 + 12.9, students will write the problem in this manner:

    3. 15.34
    4. +12.9
    5. 16.63

6. Students may believe that multiplication always results in a larger number. Additionally, students may believe that division always results in a smaller number.

expression, and then discuss the order the calculator used to evaluate the expression. Do this with four-function and scientific calculators.

2. To help students add and subtract decimals correctly, have them first estimate the sum or difference. Providing students with a decimal-place value chart will enable them to place the digits in the proper place.
3. Using models when multiplying with fractions will enable students to see that the results will be smaller.
4. Using models when dividing with fractions will enable students to see that the results will be larger.

Concepts	Competencies	Vocabulary
• Decimals • Numerical Expressions • Order of Operations	• Use whole numbers and decimals to compute accurately. • Write and interpret numerical expressions. • Evaluate expressions using the order of operations.	Additive Identity Property of 0 Algorithm Area model Array Associative Property of Addition Associative Property of Multiplication Base of an exponent Braces Brackets Commutative Property of Addition Commutative Property of Multiplication Compatible Numbers Decimal Decimal point Distributive Property Dividend Divisor Equation Equivalent fractions

Estimate
Evaluate
Exponent
Expression
Factor
Hundredth
Hundredths
Inverse operations
Long division
Multiplicative Identity
Property of 1
Multiply
Numerical expression
Order of Operations
Parentheses
Period
Place value
Powers of ten
Product
Quotient
Remainder
Sum
Tenth
Tenths
Thousandth
Thousandths
Unit Fractions
Whole numbers

Assessment(s)

Daily Practice Sheets

Oral responses

Daily Math Journal/Open Ended Responses

Pre-Test/Review

End of Chapter Test

Suggested Strategies to Support Design of Coherent Instruction

Students should be given ample opportunities to explore numerical expressions with mixed operations. This is the foundation for evaluating numerical and algebraic expressions that will include whole-number exponents in Grade 6.

There are conventions (rules) determined by mathematicians that must be learned with no conceptual basis. For example, multiplication and division are always done before addition and subtraction.

Begin with expressions that have two operations without any grouping symbols (multiplication or division combined with addition or subtraction) before introducing expressions with multiple operations. Using the same digits, with the operations in a different order, have students evaluate the expressions and discuss why the value of the expression is different. For example, have students evaluate 5 × 3 + 6 and 5 + 3 × 6. Discuss the rules that must be followed. Have students insert parentheses around the multiplication or division part in an expression. A discussion should focus on the similarities and differences in the problems and the results. This leads to students being able to solve problem situations which require that they know the order in which operations should take place.

After students have evaluated expressions without grouping symbols, present problems with one grouping symbol, beginning with parentheses, then in combination with brackets and/or braces.

Have students write numerical expressions in words without calculating the value. This is the foundation for writing algebraic expressions. Then, have students write numerical expressions from phrases without calculating them.

Because students have used various models and strategies to solve problems involving multiplication with whole numbers, they should be able to transition to using standard algorithms effectively. With guidance from the teacher, they should understand the connection between the standard algorithm and their strategies.

Connections between the algorithm for multiplying multi-digit whole numbers and strategies such as partial products or lattice multiplication are necessary for students' understanding.

You can multiply by listing all the partial products. For example:

234

× 8

32 Multiply the ones (8 × 4 ones = 32 ones)

240 Multiply the tens (8 × 3 tens = 24 tens or 240

1600 Multiply the hundreds (8 × 2 hundreds = 16 hundreds or 1600)

1872 Add the partial products

The multiplication can also be done without listing the partial products by multiplying the value of each digit from one factor by the value of each digit

from the other factor. Understanding of place value is vital in using the standard algorithm.

