Copy of Math Grade 3 - Module 1

Subject	Grade	Module	Suggested Timeline
Mathematics	3	1	6 weeks

Grade Level Summary

In grade 3, instructional time should focus on five critical areas: (1) developing understanding of multiplication and division and strategies for multiplication and division within 100 (2) developing understanding of fractions, especially unit fractions (fractions with numerator 1); (3) developing understanding of the structure of rectangular arrays and of area; (4) describing and analyzing two-dimensional shapes; and (5) solving problems involving measurement and estimation of intervals of time, money, liquid volumes, masses, and lengths of objects.

Grade Level Modules

Module 1: Multiplication and Division with Factors of 2,3,4, 5, and 10
Module 2: Problem Solving with Mass, Time, Capacity, Length, and Money
Module 3: Multiplication and Division with Factors of 6,7,8, and 9
Module 4: Multiplication and Area
Module 5: Fractions as Numbers on the Number Line
Module 6: Collecting and Displaying Data
Module 7: Word Problems with Geometry and Measurement

Module Title

Module 1: Multiplication and Division with Factors of 2,3,4,5, and 10

Module Overview

This module builds upon the foundation of multiplicative thinking with units started in grade 2. First, students concentrate on the meaning of multiplication and division and begin developing fluency for learning products and representing and solving problems involving multiplication and division involving factors of 2, 3, 4, 5, and 10. The restricted set of facts keeps learning manageable, and also provides enough examples to do one- and two-step word problems and to start measurement problems involving weight, money, length, capacities, and time in the second module.

Module Objectives

At the end of this module, students will be able to independently use their learning to:

- **Interpret and/or describe products of whole numbers fluently within 100 (up to and including 10 X 10)**
- **Interpret and/or describe whole-number quotients of whole numbers (limit dividends through 50, and limit divisors and quotients through 10).**
- **Apply the commutative property of multiplication (not identification or definition of the property)**
- **Apply the associative property of multiplication (not identification or definition of the property)**
- **Solve two-step word problems using the four operations (expressions are not explicitly stated). Limit to problems with whole numbers and having whole-number answers**
- **Represent two-step word problems using equations with a symbol standing for the unknown quantity. Limit to problems with whole numbers and having whole-number answers**
- **Assess the reasonableness of answers. Limit problems posed with whole numbers and having whole-number answers**
- **Solve two-step equations using order of operations (equation is explicitly stated with no grouping symbols)**
- **Identify arithmetic patterns (including patterns in the addition table or multiplication table) and/or explain those using properties of operations**
- **Create or match a story to a given combination of symbols and numbers**
- **Identify the missing symbol that makes a number sentence true**
- **Use multiplication (up to and including 10 X 10) and/or division (limit dividends through 50, and limit divisors and quotients through 10) to solve word problems in situations involving equal groups, arrays, and/or measurement quantities**
- **Determine the unknown whole number in a multiplication (up to and including 10 X 10) or division (limit dividends through 50), and limit divisors and quotients through 10) equation relating three whole numbers**
- **Interpret and/or model division as a multiplication equation with an unknown factor**

Standards for Mathematical Practice

MP# 1. Make sense of problems and persevere in solving them
MP# 2. Reason abstractly and quantitatively
MP# 3. Construct viable arguments and critique the reasoning of others
MP# 4. Model with mathematics
MP# 5. Use appropriate tools strategically
MP# 6. Attend to precision
MP# 7. Look for and make use of structure (Deductive Reasoning)
MP# 8. Look for and express regularity in repeated reasoning (Inductive Reasoning)

Mathematical Practices resource page on SAS

Focus Standards Addressed in this Module

CC.2.2.3.A.1

Represent and solve problems involving multiplication and division.

CC.2.2.3.A.2

Understand properties of multiplication and the relationship between multiplication and division.

CC.2.2.3.A.3

Demonstrate multiplication and division fluency.

Important Standards Addressed in this Module

CC.2.1.3.B.1

Apply place value understanding and properties of operations to perform multi-digit arithmetic.

CC.2.2.3.A.4

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

Misconceptions

1. **Students think a symbol (? or []) is always the place for the answer. This is especially true when the problem is written as 15 ÷ 3 =? or 15 = x 3.**
2. **Students also think that 3 ÷ 15 = 5 and 15 ÷ 3 = 5 are the same equations.**
3. **The use of a symbol to represent a number once cannot be used to represent another number in a different problem/situation.**

Proper Conceptions

1. ***The use of models is essential in helping students eliminate this understanding.***
2. **Presenting students with multiple situations in which they select the symbol and explain its meaning.**
3. **What the symbol represents will counter this misconception.**

Concepts

- **Place Value and Properties of Operations**

Competencies

- **Demonstrate an understanding of the relationship between multiplication and division.**
- **Demonstrate fluency.**
- **Represent and solve problems.**

Vocabulary

Division
Estimate
Multiplication
Associative Property of Addition
Quotient
Bar Model

- **Identify and explain patterns in arithmetic (including addition and subtraction).**

Base Ten Numeral Form
Base Ten Numerals
Column
Commutative Property of Addition
Compatible Numbers
Difference
Digit
Equation
Estimate
Even Number
Expanded Form
Hundreds
Factor
Multiple
Inverse Operations
Number Line
Odd number
Ones
Order of Operations
Parentheses
Place Value
Reasonableness
Regroup
Round a whole number
Row
Standard form
Subtract
Tens
Variable
Even number
Fact Family
Multiplicative Identity Property of 1
Multiply
Odd number
Product
Related facts
Row
Subtract
Sum
Zero Property of Multiplication
Product
Pattern
Unknown

Square Number
Fact Family
Array
Square numbers
Infinite

Assessment(s)

The following link represents sample questions relevant to the instruction of the module
Math Grd 3 Mod 1_Assessment_Draft 2013.docx

Multiplication Concepts Test

Multiplication Facts and Patterns Test

Quick math basic fact tests

Suggested Strategies to Support Design of Coherent Instruction

Introduce by giving students a large number of objects, or a sheet with 100+ objects on it, and have them work with a partner to create a strategy to count them. Students will likely group or skip count to find the number easier…this leads into the faster calculation of and the big idea of calculation for expediency.

Process: Gather information, Make conjecture, Formulate models, Invent Counter Examples, Build sound Arguments

Thinking/Reasoning Mathematically: Mind mapping, Identify the array that represents a multiplication problem, Perspective, Make conjecture of patterns, Formulate a pictorial representation of multiplication. "Act out" a demonstration of how: *Multiplication is Duplication, multiplication is repeated addition, multiplication involves repeated patterns.*

Formulate a model: using manipulatives, act out division situations.

Gather information to conclude/prove multiplication means "of".

Create paper models: folding 24 divided by 4 = 6; Therefore ¼ of 24 = 6

Student Journal: Explain the relationship between multiplication and division.

Frame Questions to probe students' critical thinking/conceptual understanding:

- ***Describe a context in which a total number of objects can be expressed as b X c.***
- ***Describe a context in which a number of shares or a number of groups can be expressed as 36 divided by 4.***
- ***Use drawings and equations with a symbol for the unknown number to represent the problem.***

- ***Determine the unknown number that makes the equation true in each of the equations 6 X? = 48, 4 = _ divided by 4, 5 X 5 =?***
- ***Apply properties: If 4 X 5 is known, then 5 X4 is also known.***
- ***3 X 5 X 2 can be found by 3 X 5 = 15, then 15 X 2 = 30, or by 5 X 2 = 10, then 3 X 10 = 30.***
- ***Knowing that 8 X 5 = 40 and 8 X 2 = 16, one can find 8 X 7 as 8 X (5 + 2) = (8 X 5) + (8 X 2) = 40 divided by 16 = 56.***
- ***Practice division as an unknown-factor problem. Find 32 divided by 8 by finding the number that makes 32 when multiplied by 8.***
- ***Use the relationship between multiplication and division or properties of operations to fluently multiply and divide within 100.***
- ***Represent two-step word problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.***
- ***Identify arithmetic patterns (including patterns in the addition table or multiplication table) and explain those using properties of operations.***
- ***Observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.***

They may check their thinking by asking themselves, "Does this make sense?" They listen to the strategies of others and will try different approaches. They often will use another method to check their answers. What number model could you construct to represent the problem?

