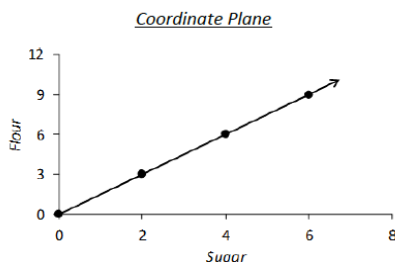


## Ratios and Proportional Relationships

In this 22-lesson module, students learn how to recognize proportional relationships given descriptions, tables containing sets of values of two related quantities, and graphs of those values on the coordinate plane. They represent proportional relationships in equations that have the form  $y=kx$ . In the latter half of the module, they extend their knowledge to find unit rates of ratios that contain rational numbers (e.g. a speed of  $\frac{1}{2}$  mile per  $\frac{1}{4}$  hour is a rate of 2 miles per hour). The module concludes with a geometric application of proportional reasoning that serves as a major foundation for geometry topics explored in Grade 8 and Grade 10. Students explore the relationships between the dimensions in scale drawings and recognize the scale factor as the constant of proportionality studied in the earlier topics of the module. They use this relationship to create scale drawings, create original drawings from a scale drawing, and create scale drawings of various scales from a given scale drawing.

## One Representation Seen in This Module

Graphical representation of a proportional relationship is a straight line that includes the point  $(0,0)$ .



## Key Words

**Proportional To:** Measures of one type of quantity are proportional to measures of a second type of quantity if there is a number  $k>0$  so that for every measure  $x$  of a quantity of the first type the corresponding measure  $y$  of a quantity of the second type is given by  $kx$ , i.e.  $y= kx$ . The number is called the *constant of proportionality*.

**Proportional Relationship:** A one-to-one matching between two types of quantities such that the measures of quantities of the first type are proportional to the measures of quantities of the second type, e.g. *a farmer sells one pound of apples for \$2 so two pounds of apples costs \$4 and three pounds of apples costs \$6, etc.*

**One-to-One Correspondence:** Two figures in the plane,  $S$  and  $S'$ , are said to be in one-to-one correspondence if there is a pairing between the points in  $S$  and  $S'$ , so that each point  $P$  of  $S$  is paired with one and only one point  $P'$  in  $S'$ , and likewise, each point  $Q'$  in  $S'$  is paired with one and only one point  $Q$  in  $S$ .

**Scale Drawing:** A scale drawing refers to a reduced-size or enlarged-size 2-dimensional picture of another 2-dimensional picture.

## Another Representation Frequently Used in this Module.

### Ratio Table

Time (h), $t$	Distance (km), $d$
0	0
1	10
2	20
3	30

## What Came Before this Module:

In Grade 6, students explored ratios, rates, and unit rates. They connected ratio and rate to whole number multiplication and division and used concepts of ratio and rate to solve problems.

## What Comes After this Module:

There is a focus on types of ratios as they can represent part-to-part relationships or part-to-whole relationships. There is often a misconception that ratios are fractions and this is of course not always the case. Students understand a rate as assigning a numerical value to the relationship between two sets of quantities using a new unit of measure derived from the units of the quantities being compared (e.g. in a relationships where distance in miles is being compared to time measure in hours, the unit of measure of the rate is miles per hour, often denoted mph).

## How can you help at home?

- ✓ Ask your child what they learned in school today and ask them to show you an example.
- ✓ Keep an open mind...your child might teach you something! Discuss the math with your child while you both make sense of it.
- ✓ Discuss a real-life example of how people determine the best buy using unit rates and reasoning skills. You can discuss the problem below or create your own problem.

A store is advertising a Back-to-School sale on pencils. A pack of 30 pencils sells for \$7.97, whereas a 12-pack of the same brand costs \$4.77. Which is the better buy? How do you know?

## Key Common Core Standards:

### Analyze proportional relationships and use them to solve real-world and mathematical problems.

- Compute unit rates associated with fractions.
- Recognize and represent proportional relationships between quantities.
- Use proportional relationships to solve multistep ratio and percent problems.

### Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

- Use variables to represent quantities in a problem and construct simple equations and inequalities to solve them.

### Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

- Solve problems involving scale drawings of geometric figures.

Our representation in the spotlight is the equation. This representation has become very familiar to your child over the course of his/her education so far. In early elementary school, your child may have seen equations such as  $7 + 4 = 11$  or  $5(2 + 1) = 15$ . In *A Story of Ratios*, students build upon their previous experiences with various equations: numeric equations (equations with numbers and symbols) and algebraic equations (equations with numbers, symbols and variables), and strengthen their understanding by engaging in problems where they use equations in various situations. This helps develop their abilities to think abstractly and algebraically and creates a strong foundation for the rigor and challenge of middle school and high school algebra. An example of this representation is below.

Write an equation that will model the relationship between the number of cups of blackberries and the number of cups of juice.

During summer vacation, Lydie spent time with her grandmother picking blackberries. They decided to make blackberry jam for their family. Her grandmother said that you must cook the berries until they become juice and then combine the juice with the other ingredients to make the jam.

Cups of Blackberries	Cups of Juice
0	0
4	$1\frac{1}{3}$
8	$2\frac{2}{3}$
12	4
24	8

Solution:

$j = \frac{1}{3}b$ , where  $j$  represents the number of cups of juice and  $b$  represents the number of cups of blackberries.

Spotlight on a representation frequently used in this module:

Equation

There are several models used in *A Story of Ratios* that will foster deep knowledge of important concepts in middle school mathematics.

In Module 1, there are three commonly used tools and representations that your child will frequently use: ratio tables, coordinate planes, and equations. Ratio tables are frequently used to show an organized list of related ratios. For example, in the ratio table above, your child is able to see how the number of cups of juice is one third of the number of cups of blackberries or if the number of cups of juice is multiplied by 3, the number of cups of blackberries is determined. In Grade 6, students recognized the multiplicative and additive structures that exist within ratio tables. The coordinate plane is another way to represent a relationship and an easy way to determine whether a relationship is proportional. Equations represent information in a clear and concise way so your child is able to quickly solve problems and make predictions.

Although each of these tools are different, they all have a similar goal of helping your child develop his/her thinking in a concrete way (manipulating something that physically exists) so he/she experiences a direct connection between the models and math symbols and is able to solve problems abstractly. In *A Story of Ratios*, your child will use the proportional reasoning skills that he/she develops in this module to propel your child into success in the modules yet to come!

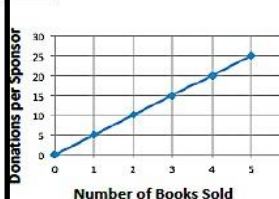
Below is a problem that shows different representations of the same proportional relationships with a description of how students can recognize it as such.

\* Taken from Lesson 6\*

**Problem:**

The school library receives money for every book sold at the school's book fair. Create a table, and then graph and explain if the quantities are proportional to each other.

Graph:



Solution:

Table:

Number of Books Sold	Donations per Sponsor (\$)
1	5
2	10
3	15
4	20
5	25

**Explanation:**

The quantities are proportional to each other because the points appear on a line that goes through the origin. Each book sold brings in \$5.00 no matter how many books are sold.

Even though the point (0,0) does not represent a ratio, interpreting the meaning of the point in the context of the problem helps students understand why it is included, i.e. if the school library does not sell any books, they will not raise any money.