In using the standard algorithm for multiplication, when multiplying the ones, 32 ones are 3 tens and 2 ones. The 2 is written in the ones place. When multiplying the tens, the 24 tens are 2 hundreds and 4 tens. But, the 3 tens from the 32 ones need to be added to these 4 tens, for 7 tens. Multiplying the hundreds, the 16 hundreds are 1 thousand and 6 hundreds. But, the 2 hundreds from the 24 tens need to be added to these 6 hundreds, for 8 hundreds.

234

× 8

1872

As students developed efficient strategies to do whole number operations, they should also develop efficient strategies with decimal operations.

Students should learn to estimate decimal computations before they compute with pencil and paper. The focus on estimation should be on the meaning of the numbers and the operations, not on how many decimal places are involved. For example, to estimate the product of 32.84 × 4.6, the estimate would be more than 120, closer to 150. Students should consider that 32.84 is closer to 30 and 4.6 is closer to 5. The product of 30 and 5 is 150. Therefore, the product of 32.84 × 4.6 should be close to 150.

Have students use estimation to find the product by using exactly the same digits in one of the factors with the decimal point in a different position each time. For example, have students estimate the product of 275 × 3.8; 27.5 × 3.8 and 2.75 × 3.8, and discuss why the estimates should or should not be the same.

Note: This section is not designed to provide a day-to-day lesson plan view; rather, it is offered as a guide to inform lesson planning.

Differentiation

Interdisciplinary Connections

Additional Resources

Textbook: Houghton Mifflin Math

Created By

Copy of Math Grade 5 - Module 3

Subject	Grade	Module	Suggested Timeline
Mathematics	5	3	7 weeks

Grade Level Summary

In grade 5, mathematics is about (1) developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to two-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; and (3) developing understanding of volume.

Grade Level Modules

Module 1: Whole Number and Decimal Fraction Place Value to the One-Thousandths
Module 2: Multi-Digit Whole Number and Decimal Fraction Operations
Module 3: Addition and Subtraction of Fractions
Module 4: Multiplication and Division of Fractions and Decimal Fractions
Module 5: Addition and Multiplication with Volume and Area
Module 6: Graph Points on the Coordinate Plane to Solve Problems

Module Title

Module 3: Addition and Subtraction of Fractions

Module Overview

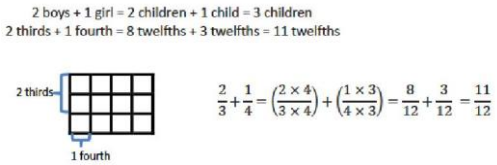
Work with place value units in the first two modules paves the path to fractions and arithmetic with fractions in Module 3 as elementary math's place value emphasis shifts to a focus on the larger set of fractional units for algebra. Like units are added to and subtracted from like units:

$1.5 + 0.8 = 1\frac{5}{10} + \frac{8}{10} = 15 \text{ tenths} + 8 \text{ tenths} = 23 \text{ tenths} = 2 \text{ and } 3 \text{ tenths} = 2\frac{3}{10} = 2.3$

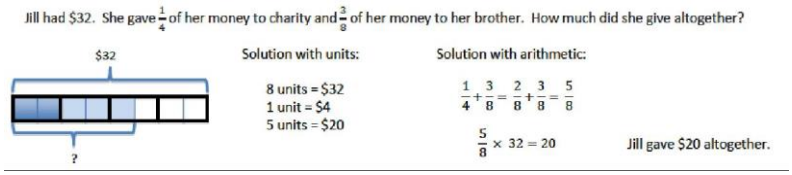
$1\frac{5}{9} + \frac{8}{9} = 14 \text{ ninths} + 8 \text{ ninths} = 22 \text{ ninths} = 2 \text{ and } 4 \text{ ninths} = 2\frac{4}{9}$

The new complexity is that if units are not equivalent, they must be changed for smaller equal units so that they can be added or subtracted. Probably the best model for showing this is the rectangular fraction model pictured below. The

equivalence is then represented symbolically as students engage in active meaning making rather than obeying the perhaps mysterious command to "multiply the top and bottom by the same number"