- **How would you describe the problem in your own words?**
- **How would you describe what you are trying to find?**
- **What do you notice about...?**
- **What information is given in the problem?**
- **Describe the relationship between the quantities.**
- **Describe what you have already tried. What might you change?**
- **Talk me through the steps you've used to this point.**
- **What steps in the process are you most confident about?**
- **What are some other strategies you might try?**
- **What are some other problems that are similar to this one?**
- **How might you use one of your previous problems to help you begin?**
- **How else might you organize...represent... show...?**
- **Help students to recognize multiplication as a means to determine the total number of objects when there are a specific number of groups with the same number of objects in each group. Multiplication requires students to think in terms of groups of things rather than individual things.**
- **Help students to see multiplication as "groups of" and problems such as 3 X 4 refer to 3 groups of 4.**
- **Multiplication is often thought of as repeated addition of equal groups. While this definition works for some sets of numbers, it is not particularly intuitive or meaningful when we think of multiplying 3by 1/2, for example,**

or 5 by -2. In such cases, it may be helpful to widen the idea of grouping to includeevaluation of part of a group. This concept is related to partitioning (which, in turn, is related to division).Ex: three groups of five students can be read as 3 • 5, or 15 students, while half a group of 10 stars can berepresented as 1/2 • 10, or 5 stars. These are examples of partitioning; each one of the three groups offive is part of the group of 15, and the group of 5 stars is part of the group of 10.

- **A second concept of multiplication is that of rate or price. Ex: If a car travels four hours at 50 miles per hour, then it travels a total of 4 X 50, or 200 miles; if CDs cost eight dollars each, then three CDs will cost 3 X $8, or $24.**
- **A third concept of multiplication is that of multiplicative comparison. Ex: Sara has four CDs,**
- **Joanne has three times as many as Sara, and Sylvia has half as many as Sara. Thus, Joanne has 3 X 4, or12 CDs and Sylvia has 1/2 X 4, or 2 CDs.**
- **Help students to express/ demonstrate learning objectives in student-friendly "I Can…" statements:**

I Can Use Multiplication and Division to Help Me Understand Math:

□□I can understand multiplication by thinking about groups of objects.

□□I can understand division by thinking about how one group can be divided into smaller groups.

□□I can use what I know about multiplication and division to solve word problems.

□□I can find the missing number in a multiplication or division equation.

□□I can figure out that if 6 x 4 = 24, then 4 x 6 = 24.

□□I can figure out 3 x 5 x 2. I can multiply 3 x 5 = 15, then 15 x 2 = 30 OR multiply 5 x 2 = 10, then 3 x 10 = 30.)

□□I can use the Distributive property of multiplication. (To figure out 8 x 7, I can think of 8 x (5 + 2) which means (8 x 5) + (8 x 2) = 40 + 16 = 56.)

□□I can find the answer to a division problem by thinking of the missing factor in a multiplication problem. (I can figure out 32 □ 8 because I know that 8 x 4 = 32.)

□□I can multiply and divide within 100 easily and quickly because I know how multiplication and division are related.

□□I can use addition, subtraction, multiplication and division to solve all kinds of word problems and then use mental math to decide if my answers are reasonable.

□□I can find patterns in multiplication tables and explain them using what I

know about how numbers work.

Note: This section is not designed to provide a day-to-day lesson plan view; rather, it is offered as a guide to inform lesson planning.

Differentiation

Interdisciplinary Connections

Additional Resources

Instructional Resources/Tools

- **SAS Voluntary Model Curriculum**
- **NLVM**
- **3rd Grade Common Core State Standards**
- **Lessons designed to "illuminate" the new NCTM Principles and Standards for School Mathematics.**
- ***PARCC* is a 19-state consortium working together to develop next-generation K-12 assessments in math.**
- **Math Lab Manipulatives**
- **Houghton Mifflin Text**
- **Simple Solutions**

Created By

Mercer 3rd Grade Team

Copy of Math Grade 3 - Module 2

Subject	Grade	Module	Suggested Timeline
Mathematics	3	2	6 weeks

Grade Level Summary

In grade 3, instructional time should focus on five critical areas: (1) developing understanding of multiplication and division and strategies for multiplication and division within 100 (2) developing understanding of fractions, especially unit fractions (fractions with numerator 1); (3) developing understanding of the structure of rectangular arrays and of area; (4) describing and analyzing two-dimensional shapes; and (5) solving problems involving measurement and estimation of intervals of time, money, liquid volumes, masses, and lengths of objects.

Grade Level Modules

Module 1: Multiplication and Division with Factors of 2,3, 4, 5, and 10
Module 2: Problem Solving with Mass, Time, Capacity, Length, and Money
Module 3: Multiplication and Division with Factors of 6,7,8, and 9
Module 4: Multiplication and Area
Module 5: Fractions as Numbers on the Number Line
Module 6: Collecting and Displaying Data
Module 7: Word Problems with Geometry and Measurement

Module Title

Module 2: Problem solving with Mass, Time, Capacity, Length, and Money

Module Overview

Module 2, which focuses on measurement, again provides students with internalization time for learning the 2, 3, 4, 5, and 10 facts as part of their fluency activities. Students can also take this time to work with place value, comparison and rounding concepts. The goal is to develop students' number sense well enough that they can build proportional bar diagrams used in solving word problems in Grade 3 and beyond (e.g., "If this bar represents 62 kg, then a bar representing 35 kg needs to be slightly longer than half the 62 kg bar…"). Drawing the relative sizes of the lengths of two bars also prepares students to locate fractions on a number line in Module 5 (where they learn to locate the points 1/3 and 1/5 on the number line relative to each other and relative to the whole unit).

Module Objectives

At the end of this module, students will be able to independently use their learning to:

- **Compare total values of combinations of coins (penny, nickel, dime, quarter) and/or dollar bills less than $5.00**
- **Make change for an amount up to $5.00 with no more than $2.00 change given (penny, nickel, dime, quarter, and dollar)**
- **Round amounts of money to the nearest dollar**
- **Tell, show and/or write time (analog) to the nearest minute**
- **Calculate elapsed time to the minute in a given situation (total elapsed time limited to 60 minutes or less)**
- **Measure and estimate liquid volumes and masses of objects using standard units (cups, pints, quarts, gallons, ounces, and pounds and metric units, grams, and kilograms**
- **Add, subtract, multiply, and divide to solve one-step word problems involving masses or liquid volumes that are given in the same units**
- **Use a ruler to measure lengths to the nearest quarter inch or centimeter**

Standards for Mathematical Practice

MP# 1. Make sense of problems and persevere in solving them.
MP# 2. Reason abstractly and quantitatively
MP# 4. Model with mathematics
MP# 5. Use appropriate tools strategically
MP# 6. Attend to precision
MP# 7. Look for and make use of structure (Deductive Reasoning)
MP# 8. Look for and express regularity in repeated reasoning. (Inductive Reasoning)

Mathematical Practices resource page on SAS

Focus Standards Addressed in this Module

CC.2.4.3.A.1

Solve problems involving measurement and estimation of temperature, liquid volume, mass or length.

CC.2.4.3.A.2

Tell and write time to the nearest minute and solve problems by calculating time intervals.

CC.2.4.3.A.3

Solve problems and make change involving money using a combination of coins and bills.

Important Standards Addressed in this Module

CC.2.1.3.B.1

Apply place value understanding and properties of operations to perform multi-digit arithmetic.

CC.2.2.3.A.1

Represent and solve problems involving multiplication and division.

