

Key Words

Perfect Square

A *perfect square* is the square of an integer.

Square Root

The *square root* of a number b is equal to a if $a^2 = b$. It is denoted by \sqrt{b} .

Cube Root

The cube root of a number b is equal to a if $a^3 = b$. It is denoted by $\sqrt[3]{b}$.

Irrational Number

Irrational numbers are numbers that are not rational.

Infinite Decimals

Infinite decimals are decimals that do not repeat nor terminate.

Rational Approximation

Rational approximation is the method for determining the approximated rational form of an irrational number.

Truncated Cone

A *truncated cone* is a solid obtained from a cone by removing the top portion above a plane parallel to the base.

Introduction to Irrational Numbers Using Geometry

In this 23-lesson module, students will begin this module with work related to the Pythagorean Theorem. Students also learn the notation related to roots and learn that in order to get the decimal expansion of a number, they must develop a deeper understanding of the long division algorithm learned in Grade 6 and Grade 7. In addition, students learn that radical expressions arise naturally in geometry and apply the Pythagorean Theorem to three-dimensional figures.

What Came Before this Module:

Students were introduced to bivariate data. Students worked with functions and used their understanding of functions to model the possible relationships of bivariate data. The work in this module set an important foundation for students' work in algebra in Grade 9 with respect to functions and statistics.

*This module is the last module in Grade 8.

Key Common Core Standards:

Know that there are numbers that are not rational, and approximate them by rational numbers.

Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions.

Work with radicals and integer exponents.

Use square root and cube root symbols to represent solutions to the equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number.

Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.

Understand and apply the Pythagorean Theorem.

Explain a proof of the Pythagorean Theorem and its converse.

Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

Know the volumes of cones, cylinders, and spheres and use them to solve real world and mathematical problems.

How can you help at home?

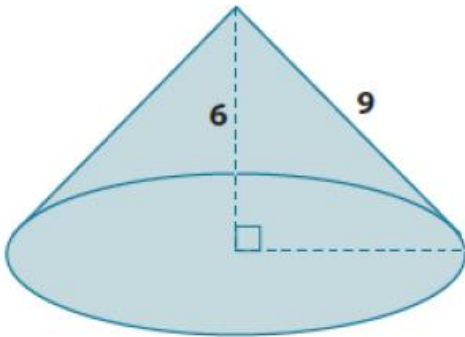
- ✓ Every day, ask your child what they learned in school and ask them to show you an example.
- ✓ Ask your child to estimate the value of $\sqrt{5}$ and explain their answer.

Solution: The value of $\sqrt{5}$ is between $\sqrt{4}$ which equals 2 and $\sqrt{9}$ which equals 3. On a number line, if you separate the space between $\sqrt{4}$ and $\sqrt{9}$ into five sections, $\sqrt{5}$ is right next to $\sqrt{4}$ so we can estimate $\sqrt{5}$ to be about 2.2.

- ✓ Ask your child to use long division to determine the decimal expansion for $\frac{54}{20}$.

*This example is taken from Lesson 19: Cones and Spheres.

Determine the exact volume of the cone shown below.



Solution:

Let r be the radius of the base.

$$6^2 + r^2 = 9^2$$

$$36 + r^2 = 81$$

$$r^2 = 45$$

The area of the base is 45π .

$$V = \frac{1}{3}Bh$$

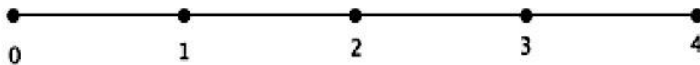
$$V = \frac{1}{3}45\pi(6)$$

$$V = 90\pi$$

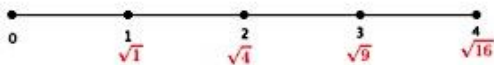
The volume of the cone is 90π units³.

*This is an example from Lesson 2: Square Roots.

1. Place the numbers $\sqrt{1}$, $\sqrt{4}$, $\sqrt{9}$, and $\sqrt{16}$ on the number line. Be prepared to explain your reasoning.



Solution:



2. Place the numbers $\sqrt{2}$ and $\sqrt{3}$ on the number line. Be prepared to explain your reasoning.

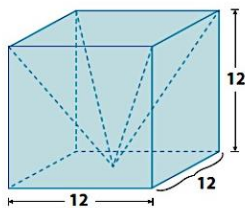
Solution:



Solutions are shown below in red. Students should reason that the numbers $\sqrt{2}$ and $\sqrt{3}$ belong on the number line between $\sqrt{1}$ and $\sqrt{4}$. They could be more specific by saying that if you divide the segment between integers 1 and 2 into three equal parts, then $\sqrt{2}$ would be at the first division and $\sqrt{3}$ would be at the second division and $\sqrt{4}$ is already at the third division, 2, on the number line. Given that reasoning, students should be able to estimate the value of $\sqrt{2} \approx 1\frac{1}{3}$.

*This is an example from Lesson 21: Volume of Composite Solids.

1. Write an expression that can be used to find the volume of the prism with the pyramid portion removed. Explain what each part of your expression represents.



2. What is the volume of the prism shown above with the pyramid portion removed?

Solution:

Solution: $12^3 - \frac{1}{3}(12^3)$

The expression 12^3 is the volume of the cube and $\frac{1}{3}(12^3)$ is the volume of the pyramid. Since the pyramid's volume is being removed from the cube, then we subtract the volume of the pyramid from the cube.

The volume of the prism is

$$V = 12^3 \\ = 1,728$$

The volume of the pyramid is

$$V = \frac{1}{3}(1,728) \\ = 576$$

The volume of the prism with the pyramid removed is 1,152 units³.