Relating different fractional units to one another requires extensive work with area and number line diagrams. Tape diagrams are used often in word problems. Tape diagrams, which students began using in the early grades and which become increasingly useful as students applied them to a greater and greater variety of word problems, hit their full strength as a model when applied to fraction word problems. At the heart of a tape diagram is the now-familiar idea of forming units. In fact, forming units to solve word problems is one of the most powerful examples of the unit theme and is particularly helpful for understanding fraction arithmetic,as in the following example:



Module Objectives

At the end of this module, students will be able to independently use their learning to :

- **Add and subtract fractions (including mixed numbers) with unlike denominators**
- **Solve problems using computation of fractions by using information presented in line plots**

Standards for Mathematical Practice

MP# 1. Make sense of problems and persevere in solving them
MP# 2. Reason abstractly and quantitatively
MP# 4. Model with mathematics
MP# 5. Use appropriate tools strategically
MP# 6. Attend to precision
MP# 7. Look for and make use of structure (Deductive Reasoning)
MP# 8. Look for and express regularity in repeated reasoning

Focus Standards Addressed in this Module

CC.2.1.5.C.1

Use the understanding of equivalency to add and subtract fractions.

Important Standards Addressed in this Module

CC.2.4.5.A.4

Solve problems involving computation of fractions using information provided in a line plot.

Misconceptions	Proper Conceptions
1.Students often mix models when adding, subtracting or comparing fractions. Students will use a circle for thirds and a rectangle for fourths when comparing fractions with thirds and fourths.	1.Remind students that the representations need to be from the same whole models with the same shape and size.

Concepts	Competencies	Vocabulary
•Fractions	•Add, Subtract, Multiply and Divide fractions to solve problems. •Explain operations as they pertain to fractions. •Solve problems involving computation with fractions using information obtained from data displays.	Addend Associative Property of Addition Benchmark fractions Common denominators Common multiples Commutative Property of Addition Denominator Difference Equivalent fractions Estimate Fraction Fraction greater than 1 Fraction less than 1 Like denominators Lowest terms Minuend Mixed number Numerator Reasonableness Simplest form Simplify Subtrahend Sum Unit Fractions Unlike denominators

Assessment(s)

Daily Practice Sheets

Oral Responses

Daily Math Journal/Open Ended Responses

Pre-Test/Review

End of Chapter Test

Suggested Strategies to Support Design of Coherent Instruction

Add and Subtract Fractions

To add or subtract fractions with unlike denominators, students use their understanding of equivalent fractions to create fractions with the same denominators. Start with problems that require the changing of one of the fractions and progress to changing both fractions. Allow students to add and subtract fractions using different strategies such as number lines, area models, fraction bars or strips. Have students share their strategies and discuss commonalities in them.

Students need to develop the understanding that when adding or subtracting fractions, the fractions must refer to the same whole. Any models used must refer to the same whole. Students may find that a circular model might not be the best model when adding or subtracting fractions.

As with solving word problems with whole number operations, regularly present word problems involving addition or subtraction of fractions. The concept of adding or subtracting fractions with unlike denominators will develop through solving problems. Mental computations and estimation strategies should be used to determine the reasonableness of answers. Students need to prove or disprove whether an answer provided for a problem is reasonable.

Estimation is about getting useful answers, it is not about getting the right answer. It is important for students to learn which strategy to use for estimation. Students need to think about what might be a close answer.

Line Plot

Using a line plot to solve problems involving operations with unit fractions now includes multiplication and division. Revisit using a number line to solve multiplication and division problems with whole numbers. In addition to knowing how to use a number line to solve problems, students also need to know which operation to use to solve problems.

Use the tables for common addition and subtraction, and multiplication and division situations as a guide to the types of problems students need to solve without specifying the type of problem. Allow students to share methods used to solve the problems. Also have students create problems to show their understanding of the meaning of each operation.

Note: This section is not designed to provide a day-to-day lesson plan view; rather, it is offered as a guide to inform lesson planning.