CC.2.2.3.A.4

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

Misconceptions

1. **Students may read the mark on a scale that is below a designated number on the scale as if it was the next number. For example, a mark that is one mark below 80 grams maybe read as 81 grams. Students realize it is one away from 80, but do not think of it as 79grams.**
2. **Avoid the use of paper plate clocks. This is not adequately represented on student made clocks.**
3. **Students forget to label the measurement or choose the incorrect unit.**
4. **Students often focus on size to determine estimates of mass. They can be confused by a big fluffy object and a tiny dense object.**
5. **Students confuse whether to use metric or standard units and do not line up the ruler at the appropriate mark.**
6. **Students have difficulty making reasonable estimations.**

Proper Conceptions

1. **Students need to practice skip counting by a variety of intervals. They need to make sense of how the scale needs to be interpreted based on the labeled numbers on the scale.**
2. **Students need to see the actual relationship between the hour and the minute hand. Model explicitly.**
3. **Students need to have visual hooks of benchmarks established from experiences, which build conceptual understanding. Example: Taping a square yard on the floor, acting out a square foot using arms or yarn. Putting cubic units into a rectangular prism.**
4. **Because students cannot tell actual mass until they have handled an object, it is important that teachers do not ask students to estimate the mass of objects until they have had the opportunity to lift the objects and then make an estimate of the mass.**
5. **After modeling explicitly with the group, follow up with small group formative assessment, and then individual assessment of using a ruler correctly. First, have**

students estimate, then measure accurately to check for proper alignment, etc.

6. Students need to have practice unitizing large areas. (They need to have established reference or benchmark for a mg (grain of sand weight), an inch (a fingertip), etc.

Concepts	Competencies	Vocabulary
• Measurement • Time • Money (Coins and Bills)	• Make estimations. • Distinguish between linear and area measurements. • Solve problems. • Tell and write time to nearest minute. • Calculate time intervals. • Make change using combination of coins and bills.	Estimate Liquid volume Mass Length Temperature Celsius Fahrenheit Degree Interval Change Value Elapsed time Second Minute Hour Coin values Round Benchmark Time Weight Capacity Inches, feet, yards Cups, pints, quarts, gallons, ounces, pounds, metric units, grams, and kilograms Appropriate unit of measure Estimating measures Exact/estimate

Assessment(s)

Money Test

Time Test

Metric Measurement Test

Customary Measurement Test

Suggested Strategies to Support Design of Coherent Instruction

- **Process: Gather information, make conjecture, formulate models, invent counter examples, build sound arguments.**
- **Reading Strategies: Think aloud, Predicting, Visualization, Questioning, Compare/contrast, Imagery**
- **Math Word Wall: Engage in activity, visual + word definition placed on wall and referred to throughout year. (memory hook).**
- **Manipulative: Rulers, Tape measures, yard sticks, meter sticks, containers representing various weights, capacities, volumes and calculators.**
- **M & Ms., coupons.**
- **Speaking: Students explain methods for making a "reasonable estimate" (benchmarks, unitizing)**
- **Whole group explicit instruction of measuring, small group centers in which an adult assesses ability to measure accurately, followed by individual demonstration of ability to estimate and to measure accurately.**
- **Estimating with money, Coupon math, Department Store math.**

Provide opportunities for students to use appropriate tools to measure and estimate liquid volumes in liters only and masses of objects in grams and kilograms. Students need practice in reading the scales on measuring tools since the markings may not always be in intervals of one. The scales may be marked in intervals of two, five or ten.

Allow students to hold gram and kilogram weights in their hand to use as a benchmark. Use water colored with food coloring so that the water can be seen in a beaker. Students should estimate volumes and masses before actually finding the measuring. Show students a group containing the same kind of objects. Then, show them one of the objects and tell them its weight. Fill a container with more objects and ask students to estimate the weight of the objects. Use similar strategies with liquid measures. Be sure that students have opportunities to pour liquids into different size containers to see how much liquid will be in certain whole liters. Show students containers and ask, "How many liters do you think will fill the container?" If making several estimates, students should make an estimate, then the measurement and continue the process of estimating measure rather than all estimates and then all measures. It is important to provide feedback to students on their estimates by using measurement as a way of gaining feedback on estimates.

Appropriate use of tools and appropriate units of measure should be part of the

classroom discussion.

Instructional Strategies

- **§ Skip-count up and down by $10 between 45 and 125.**
- **(45, 55, 65, 75, 85, 95, 105, 115, 125).**
- **§ Practice "making change" by counting on from an amount up to a specified total.**
- **§ "More" and "Less" games may also be played with money (See Base-Ten Blocks).**
- **§ Play equivalency games. How many $5 bills in a $10 bill? A $20 bill? A $100 bill? etc.**

Manipulative: Student Clocks, Play money, Time and Money Flashcards, Telling Time Chips, Bingo, Quizmo, Calculators, Overhead clocks.

Reading Strategies: Think a loud, draw conclusion, graphic organizers, think/pair/share, think pad

- **Using an appropriate benchmark, students estimate lengths of objects in room.**
- **Decide upon most appropriate tool and appropriate measure.**
- **Students should justify reasoning actively, visually, and through verbal and written explanation using explicit mathematical vocabulary.**
- **Have students describe situations, orally and written, when an exact measure is needed and when an estimate is appropriate.**
- **Write a paragraph to describe the importance of measurement in real life.**
- **Have students describe situations, orally and written, when they would use benchmarks or would unitize in real life.**
- **Have students estimate, estimate, estimate…and then measure, measure, measure, precisely!**
- **Ask yourself how you are incorporating writing, speaking, listening, reading, thinking (reasoning, analyzing, problem solving, decision making, and critical thinking) and technology into your lessons daily.**
- **Ask yourself how the mass, time, capacity, length, and money concepts students are exploring in this unit will lead them to an understanding of later algebra, geometry, trigonometry, and calculus courses in the future.**

Note: This section is not designed to provide a day-to-day lesson plan view; rather, it is offered as a guide to inform lesson planning.

Differentiation

Interdisciplinary Connections

Additional Resources

Instructional Resources/Tools

- **SAS Voluntary Model Curriculum**

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Created By

Mercer 3rd Grade

Copy of Math Grade 3 - Module 3

Subject	Grade	Module	Suggested Timeline
Mathematics	3	3	7 weeks

Grade Level Summary

In grade 3, instructional time should focus on five critical areas: (1) developing understanding of multiplication and division and strategies for multiplication and division within 100 (2) developing understanding of fractions, especially unit fractions (fractions with numerator 1); (3) developing understanding of the structure of rectangular arrays and of area; (4) describing and analyzing two-dimensional shapes; and (5) solving problems involving measurement and estimation of intervals of time, money, liquid volumes, masses, and lengths of objects.

Grade Level Modules

Module 1: Multiplication and Division with Factors of 2,3, 4, 5, and 10
Module 2: Problem Solving with Mass, Time, Capacity, Length, and Money
Module 3: Multiplication and Division with Factors of 6,7,8, and 9
Module 4: Multiplication and Area
Module 5: Fractions as Numbers on the Number Line
Module 6: Collecting and Displaying Data
Module 7: Word Problems with Geometry and Measurement

Module Title

Module 3: Multiplication and Division with Factors of 6,7,8, and 9

Module Overview

Students learn the remaining multiplication and division facts in Module 3 as they continue to develop their understanding of multiplication and division strategies within 100 and use those strategies to solve two-step word problems. The “2, 3, 4, 5 and 10 facts” module (Module 1) and the “6, 7, 8 and 9 facts” module (Module 3) both provide important, sustained time for work in understanding the structure of rectangular arrays to prepare students for area in Module 4. This work is necessary because students initially find it difficult to distinguish the different squares in a rectangular array area model (the third array in the picture below), count them and recognize that the count is related to multiplication. Modules 1 and 3 slowly build up to a rectangular array area model using hands-on rectangular arrays (i.e., a Rekenrek) and/or pictures of rectangular arrays involving objects only (stars, disks, etc.)—all in the context

of learning multiplication and division.