Differentiation

Interdisciplinary Connections

Additional Resources

- **Textbook: Houghton Mifflin Math**

Created By

Copy of Math Grade 5 - Module 4

Subject	Grade	Module	Suggested Timeline
Mathematics	5	4	7 weeks

Grade Level Summary

In grade 5, mathematics is about (1) developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to two-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; and (3) developing understanding of volume.

Grade Level Modules

Module 1: Whole Number and Decimal Fraction Place Value to the One-Thousandths
Module 2: Multi-Digit Whole Number and Decimal Fraction Operations
Module 3: Addition and Subtraction of Fractions
Module 4: Multiplication and Division of Fractions and Decimal Fractions
Module 5: Addition and Multiplication with Volume and Area
Module 6: Graph Points on the Coordinate Plane to Solve Problems

Module Title

Module 4: Multiplication and Division of Fractions and Decimal Fractions

Module Overview

Near the end of Module 4 students know enough about fractions and whole number operations to begin to explore multi-digit decimal multiplication and division. In multiplying 2.1 x 3.8, for example, students now have multiple skills and strategies that they can use to locate the decimal point in the final answer.

Similar strategies enrich students' understanding of division and help them to see multi-digit decimal division as whole number division in a different unit. For example, we divide to find, "How many groups of 3 apples are there in 45 apples?" and write 45 apples ÷ 3 apples = 15. Similarly, 4.5 ÷ 0.3 can be written as "45 tenths ÷ 3 tenths" with the same answer: There are 15 groups of 0.3 in 4.5. This idea was used to introduce fraction division earlier in the module, thus gluing division to whole numbers, fractions and decimals together through an

understanding of units.

Module Objectives

At the end of this module, students will be able to independently use thier learning to:

- **Solve word problems, including division of whole numbers, leading to answers in the form of fractions**
- **Multiply a fraction and mixed numbers by a fraction**
- **Demonstrate an understanding of multiplication as scaling/resizing**
- **Divide unit fractions by whole numbers and whole numbers by unit fractions**
- **Convert among different sized measurement units within a given measurement system using a provided table of equivalencies**
- **Solve problems involving computation of fractions by using information presented in line plots**
- **Display and interpret data shown in tallies, tables, charts, pictographs, bar graphs, and line graphs**
- **Display and interpret data using the title, appropriate scale, and labels**

Standards for Mathematical Practice

MP# 1. Make sense of problems and persevere in solving them
MP# 2. Reason abstractly and quantitatively
MP# 4. Model with mathematics
MP# 5. Use appropriate tools strategically
MP# 6. Attend to precision
MP# 7. Look for and make use of structure (Deductive Reasoning)
MP# 8. Look for and express regularity in repeated reasoning

Focus Standards Addressed in this Module

CC.2.1.5.C.2

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Important Standards Addressed in this Module

CC.2.4.5.A.1

Solve problems using conversions within a given measurement system.

CC.2.4.5.A.2

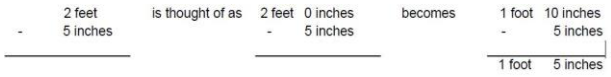
Represent and interpret data using appropriate scale.

CC.2.4.5.A.4

Solve problems involving computation of fractions using information provided in a line plot.

Misconceptions

1. **Students may believe that multiplication always results in a larger number.**
2. **Additionally, students may believe that division always results in a smaller number.**
3. **When solving problems that require renaming units, students use their knowledge of renaming the numbers as with whole numbers. Students need to pay attention to the unit of measurement which dictates the renaming and the number to use. The same procedures used in renaming whole numbers should not be taught when solving problems involving measurement conversions. For example, when subtracting 5 inches from 2 feet, students may take one foot from the 2 feet and use it as 10 inches. Since there were no inches with the 2 feet, they put 1 with 0 inches and make it 10 inches.**
4. **Since there were no inches with the 2 feet, they put 1 with 0 inches and make it 10 inches.**



Proper Conceptions

1. **Using models when multiplying with fractions will enable students to see that the results will be smaller.**
2. **Using models when dividing with fractions will enable students to see that the results will be larger.**

Concepts

- **Fractions**
- **Data Displays**

Competencies

- **Add, Subtract, Multiply and Divide fractions to solve problems.**
- **Explain operations as they pertain to fractions.**
- **Organize and display data in order to answer questions.**
- **Represent and interpret**

Vocabulary

Area
Area model
Array
Capacity
Centimeter
Compatible numbers
Cup

data using appropriate scale.

- **Solve problems involving computation with fractions using information obtained from data displays.**

Customary system
Decimal
Decimeter
Dekameter
Denominator
Distributive Property
Dividend
Divisor
Elapsed time
Equation
Equivalent fractions
Estimate
Factor
Fluid ounce
Foot
Fraction bar
Fraction greater than 1
Fraction less than 1
Gallon
Gram
Inch
Inverse operations
Kilogram
Kilometer
Liter
Long division
Mass
Measurement Systems
Measurement Units
Meter
Metric system
Mile
Milligram
Milliliter
Millimeter
Mixed number
Multiplicative Identity Property of 1
Numerator
Ounce
Partial quotients
Pint
Place value
Pound
Product
Quart

Quotient
Rectangle
Remainder
Scaling (resizing)
Simplest form
Simplify
Square unit
Ton
Unit Fraction
Weight
Whole numbers
Yard

Assessment(s)

Daily Practice Sheets

Oral Responses

Daily Math Journal/Open Ended Responses

Pre-test/Review

End of the Chapter Test

Suggested Strategies to Support Design of Coherent Instruction

Connect the meaning of multiplication and division of fractions with whole-number multiplication and division. Consider area models of multiplication and both sharing and measuring models for division.

In multiplying 2.1 x 3.8, for example, students now have multiple skills and strategies that they can use to locate the decimal point in the final answer:

- **Unit awareness: 2.1 x 3.8 = 21 tenths x 38 tenths = 798 hundredths**
- **Estimation (through rounding): 2.1 x 3.8 ≈ 2 x 4 = 8, so 2.1 x 3.8 = 7.98**
- **Fraction multiplication: 21/10 x 38/10 = (21 x 38)/(10 x 10)**

Similar strategies enrich students' understanding of division and help them to see multi-digit decimal division as whole number division in a different unit. For example, we divide to find, "How many groups of 3 apples are there in 45 apples?" and write 45 apples ÷ 3 apples = 15. Similarly, 4.5 ÷ 0.3 can be written as "45 tenths ÷ 3 tenths" with the same answer: There are 15 groups of 0.3 in 4.5. This idea was used to introduce fraction division earlier in the module, thus gluing division to whole numbers, fractions, and decimals together through an understanding of units.

Encourage students to use models or drawings to multiply or divide with

fractions. Begin with students modeling multiplication and division with whole numbers. Have them explain how they used the model or drawing to arrive at the solution.

Models to consider when multiplying or dividing fractions include, but are not limited to: area models, using rectangles or squares, fraction strips/bars, and sets of counters.

Use calculators or models to explain what happens to the result of multiplying a whole number by a fraction (3 x 12, 4 x 12, 5 x 12…and 4 x 12, 4 x 13, 4 x 14…) and when multiplying a fraction by a number greater than 1.

Use calculators or models to explain what happens to the result when dividing a unit fraction by a non-zero whole number (18 ÷ 4, 18 ÷ 8, 18 ÷16) and what happens to the results when dividing a whole number by a unit fraction (4 ÷14, 8 ÷ 14, 12 ÷ 14).

Present problem situations and have students use models and equations to solve the problem. It is important for all students to develop understanding of multiplication and division of fractions through contextual situations.

Note: This section is not designed to provide a day-to-day lesson plan view; rather, it is offered as a guide to inform lesson planning.