Module Objectives

At the end of this module, students will be able to independently use their learning to:

- **Interpret and/or describe products of whole numbers fluently within 100 (up to and including 10 X 10)**
- **Interpret and/or describe whole-number quotients of whole numbers (limit dividends through 50, and limit divisors and quotients through 10)**
- **Apply the commutative property of multiplication (not identification or definition of the property)**
- **Apply the associative property of multiplication (not identification or definition of the property)**
- **Solve two-step word problems using the four operations (expressions are not explicitly stated). Limit to problems with whole numbers and having whole-number answers.**
- **Represent two-step word problems using equations with a symbol standing for the unknown quantity. Limit to problems with whole numbers and having whole-number answers.**
- **Assess the reasonableness of answers. Limit problems posed with whole numbers and having whole-number answers.**
- **Solve two-step equations using order of operations (equation is explicitly stated with no grouping symbols).**
- **Identify arithmetic patterns (including patterns in the addition table or multiplication table) and/or explain them using properties of operations.**
- **Create or match a story to a given combination of symbols and numbers.**
- **Identify the missing symbol that makes a number sentence true.**
- **Use multiplication (up to and including 10 X 10) and/or division (limit dividends through 50, and limit divisors and quotients through 10) to solve word problems in situations involving equal groups, arrays, and/or measurement quantities.**
- **Determine the unknown whole number in a multiplication (up to and including 10 X 10) or division (limit dividends through 50), and limit divisors and quotients through 10) equation relating three whole numbers.**
- **Interpret and/or model division as a multiplication equation with an unknown factor**

Standards for Mathematical Practice

MP# 1. Make sense of problems and persevere in solving them
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MP# 7. Look for and make use of structure (Deductive Reasoning)
MP# 8. Look for and express regularity in repeated reasoning (Inductive Reasoning)

Mathematical Practices resource page on SAS

Focus Standards Addressed in this Module

CC.2.2.3.A.1

Represent and solve problems involving multiplication and division.

CC.2.2.3.A.2

Understand properties of multiplication and the relationship between multiplication and division.

CC.2.2.3.A.3

Demonstrate multiplication and division fluency.

Important Standards Addressed in this Module

CC.2.1.3.B.1

Apply place value understanding and properties of operations to perform multi-digit arithmetic.

CC.2.2.3.A.2

Understand properties of multiplication and the relationship between multiplication and division.

CC.2.2.3.A.4

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

Misconceptions

1. **Students think a symbol (? or []) is always the place for the answer. This is especially true when the problem is written as 18/ 3 =? or 18= ? X 3.**
2. **Students also think that 3 ÷ 18 = 6 and 18 ÷ 3 = 6 are the same equations.**
3. **The use of a symbol to represent a number once cannot be used to represent another number in a different problem/situation.**
4. **Multiplication and Division do not BOTH follow the commutative**

Proper Conceptions

1. **The use of models is essential in helping students eliminate this understanding.**
2. **Presenting students with multiple situations in which they select the symbol and explain the meaning.**
3. **What the symbol represents will counter this misconception.**
4. **Examples of the commutative and associative properties for multiplication must be explored, and then use inquiry to have students determine if this rule will**

and associative property. Some students may both properties apply to both operations.

apply to division.

Concepts	Competencies	Vocabulary
• **Multiplication and Division**	• **Demonstrate an understanding of properties of multiplication.** • **Demonstrate an understanding of the relationship between multiplication and division.** • **Demonstrate fluency.**	**Division** **Estimate** **Multiplication** **Factor** **Product** **Pattern** **Multiple** **Unknown** **Quotient** **Commutative** **Associative** **Area Model** **Equation, Number Model, Number Sentence**

Assessment(s)

Multiplication Quick Math Tests

Division Quick Math Tests

Multiplication Concepts Test

Multiplication Facts and Patterns Test

Division Concepts Test

Division Facts and Patterns

Suggested Strategies to Support Design of Coherent Instruction

Frame Questions to probe students' critical thinking/conceptual understanding:

- ***Describe a context in which a total number of objects can be expressed as b X c.***
- ***Describe a context in which a number of shares or a number of groups can be expressed as 36 divided by 9.***
- ***Use drawings and equations with a symbol for the unknown number to represent the problem.***
- ***Determine the unknown number that makes the equation true in each of the equations 6 X? = 48, 7 = _ divided by 4, 6 X 7 =?***
- ***Apply properties: If 6 X 7 is known, then 7 X 6 is also known.***

- ***3 X 6 X 2 can be found by 3 X 6 = 18, then 18 X 2 = 36, or by 6 X 2 = 12, then 3 X 12 = 36.***
- ***Knowing that 8 X 5 = 40 and 8 X 2 = 16, one can find 8 X 7 as 8 X (5 + 2) = (8 X 5) + (8 X 2) = 40 divided by 16 = 56.***
- ***Practice division as an unknown-factor problem. Find 32 divided by 8 by finding the number that makes 32 when multiplied by 8.***
- ***Use the relationship between multiplication and division or properties of operations to fluently multiply and divide within 100.***
- ***Represent two-step word problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.***
- ***Identify arithmetic patterns (including patterns in the addition table or multiplication table) and explain those using properties of operations.***
- ***Observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.***
- **Help students to recognize multiplication as a means to determine the total number of objects when there are a specific number of groups with the same number of objects in each group. Multiplication requires students to think in terms of groups of things rather than individual things.**
- **Help students to see multiplication as "groups of" and problems such as 8 X 9 refer to 8 groups of 9. Multiplication is often thought of as repeated addition of equal groups. While this definition works for some sets of numbers, it is not particularly intuitive or meaningful when we think of multiplying 3 by 1/2, for example, or 5 by -2. In such cases, it may be helpful to widen the idea of grouping to include evaluation of part of a group. This concept is related to partitioning (which, in turn, is related to division). Ex: three groups of five students can be read as 3 • 5, or 15 students, while half a group of 10 stars can be represented as 1/2 • 10, or 5 stars. These are examples of partitioning; each one of the three groups of five is part of the group of 15, and the group of 5 stars is part of the group of 10.**

I Can Use Multiplication and Division to Help Me Understand Math:

□□I can understand multiplication by thinking about groups of objects.

□□I can understand division by thinking about how one group can be divided into smaller groups.

□□I can use what I know about multiplication and division to solve word problems.

□□I can find the missing number in a multiplication or division equation.

□□I can figure out that if 6 x 4 = 24, then 4 x 6 = 24.

□□I can figure out 3 x 5 x 2. I can multiply 3 x 5 = 15, then 15 x 2 = 30 OR

multiply 5 x 2 = 10, then 3 x 10 = 30.)

□□I can use the Distributive property of multiplication. (To figure out 8 x 7, I can think of 8 x (5 + 2) which means (8 x 5) + (8 x 2) = 40 + 16 = 56.)

□□I can find the answer to a division problem by thinking of the missing factor in a multiplication problem. (I can figure out 32 □ 8 because I know that 8 x 4 = 32.)

□□I can multiply and divide within 100 easily and quickly because I know how multiplication and division are related.

□□I can use addition, subtraction, multiplication and division to solve all kinds of word problems and then use mental math to decide if my answers are reasonable.

□□I can find patterns in multiplication tables and explain them using what I know about how numbers work.

- **Students need to explore/conceptualize/identify/model Multiplication Situations**

Product	**Group Size Unknown** **(“How many in each group? Division)**		**Number of Groups Unknown** **(“How many groups?” Division)**
3 × 6 =?	**3 ×? – 18, and 18 ÷3 =?**		**? × 6 = 18, and 18 ÷ 6 =?**
Equal Groups	**There are 3 bags with 6 plums in each bag. How many plums are there in all?** ***Measurement example*. You need 3 lengths of string, each6 inches long. How much string will you need altogether?**	**If 18 plums are shared equally into 3 bags, then how many plums will be in each bag?** ***Measurement example*. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?**	**If 18 plums are to be packed 6to a bag, then how many bags are needed?** ***Measurement example*. You have 18 inches of string, which you will cut into pieces that are6 inches long. How many pieces of string will you have?**
Arrays1, **Area2**	**There are 3 rows of apples with 6 apples in each row.**	**If 18 apples are arranged into 3equal rows, how many**	**If 18 apples are arranged into equal rows of 6 apples, how many rows**

	How many apples are there? ***Area example*. What is the area of a 3 cm by 6 cm rectangle?**	**apples will be in each row?** ***Area example*. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?**	**will there be?** ***Area example*. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?**
Compare	**A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost?** ***Measurement example*. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3times as long?**	**A red hat costs $18 and that is3 times as much as a blue hat costs. How much does a blue hat cost?** ***Measurement example*. A rubber band is stretched to be18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?**	**A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat?** ***Measurement example*. A rubber band was 6 cm long at first. Now it is stretched to be18 cm long. How many times as long is the rubber band now as it was at first?**
General	***a* ×*b* =?**	***a* ×? = *p*, and *pea* =?**	**?×*b* = *p*, and *pub* =?**

- **Provide a variety of contexts and tasks so that students will have more opportunity to develop and use thinking strategies to support and reinforce learning of basic multiplication and division facts.**
- **Have students create multiplication problem situations in which they interpret the product of whole numbers as the total number of objects in a group and write as an expression. Also, have students create division-problem situations in which they interpret the quotient of whole numbers as the number of shares.**
- **Students can use known multiplication facts to determine the unknown fact in a multiplication or division problem. Have them write a multiplication or division equation and the related multiplication or division equation. For example, to determine the unknown whole number in 27 ÷? = 3, they should ask themselves questions such as, "How many 3s are in 27?" or "3 times what number is 27?" Have them justify their thinking with models or drawings.**

- **Using a multiplication table, highlight a row of numbers and ask students what they notice about the**

High-lighted numbers. Explain a pattern using properties of operations.

When (commutative property) one changes the order of the factors they will still get the same product, example 6 x 5 = 30 and 5 x 6 = 30.

Teacher: What pattern do you notice when 2, 4, 6, 8, or 10 are multiplied by any number (even or odd)?

Student: The product will always be an even number.

Teacher: Why?

- **Engage students in active learning by having students "act out" multiplication and division physically (lining up rows of students, etc.). Have students visualize the process of multiplication as "duplication" (Visualize a copy machine putting in two copies and duplicating it five times).**
- **Following acting out, have students work with manipulative, while introducing vocabulary.**
- **Connect the activity in which students engage during the exploratory stage to a word wall. On the word wall card: Draw a picture of the activity (visual memory hook) with the vocabulary to activate students' prior knowledge. This will assist students throughout the school year to remember the vocabulary definition.**
- **Blend physical, visual, and vocabulary to cement their learning.**

Note: This section is not designed to provide a day-to-day lesson plan view; rather, it is offered as a guide to inform lesson planning.

Differentiation

Interdisciplinary Connections

Additional Resources

Instructional Resources/Tools

- **SAS Voluntary Model Curriculum**
- **NLVM**
- **3rd Grade Common Core State Standards**
- **Lessons designed to "illuminate" the new NCTM Principles and Standards for School Mathematics.**
- ***PARCC* is a 19-state consortium working together to develop next-generation K-12 assessments in math.**
- **Math Lab Manipulatives**
- **Houghton Mifflin Text**
- **Simple Solutions**

Created By

Mercer 3rd Grade

Copy of Math Grade 3 - Module 4

Subject	Grade	Module	Suggested Timeline
Mathematics	3	4	5 weeks

Grade Level Summary

In grade 3, instructional time should focus on five critical areas: (1) developing understanding of multiplication and division and strategies for multiplication and division within 100 (2) developing understanding of fractions, especially unit fractions (fractions with numerator 1); (3) developing understanding of the structure of rectangular arrays and of area; (4) describing and analyzing two-dimensional shapes; and (5) solving problems involving measurement and estimation of intervals of time, money, liquid volumes, masses, and lengths of objects.

Grade Level Modules

Module 1: Multiplication and Division with Factors of 2,3,4, 5, and 10
Module 2: Problem Solving with Mass, Time, Capacity, Length, and Money
Module 3: Multiplication and Division with Factors of 6,7,8, and 9
Module 4: Multiplication and Area
Module 5: Fractions as Numbers on the Number Line
Module 6: Collecting and Displaying Data
Module 7: Word Problems with Geometry and Measurement

Module Title

Module 4: Multiplication and Area

Module Overview

By Module 4, students are ready to investigate area and the formula for the area of a rectangle. They measure the area of a shape by finding the total number of same-size units of area required to cover the shape without gaps or overlaps. When that shape is a rectangle with whole number side lengths, it is easy to partition the rectangle into squares with equal areas (as in Module 3).

Module Objectives

At the end of this module, students wil be able to idependently use their learning to:

- **Measure areas by counting unit squares (square centimeter, square meter, square inch, square foot, and non-standards units)**
- **Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real-world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning**

Standards for Mathematical Practice

MP# 1. Make sense of problems and persevere in solving them
MP# 2. Reason abstractly and quantitatively
MP# 3. Construct viable arguments and critique the reasoning of others
MP# 4. Model with mathematics
MP# 5. Use appropriate tools strategically
MP# 6. Attend to precision
MP# 7. Look for and make use of structure (Deductive Reasoning)
MP# 8. Look for and express regularity in repeated reasoning (Inductive Reasoning)

Mathematical Practices resource page on SAS

Focus Standards Addressed in this Module

CC.2.4.3.A.5

Determine the area of a rectangle and apply the concept to multiplication and to addition.

Important Standards Addressed in this Module

CC.2.2.3.A.1

Represent and solve problems involving multiplication and division.

CC.2.2.3.A.2

Understand properties of multiplication and the relationship between multiplication and division.

CC.2.2.3.A.4

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

Misconceptions

1. **Students confuse area and perimeter.**
2. **Students choose the wrong operation to calculate area or perimeter.**

Proper Conceptions

1. **Students must be explicitly taught that perimeter is the "edge" of a color tile when building a rectangular array, while area is represented by the**

3. **Students do not label square units and units correctly.**

surface that covers a color tile. Students count the edges around a shape to find the perimeter in linear units, while they count the squares to find the number of square units that cover a shape.

2. **After creating a rectangle with a rubber band on a geoboard, students can count the distance from "peg to peg" to find perimeter of the shape, while counting the squares (4 peg square sections) to find the area.**
3. **Once students understand conceptually the difference between area and perimeter, gradually move them to discovering patterns/short cuts to develop the formula for area of a rectangle. Follow up all activities with a word wall card, labeled with a visual of the perimeter or area activity in which the students were engaged.**
4. **Discussion must clearly identify when to multiply and why and when to add and why. Probe students thinking with questions.**
5. **Much practice must be done with manipulatives in order for students to understand that "flat squares" that cover a shape need to be labeled in "square units" and distance around the shape are the total of the edges or "units."**

Concepts	Competencies	Vocabulary
• **Place Value and Properties of Operations** • **Patterns**	• **Demonstrate an understanding of properties of multiplication.** • **Represent and solve problems.** • **Identify and explain patterns in arithmetic (including addition and**	**Factor** **Product** **Multiple** **Rectangular array** **Square unit** **Length** **Width**

	subtraction). • **Determine the area of a rectangle as it relates to multiplication and addition.** • **Distinguish between linear and area measurements.**	**Rows** **Columns** **Commutative (in relation to L X W= W X L)** **Dimension** **Square numbers** **Equation** **Patterns** **Repeated addition** **Fact family** **Area** **Perimeter**

Assessment(s)

Perimeter and Area Test

Solid Planes and Figures Test

Mulitiplication Concepts and Patterns

Perimeter and Area Projects

Suggested Strategies to Support Design of Coherent Instruction

Questions to develop critical thinking:

How would you describe the problem in your own words?

How would you describe what you are trying to find?

What do you notice about...?