Differentiation

Interdisciplinary Connections

Additional Resources

- **Textbook: Houghton Mifflin Math**

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Copy of Math Grade 5 - Module 5

Subject	Grade	Module	Suggested Timeline
Mathematics	5	5	6 weeks

Grade Level Summary

In grade 5, mathematics is about (1) developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to two-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; and (3) developing understanding of volume.

Grade Level Modules

Module 1: Whole Number and Decimal Fraction Place Value to the One-Thousandths
Module 2: Multi-Digit Whole Number and Decimal Fraction Operations
Module 3: Addition and Subtraction of Fractions
Module 4: Multiplication and Division of Fractions and Decimal Fractions
Module 5: Addition and Multiplication with Volume and Area
Module 6: Graph Points on the Coordinate Plane to Solve Problems

Module Title

Module 5: Addition and Multiplication with Volume and Area

Module Overview

Through the daily use of area models, the fraction module prepares students for an in-depth discussion of area and volume in Module 5. But the module on area and volume also reinforces work done in the fraction module: Now, questions about how the area changes when a rectangle is scaled by a whole or fractional scale factor may be asked. Measuring volume once again highlights the unit theme, as a unit cube is chosen to represent a volume unit and used to measure the volume of simple shapes composed out of rectangular prisms.

Module Objectives

At the end of this module, students will be able to independently use their learning to :

- **Classify two dimensional figures in a hierarchy based on properties**
- **Apply the formulas V = l x w x h and V = B x h for rectangular prisms to find volumes of right rectangular prisms with whole number edge lengths in the context of solving real world and mathematical problems given the appropriate formula**
- **Find volumes of solid figures composed of two non-overlapping right rectangular prisms**

Standards for Mathematical Practice

MP# 1. Make sense of problems and persevere in solving them
MP# 2. Reason abstractly and quantitatively
MP# 4. Model with mathematics
MP# 5. Use appropriate tools strategically
MP# 6. Attend to precision
MP# 7. Look for and make use of structure (Deductive Reasoning)
MP# 8. Look for and express regularity in repeated reasoning

Focus Standards Addressed in this Module

CC.2.3.5.A.2

Classify two-dimensional figures into categories based on an understanding of their properties.

CC.2.3.5.A.5

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Important Standards Addressed in this Module

Misconceptions	Proper Conceptions
1. **Students think that when describing geometric shapes and placing them in subcategories, the last category is the only classification that can be used.**	1. **Clarify the properties of each classification, reinforcing the idea that shapes can "fit" into more than one classification based on properties.**

Concepts	Competencies	Vocabulary
• **Two Dimensional Figures** • **Measurement** • **Volume**	• **Relate volume to multiplication and to addition.** • **Classify two-dimensional figures based on their properties.**	**Acute triangle** **Associative Property of Multiplication** **Attribute** **Base of a solid figure** **Congruent**

Cubic unit
Decagon
Decagonal prism
Equilateral triangle
Formula
Isosceles triangle
Hierarchy
Heptagon
Hexagon
Hexagonal prism
Lateral face
Measurement Systems
Measurement Unit
Nonagon
Obtuse triangle
Octagon
Octagonal prism
Parallel lines
Parallelogram
Pentagon
Pentagonal prism
Pentagonal pyramid
Perpendicular lines
Polygon
Polyhedron
Prism
Pyramid
Quadrilateral
Rectangle
Regular polygon
Rhombus
Right rectangular prism
Right triangle
Scalene triangle
Solid figure
Three-dimensional figures
Trapezoid
Two-dimensional figures
Unit cube
Volume

Assessment(s)

Daily Practice Sheets

Oral Responses

Daily Math Journal/Open Ended Responses

Pre-Test/Review

End of Chapter Tests

Suggested Strategies to Support Design of Coherent Instruction

Instructional Strategies for Volume

Volume refers to the amount of space that an object takes up and is measured in cubic units such as cubic inches or cubic centimeters.

Students need to experience finding the volume of rectangular prisms by counting unit cubes, in metric and standard units of measure, before the formula is presented. Provide multiple opportunities for students to develop the formula for the volume of a rectangular prism with activities similar to the one described below.

Give students one block (a l- or 2- cubic centimeter or cubic-inch cube), a ruler with the appropriate measure based on the type of cube, and a small rectangular box. Ask students to determine the number of cubes needed to fill the box. Have students share their strategies with the class using words, drawings or numbers. Allow them to confirm the volume of the box by filling the box with cubes of the samesize.
By stacking geometric solids with cubic units in layers, students can begin understanding the concept of how addition plays a part in finding volume. This will lead to an understanding ofthe formula for the volume of a right rectangular prism, b x h, where b is the area of the base. A right rectangular prism has three pairs of parallel faces that are all rectangles.

Have students build a prism in layers. Then, have students determine the number of cubes in the bottom layer and share their strategies. Students should use multiplication based on their knowledge of arrays and its use in multiplying two whole numbers.

Ask what strategies can be used to determine the volume of the prism based on the number of cubes in the bottom layer. Expect responses such as "adding the same number of cubes in each layer as were on the bottom layer" or multiply the number of cubes in one layer times the number of layers.

Instructional Resources/Tools

Cubes
Rulers (marked in standard or metric units)
Grid paper

Instructional Strategies for Two-Dimensional Shapes:

This cluster builds from Grade 3 when students described, analyzed and compared properties of two-dimensional shapes. They compared and classified shapes by their sides and angles, and connected these with definitions of shapes. In Grade 4 students built, drew and analyzed two-dimensional shapes to deepen their understanding of the properties of two-dimensional shapes. They looked at the presence or absence of parallel and perpendicular lines or the presence or absence of angles of a specified size to classify two-dimensional shapes. Now, students classify two-dimensional shapes in a hierarchy based on properties. Details learned in earlier grades need to be used in the descriptions of the attributes of shapes. The more ways that students can classify and discriminate shapes, the better they can understand them. The shapes are not limited to quadrilaterals.

Students can use graphic organizers such as flow charts or T-charts to compare and contrast the attributes of geometric figures. Have students create a T-chart with a shape on each side. Have them list attributes of the shapes, such as number of side, number of angles, types of lines, etc. they need to determine what's alike or different about the two shapes to get a larger classification for the shapes.

Pose questions such as, "Why is a square always a rectangle?" and "Why is a rectangle not always a square?"

Note: This section is not designed to provide a day-to-day lesson plan view; rather, it is offered as a guide to inform lesson planning.

Differentiation

Interdisciplinary Connections

Additional Resources

- **Textbook: Houghton Mifflin Math**

Created By

Copy of Math Grade 5 - Module 6

Subject	Grade	Module	Suggested Timeline
Mathematics	5	6	6 weeks

Grade Level Summary

In grade 5, mathematics is about (1) developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to two-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; and (3) developing understanding of volume.

Grade Level Modules

Module 1: Whole Number and Decimal Fraction Place Value to the One-Thousandths
Module 2: Multi-Digit Whole Number and Decimal Fraction Operations
Module 3: Addition and Subtraction of Fractions
Module 4: Multiplication and Division of Fractions and Decimal Fractions
Module 5: Addition and Multiplication with Volume and Area
Module 6: Graph Points on the Coordinate Plane to Solve Problems

Module Title

Module 6: Graph Points on the Coordinate Plane to Solve Problems

Module Overview

Scaling is revisited in the last module on the coordinate plane. Ever since the growth and shrinking patterns were first introduced in Kindergarten, students have been using bar graphs to display data and patterns. Extensive bar-graph work has set the stage for line plots, which are both the natural extension of bar graphs and the precursor to linear functions. It is in this final module of K-5 that a simple line plot of a straight line is presented on a coordinate plane and students are asked about the scaling relationship between the increases in the units of the vertical axis for 1 unit of increase in the horizontal axis. This is the first hint of slope and marks the beginning of the major theme of middle school: ratios and proportions.