What information is given in the problem?

Describe the relationship between the quantities.

Describe what you have already tried. What might you change?

Talk me through the steps you've used to this point.

What steps in the process are you most confident about?

What are some other strategies you might try?

What are some other problems that are similar to this one?

How might you use one of your previous problems to help you begin?

How else might you organize...represent... show...?

What do the numbers used in the problem represent?

What is the relationship of the quantities?

How is _____ related to _____?

What is the relationship between _____and _____?

What does_____mean to you? (e.g. symbol, quantity, diagram)

What properties might we use to find a solution?

How did you decide in this task that you needed to use...?

Could we have used another operation or property to solve this task? Why or why not?

What number model could you construct to represent the problem?

What are some ways to represent the quantities?

What is an equation or expression that matches the diagram, number line.., chart..., table..?

Where did you see one of the quantities in the task in your equation or expression?

How would it help to create a diagram, graph, table...?

What are some ways to visually represent...?

What formula might apply in this situation?

What mathematical tools could we use to visualize and represent the situation?

What information do you have?

What do you know that is not stated in the problem?

What approach are you considering trying first?

What estimate did you make for the solution?

In this situation would it be helpful to use...a graph...,

number line..., ruler..., diagram..., calculator..., manipulative?

Why was it helpful to use...?

What can using a _____ show us that _____may not?

In what situations might it be more informative or helpful to use...?

What observations do you make about...?

What do you notice when...?

What parts of the problem might you eliminate..., simplify...?

What patterns do you find in...?

How do you know if something is a pattern?

What ideas that we have learned before were useful in solving this problem?

What are some other problems that are similar to this one?

How does this relate to...?

In what ways does this problem connect to other mathematical concepts?

Explain how this strategy work in other situations?

Is this always true, sometimes true or never true?

How would we prove that...?

What do you notice about...?

What is happening in this situation?

What would happen if...?

Is there a mathematical rule for...?

What predictions or generalizations can this pattern support?

What mathematical consistencies do you notice ?

Sets of counters, number lines to skip count followed by multiplication and arrays/area models will aid students in solving problems involving multiplication and division. Allow students to model problems using these tools. Students should represent the model used as a drawing or equation to find the solution. Show a variety of models of multiplication. Through inquiry of rectangular arrays comprised of square units, encourage students to find a "shortcut"/develop a strategy for finding the total number of square units (discovering the formula for area of a rectangle) . Explore perfect squares and the reasoning behind "square units."

Manipulatives are key to students' conceptual understanding of area and perimeter. Have students "touch" the flat area that covers his/her desk and trace the perimeter of the desk with his/her finger. Walk the perimeter of the room. Relate to "real world" context of carpeting for the room, paint for the walls, etc.

Activity: Using 24 color tiles, how many rectangles can you make? What is the area in square units of each rectangle?

How many rectangles can you make with 12 color tiles? Have students chart the area versus the perimeter of each rectangle, and then discover pa

The commutative property (order property) states that the order of numbers does not matter when adding or multiplying numbers. For example, if a student knows that 5 x 4 = 20, then they also know that 4 x 5 = 20.

The array below could be described as a 5 x 4 array for 5 columns and 4 rows, or a 4 x 5 array for 4 rows and 5 columns. There is no "fixed" way to write the dimensions of an array as rows x columns or columns x rows.

Students should have flexibility in being able to describe both dimensions of an array.

XXXX XXXXX

XXXX XXXXX

XXXX XXXXX

XXXX XXXXX

XXXX

This can lead into a discussion of square units in relation to area. Have students defend their reasoning for whether or not the 2 arrays cover the same area.

During activities relating multiplication and area, have students use inquiry to discover patterns: Gather information, make conjecture, formulate models, invent counter examples, and build sound arguments.

Employ reading strategies during instruction: Think alouds, predicting, visualization, questioning, compare and contrast, and imagery.

Other technology/manipulatives: rulers, tape measures, yard sticks, meter sticks. NLVM (National Library of Virtual Manipulatives online will allow exploratory activities with color tiles to build conceptual understanding of the relationship between area and multiplication.)

Include student speaking, listening, reading, and thinking (reasoning, analyzing, problem solving, decision making, critical thinking) and technology activities to build understanding. Ask yourself how area and multiplication at the third grade level are building blocks to future algebra, geometry, trigonometry, and calculus courses, and keep this vision in mind, as you instruct.

Create real life examples of using area and multiplication: Example covering a table for Thanksgiving dinner: Create a model of the number of square feet it would take to cover the table. (Extension: Building a model using Scale could be included where a 1 X 1 inch color tile = 1 square foot).

Note: This section is not designed to provide a day-to-day lesson plan view; rather, it is offered as a guide to inform lesson planning.

Differentiation

Interdisciplinary Connections

Additional Resources

Instructional Resources/Tools

- **SAS Voluntary Model Curriculum**
- **NLVM**
- **3rd Grade Common Core State Standards**
- **Lessons designed to "illuminate" the new NCTM Principles and Standards for School Mathematics.**
- ***PARCC* is a 19-state consortium working together to develop next-generation K-12 assessments in math.**
- **Math Lab Manipulatives**

- **Houghton Mifflin Text**
- **Simple Solutions**

Created By

Mercer 3rd Grade

Copy of Math Grade 3 - Module 5

Subject	Grade	Module	Suggested Timeline
Mathematics	3	5	6 weeks

Grade Level Summary

In grade 3, instructional time should focus on five critical areas: (1) developing understanding of multiplication and division and strategies for multiplication and division within 100 (2) developing understanding of fractions, especially unit fractions (fractions with numerator 1); (3) developing understanding of the structure of rectangular arrays and of area; (4) describing and analyzing two-dimensional shapes; and (5) solving problems involving measurement and estimation of intervals of time, money, liquid volumes, masses, and lengths of objects.

Grade Level Modules

Module 1: Multiplication and Division with Factors of 2,3,4, 5, and 10
Module 2: Problem Solving with Mass, Time, Capacity, Length, and Money
Module 3: Multiplication and Division with Factors of 6,7,8, and 9
Module 4: Multiplication and Area
Module 5: Fractions as Numbers on the Number Line
Module 6: Collecting and Displaying Data
Module 7: Word Problems with Geometry and Measurement

Module Title

Module 5: Fractions as Numbers on the Number Line

Module Overview

The goal of Module 5 is for students to transition from thinking of fractions as parts of a figure to points on a number line. To make that jump, students think of fractions as being constructed out of unit fractions: “1 fourth” is the length of a segment on the number line such that the length of four concatenated fourth segments on the line equals 1 (the whole). Once the unit “1 fourth” has been established, counting them is as easy as counting whole numbers: 1 fourth, 2 fourths, 3 fourths, 4 fourths, 5 fourths, etc. Students also compare fractions, find equivalent fractions in special cases, and solve problems that involve comparing fractions.

Module Objectives

At the end of this module, students will be able to independently use thier learning to:

- **Demonstrate that when a whole or set is partitioned into y equal parts, the fraction 1/y represents 1 part of the whole and/or the fraction x/y represents x equal parts of the whole (limit the denominators to 2,3,4,6,and 8; limit numerators to whole numbers less than the denominator; no simplification necessary)**
- **Represent fractions on a number line (limit the denominators to 2,3,4,6, and 8; limit numerators to whole numbers less than the denominator; no simplification necessary)**
- **Recognize and generate simple equivalent fractions (limit the denominators to 1,2,3,4,6, and 8; limit numerators to whole numbers less than the denominator)**
- **Express whole numbers as fractions, and/or generate fracions that are equivalent to whole numbers (limit the denominators to 1,2,3,4,6,and 8; limit numerators to whole numbers less than the denominator)**
- **Compare two fractions with the same denominator (limit the denominators to 1,2,3,4,6,and 8), using the symbols >, =, or**
- **Round two- and three-digit whole numbers to the nearest ten or hundred, respectively**
- **Add two- and three-digit whole numbers (limit sums from 100 through 1,000), and/or subtract two- and three-digit numbers from three-digit whole numbers.**
- **Multiply one-digit whole numbers by two-digit multiples of 10 (from 10 through 90)**
- **Order a set of whole numbers from least to greatest or greatest to least (up through 9,999; limit sets to no more than four numbers)**

Standards for Mathematical Practice

MP# 1. Make sense of problems and persevere in solving them
MP# 2. Reason abstractly and quantitatively
MP# 3. Construct viable arguments and critique the reasoning of others
MP# 4. Model with mathematics
MP# 5. Use appropriate tools strategically
MP# 6. Attend to precision
MP# 7. Look for and make use of structure (Deductive Reasoning)
MP# 8. Look for and express regularity in repeated reasoning (Inductive Reasoning)

Mathematical Practices resource page on SAS

Focus Standards Addressed in this Module

CC.2.1.3.C.1

Explore and develop an understanding of fractions as numbers.

CC.2.2.3.A.2

Understand properties of multiplication and the relationship between multiplication and division.

Important Standards Addressed in this Module

CC.2.1.3.B.1

Apply place value understanding and properties of operations to perform multi-digit arithmetic.

Misconceptions

1. **The idea that the smaller the denominator, the smaller the piece or part of the set, or the larger the denominator, the larger the piece or part of the set, is based on the comparison that in whole numbers, the smaller a number, the less it is, or the larger a number, the more it is.**
2. **Students think all shapes can be divided the same way (three-eighths, and so on).**
3. **Students may think that because a shape is divided into 3 parts the parts are thirds. (All parts must be congruent).**

Proper Conceptions

1. **The use of different models, such as fraction bars and number lines, allows students to compare unit fractions to reason about their sizes.**
2. **Present shapes other than circles, squares or rectangles to prevent students from overgeneralizing that all shapes can be divided the same way. For example, have students fold a (non-equilateral) triangle into eighths.**
3. **Provide oral directions for folding the triangle: Note: This will work for an equilateral triangle. Ask students: Does this work for all types of triangles?**

- **Fold the triangle into half by folding the left vertex (at the base of the triangle) over to meet the right vertex.**
- **Fold in this manner two more times.**
- **Have students label each eighth using fractional notation. Then, have students count the fractional parts in the triangle (one-eighth, two-eighths,**
- **Example/nonexample strategy to**

show equal portions of a number line or other shape.

Concepts	Competencies	Vocabulary
• Fractions	• Represent fractions on a number line. • Represent and generate equivalent fractions. • Compare fractions with the same numerator or same denominator. • Partition two-dimensional shapes into equal parts. • Express the area of a partition as a unit fraction of the whole.	Numerator Denominator Division Equivalent Fractions Fraction bar Number Line and Open Number Line Symmetric Symmetry Equal parts Equal length Distance Part Whole Increment

Assessment(s)

Fractions Concepts Test

Work with Fractions Test

Suggested Strategies to Support Design of Coherent Instruction

Reading Strategies: Reading for information: problem solving, visualization, think alouds, think/pair/share, make predictions, draw conclusions, response journal, restate the question, self questioning, prior knowledge, paraphrasing.

Manipulatives: Fraction bars, colored chips, Hershey bars, pattern blocks, tape measures, rulers, string, thermometers, rulers

WALK a number line on the floor.

Encourage students to create "open" number lines to model their thinking.

¼ should be connected to other units: Example: quarter of a dollar, quarter past noon, 4 quarts in a gallon = ¼ gallon, etc.

Do the same with other fractions: Connect all units in math and other content areas to make "real life' connections for students.

Relate fractional distance as students are walking down the hallway. Change the denominator each day. Day 1: we are ½ the way to gym class. Day 2: We are ¾ of the way there. Encourage estimation: Stop when we are 7/8 of the way there.

Have students explain reasoning orally, written, and visually: What fractions are close to ½? to 1 whole? To 2 wholes? Allow students to be flexible in their thinking…if they come up with "improper fractions" or "negative fractions" encourage them to explain their reasoning.

Using a blank open number line, have students label fractions, or using a clothesline, have students clip fractions to it. Challenge them to place equivalent fractions in the correct position.

Relate to measurement: Temperature on a thermometer and increments on a ruler

Create paper model strips and fold paper to model fractional lengths.

Implement Inquiry: Gather information, make conjectures, formulate models, invent counter examples, build sound arguments.

Note: This section is not designed to provide a day-to-day lesson plan view; rather, it is offered as a guide to inform lesson planning.

Differentiation

Interdisciplinary Connections

Additional Resources

- **Instructional Resources/Tools**
  - **SAS Voluntary Model Curriculum**
  - **NLVM**
  - **3rd Grade Common Core State Standards**
  - **Lessons designed to "illuminate" the new NCTM Principles and Standards for School Mathematics.**
  - ***PARCC* is a 19-state consortium working together to develop next-generation K-12 assessments in math.**
  - **Math Lab Manipulatives**
  - **Houghton Mifflin Text**
  - **Simple Solutions**

Created By

Mercer 3rd Grade

Copy of Math Grade 3 - Module 6

Subject	Grade	Module	Suggested Timeline
Mathematics	3	6	5 weeks

Grade Level Summary

In grade 3, instructional time should focus on five critical areas: (1) developing understanding of multiplication and division and strategies for multiplication and division within 100 (2) developing understanding of fractions, especially unit fractions (fractions with numerator 1); (3) developing understanding of the structure of rectangular arrays and of area; (4) describing and analyzing two-dimensional shapes; and (5) solving problems involving measurement and estimation of intervals of time, money, liquid volumes, masses, and lengths of objects.

Grade Level Modules

Module 1: Multiplication and Division with Factors of 2,3,4, 5, and 10
Module 2: Problem Solving with Mass, Time, Capacity, Length, and Money
Module 3: Multiplication and Division with Factors of 6,7,8, and 9
Module 4: Multiplication and Area
Module 5: Fractions as Numbers on the Number Line
Module 6: Collecting and Displaying Data
Module 7: Word Problems with Geometry and Measurement

Module Title

Module 6: Collecting and Displaying Data

Module Overview

In Module 6, students leave the world of exact measurements behind. By applying their knowledge of fractions from Module 5, they estimate lengths to the nearest halves and fourths of an inch and record that information in bar graphs and line plots. This module also prepares students for the multiplicative comparison problems of grade 4 by asking students “how many more” and “how many less” questions of scaled bar graphs.

Module Objectives

At the end of this module, students will be able to independently use their learning to:

- **Complete a scaled pictograph and a scaled bar graph to represent a data set with several categories (scales limited to 1,2,5, and 10)**
- **Solve one- and two-step problems using information to interpret data presented in scaled pictographs and scaled bar graphs (scales limited to 1,2, 5, and 10)**
- **Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Display the data by making a line plot, where the horizontal scale is marked in appropriate units – whole numbers, halves, or quarters**
- **Translate information from one type of display to another. Limit to pictographs, tally charts, bar graphs, and tables**

Standards for Mathematical Practice

MP# 1. Make sense of problems and persevere in solving them
MP# 2. Reason abstractly and quantitatively
MP# 3. Construct viable arguments and critique the reasoning of others
MP# 4. Model with mathematics
MP# 5. Use appropriate tools strategically
MP# 6. Attend to Precision
MP# 7. Look for and make use of structure (Deductive Reasoning)
MP# 8. Look for and express regularity in repeated reasoning (Inductive Reasoning)

Mathematical Practices resource page on SAS

Focus Standards Addressed in this Module

CC.2.4.3.A.4

Represent and interpret data using tally charts, tables, pictographs, line plots, and bar graphs.

Important Standards Addressed in this Module

CC.2.4.3.A.1

Solve problems involving measurement and estimation of temperature, liquid volume, mass or length.

Misconceptions

1. **Although intervals on a bar graph are not in single units, students count each square as one.**

Proper Conceptions

1. **To avoid this error, have students include tick marks between each interval. Students should begin each scale with 0.**
2. **They should think of skip-counting when determining the**

value of a bar since the scale is not in single units.