Module Objectives

At the end of this module, students will be able to independently use their learning to :

- **Generate two numerical patterns using two given rules**
- **Identify apparent relationships between corresponding terms of two patterns with the same starting numbers that follow different rules**
- **Identify parts of the coordinate plane (x-axis, y-axis, and the origin) and the ordered pair (x-coordinate and y-coordinate). Limit the coordinate plane to quadrant I**
- **Represent real-world and mathematical problems by plotting points in quadrant I of the coordinate plane, and interpret coordinate values of points in the context of a situation**
- **Solve problems involving computation of fractions by using information presented in line plots**
- **Display and interpret data shown in tallies, tables, charts, pictographs, bar graphs, and line graphs**
- **Display and interpret data using a title, appropriate scale, and labels**

Standards for Mathematical Practice

MP# 1. Make sense of problems and persevere in solving them
MP# 2. Reason abstractly and quantitatively
MP# 4. Model with mathematics
MP# 5. Use appropriate tools strategically
MP# 6. Attend to precision
MP# 7. Look for and make use of structure (Deductive Reasoning)
MP# 8. Look for and express regularity in repeated reasoning

Focus Standards Addressed in this Module

CC.2.2.5.A.4

Analyze patterns and relationships using two rules.

CC.2.3.5.A.1

Graph points in the first quadrant on the coordinate plane and interpret these points when solving real world and mathematical problems.

Important Standards Addressed in this Module

CC.2.4.5.A.2

Represent and interpret data using appropriate scale.

CC.2.4.5.A.4

Solve problems involving computation of fractions using information provided in a line plot.

Misconceptions	Proper Conceptions
1.Students reverse the points when plotting them on the coordinate plane. They count up first on the y-axis and then over on the x-axis.	1.The location of every point in the plane has a specific place. Have students plot points where the numbers are reversed, such as (4, 5) and (5, 4). Begin with students providing a verbal description of how to plot each point. Then, have them follow the verbal description and plot each point.

Concepts	Competencies	Vocabulary
•Patterns •Coordinate Plane •Measurement •Data Displays	•Generate, analyze and compare patterns. •Plot points in quadrant I. •Describe and interpret points given an ordered pair. •Identify parts of a coordinate grid. •Organize and display data in order to answer questions. •Represent and interpret data using appropriate scale. •Solve problems involving computation with fractions using information obtained from data displays.	Axis (axes) Coordinate plane Coordinate system Coordinates Corresponding terms Data Fraction Intersect Interval Line graph Line plot Ordered pair Origin Perpendicular Plane Quadrants Scale Sequence Unit fraction X-axis X-coordinate Y-axis Y-coordinate

Assessment(s)

Daily Practice Sheets

Oral Responses

Daily Math Journal/Open Ended Responses

Pre-Test/Review

End of Chapter Test

Suggested Strategies to Support Design of Coherent Instruction

Students need to understand the underlying structure of the coordinate system and see how axes make it possible to locate points anywhere on a coordinate plane. This is the first time students are working with coordinate planes, and only in the first quadrant. It is important that students create the coordinate grid themselves. This can be related to two number lines and reliance on previous experiences with moving along a number line.

Multiple experiences with plotting points are needed. Provide points plotted on a grid and have students name and write the ordered pair. Have students describe how to get to the location. Encourage students to articulate directions as they plot points.

Present real-world and mathematical problems and have students graph points in the first quadrant of the coordinate plane. Gathering and graphing data is a valuable experience for students. It helps them to develop an understanding of coordinates and what the overall graph represents. Students also need to analyze the graph by interpreting the coordinate values in the context of the situation.

Note: This section is not designed to provide a day-to-day lesson plan view; rather, it is offered as a guide to inform lesson planning.

Differentiation

Interdisciplinary Connections

Additional Resources

- **Textbook: Houghton Mifflin Math**

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