Concepts	Competencies	Vocabulary
• Data Displays	• Solve problems. • Make estimations. • Represent and interpret data using various displays.	Bar graph Chart Difference Graph Increment Interval Line plot graph Pictograph Round Data Greatest Least

Assessment(s)

Make and Interpret Graphs/Analyzing Data Graph

Suggested Strategies to Support Design of Coherent Instruction

Examples of Common Graphing Situations :

• Pose a question: Student should come up with a question. What is the typical genre read in our class?

• Collect and organize data: student survey

• Pictographs: Scaled pictographs include symbols that represent multiple units. Below is an example of a pictograph with symbols that represent multiple units. Graphs should include a title, categories, category label, key, and data. How many more books did Juan read than Nancy?

Evaluate data and explain choices and conclusions.

Make inferences from collected data.

Manipulatives; String, Cereal, Flip Chips, popsicle sticks, straws to be used for collecting data or for creating graphs.

Note: This section is not designed to provide a day-to-day lesson plan view; rather, it is offered as a guide to inform lesson planning.

Differentiation

Interdisciplinary Connections

Additional Resources

Instructional Resources/Tools

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  - **SAS Voluntary Model Curriculum**
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  - ***PARCC* is a 19-state consortium working together to develop next-generation K-12 assessments in math.**
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Created By

Mercer 3rd Grade

Copy of Math Grade 3 - Module 7

Subject	Grade	Module	Suggested Timeline
Mathematics	3	7	4 weeks

Grade Level Summary

In grade 3, instructional time should focus on five critical areas: (1) developing understanding of multiplication and division and strategies for multiplication and division within 100 (2) developing understanding of fractions, especially unit fractions (fractions with numerator 1); (3) developing understanding of the structure of rectangular arrays and of area; (4) describing and analyzing two-dimensional shapes; and (5) solving problems involving measurement and estimation of intervals of time, money, liquid volumes, masses, and lengths of objects.

Grade Level Modules

Module 1: Multiplication and Division with Factors of 2,3,4, 5, and 10
Module 2: Problem Solving with Mass, Time, Capacity, Length, and Money
Module 3: Multiplication and Division with Factors of 6,7,8, and 9
Module 4: Multiplication and Area
Module 5: Fractions as Numbers on the Number Line
Module 6: Collecting and Displaying Data
Module 7: Word Problems with Geometry and Measurement

Module Title

Module 7: Word Problems with Geometry and Measurement

Module Overview

The year rounds out with plenty of time to solve two-step word problems involving the four operations, and to improve fluency for concepts and skills initiated earlier in the year. In Module 7, students also describe, analyze, and compare properties of two-dimensional shapes. By now, students have done enough work with both linear and area measurement models to study that there is no relationship in general between the perimeter and area of a figure, one of the concepts of the last module.

Module Objectives

At the end of this module, students wil be able to independently use their

learning to:

- **Explain that shapes in different categories may share attributes, and that the shared attributes can define a larger category**
- **Recognize rhombi, rectangles, and squares as examples of quadrilaterals, and/or draw examples of quadrilaterals that do not belong to any of these subcategories**
- **Partition shapes into parts with equal areas Express the area of each part as a unit fraction of the whole**
- **Solve two-step problems using the four operations (expressions are not explicitly states) Limit to problems with whole numbers and having whole-number answers**
- **Solve two-step equations using order of operations (equation is explicitly stated with no grouping symbols)**
- **Use a ruler to measure lengths to the nearest quarter inch or centimeter**
- **Solve real-world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, exhibiting rectangles with the same perimeter and different areas, and exhibiting rectangles with the same area and different perimeters. Use the same units throughout the problem**

Standards for Mathematical Practice

MP# 1. Make sense of problems and persevere in solving them
MP# 2. Reason abstractly and quantitatively
MP# 3. Construct viable arguments and critique the reasoning of others
MP# 4. Model with mathematics
MP# 5. Use appropriate tools strategically
MP# 6. Attend to precision
MP# 7. Look for and make use of structure (Deductive Reasoning)
MP# 8. Look for and express regularity in repeated reasoning (Inductive Reasoning)

Mathematical Practices resource page on SAS

Focus Standards Addressed in this Module

CC.2.3.3.A.1

Identify, compare, and classify shapes and their attributes.

CC.2.3.3.A.2

Use the understanding of fractions to partition shapes into parts with equal areas and express the area of each part as a unit fraction of the whole.

CC.2.4.3.A.6

Solve problems involving perimeters of polygons and distinguish between linear and area measures.

Important Standards Addressed in this Module

CC.2.2.3.A.4

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

CC.2.4.3.A.1

Solve problems involving measurement and estimation of temperature, liquid volume, mass or length.

Misconceptions

1. **Students think that when they are presented with a drawing of a rectangle with only two of theside lengths shown or a problem situation with only two of the side lengths provided, these are the only dimensions they should add to find the perimeter.**
2. **Students may identify a square as a "non-rectangle" or a "non-rhombus" based on limited images they see. They do not recognize that a square is a rectangle because it has all of the properties of a rectangle. They may list properties of each shape separately, but not see the interrelationships between the shapes. For example, students do not look at the properties of a square that are characteristic of other figures as well.**

Proper Conceptions

1. **Encourage students to include the appropriate dimensions on the other sides of the rectangle. With problem situations, encourage students to make a drawing to represent the situation in order to find the perimeter.**
2. **Using straws to make four congruent figures have students change the angles to see the relationships between a rhombus and a square. As students develop definitions for these shapes, relationships between the properties will be understood.**

Concepts

- **Two and Three Dimensional Figures**

Competencies

- **Identify and classify shapes and their attributes.**
- **Compare shapes.**

Vocabulary

Point
Line
Ray
Line segment
Plane
Intersecting
Parallel
Right angle
Solid figure

Vertex
Symmetry
Flip
Slide
Turn
Symmetrical
Acute
Obtuse
Straight
Polygon
Congruent
Perimeter
Area
Volume
Face
Edge
Prism
Pyramid
Sphere
Cylinder
Cube
Cone
Angle
Circle
Square
Triangle
Rectangle
Pentagon
Hexagon
Octagon

Assessment(s)

Geometry Test

Congruence, symmetry and Transformations Test

Suggested Strategies to Support Design of Coherent Instruction

Reading Strategies: Think aloud, Analogies, Graphic Organizers and Charts to organize shapes according to their attributes, Journal Writing, Imagery, Making predictions, paraphrasing

Technology/Manipulative: Paper folding: Reflection on a line of symmetry, geoboards, pattern blocks, color tiles, polygon tiles, Three dimensional space figures, wooden cubes, nets

Writing and speaking: Students present a model and explain geometric attributes

Students create a design using geometric figures and present its attributes.

Create on geoboards and reproduce design using dot paper.

Journal writing: Explain the difference between the area and the perimeter of your creation.

Identify and describe geometric figures in real life. Have a scavenger hunt.

Identify geometry in nature.

Identify jobs in construction and design.

Thinking/Reasoning mathematically:

Relate square/cube

Relate triangle/pyramid

Relate rectangle/prism

Identify relationships of polygons

Discover visual patterns

Classify shapes according to characteristics

Young students can learn and internalize geometric vocabulary through visual and active learning;

Create a chart broken into:

1. **My creation (After creating, draw a picture in math journal)**
2. **(students create geometric figure on a geoboard, with clay, with straws, or other manipulative) List vocabulary word next to the geometric figure.**
3. **Place a word wall card on the wall for future reference: The card should depict the vocabulary word and the visual or the activity in which the students engaged as a memory hook.**
4. **"Act out" the geometric figure. Many vocabulary words can be acted out using arms and legs: Review throughout the school year by getting students out of their seats to act out figures.**

Note: This section is not designed to provide a day-to-day lesson plan view; rather, it is offered as a guide to inform lesson planning.

Differentiation

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