



Grade 8 Pre-Algebra - Unit D - Exploring Number Systems

Unit Focus

After an initial review of exponential expressions, students will examine powers of 10, formulating the rules $10^n \cdot 10^m = 10^{n+m}$, $(10^n)^m = 10^{n \cdot m}$, and for $n > m$, $\frac{10^n}{10^m} = 10^{n-m}$ where n and m are positive integers. After working with these powers of 10, they consider what the value of 10^0 should be and define 10^0 to be 1. Students consider what happens when the exponent rules are used on exponential expressions with base 10 and negative integer exponents and define 10^{-n} to be $\frac{1}{10^n}$. Next, students return to powers of 10 as a prelude to the introduction of scientific notation. Students consider differences in magnitude of powers of 10 and use powers of 10 and multiples of powers of 10 to describe magnitudes of quantities as well as very small quantities. Students discover the rules for multiplying and dividing numbers written in scientific notation as well as adding and subtracting numbers in scientific notation. This type of reasoning appears again in high school when students extend the rules of exponents to make sense of exponents that are not integers.

Students learn and use definitions for “rational number” and “irrational number.” They plot rational numbers, square roots, and cube roots on the number line. They use the meaning of “square root,” understanding that if a given number p is the square root of n , then $x^2 = n$. Additionally, they understand that if a given number x is the square root of n and n is between m and p , then x^2 is between the square root of m and p . This thought process extends itself to cube roots as well. Students learn (without proof) that the *square root of 2* is irrational. They use the Pythagorean Theorem in two and three dimensions, e.g., to determine lengths of diagonals of rectangles and right rectangular prisms and to estimate distances between points in the coordinate plane. Students work with decimal representations of rational numbers and decimal approximations of irrational numbers.

Stage 1: Desired Results - Key Understandings

Standard(s)	Transfer	
Standards - Common Core - Mathematics: 8 - Know that there are numbers that are not rational, and approximate them by rational numbers. - Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number. (CCSS.MATH.CONTENT.8.NS.A.1) - Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and	<i>Students will be able to independently use their learning to...</i> T1 Identify and generalize patterns and structure in numbers, expressions, data and objects. (MP 7) T2 Apply an understanding of known patterns to new problems and make connections between concepts. (MP 8)	
	Meaning	
	Understanding(s)	Essential Question(s)
	<i>Students will understand that...</i> U1 Mathematicians see patterns to make generalizations about structures and relationships. U2 Mathematicians use their knowledge from patterns and structures to apply efficient strategies to solve a problem.	<i>Students will keep considering...</i> Q1 What generalizations can be made from this pattern? Q2 How can we apply knowledge of a pattern to similar problems? Q3 What is a way of expressing the recurring pattern?

Stage 1: Desired Results - Key Understandings

	Acquisition of Knowledge and Skill	
	Knowledge	Skill(s)
1.5, and explain how to continue on to get better approximations. (CCSS.MATH.CONTENT.8.NS.A.2) - Expressions and Equations Work with radicals and integer exponents. - Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = (1/3)^3 = 1/27$. (CCSS.MATH.CONTENT.8.EE.A.1) - Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational. (CCSS.MATH.CONTENT.8.EE.A.2) - Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. For example, estimate the population of the United States as 3 times 108 and the population of the world as 7 times 109, and determine that the world population is more than 20 times larger. (CCSS.MATH.CONTENT.8.EE.A.3) - Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology (CCSS.MATH.CONTENT.8.EE.A.4) - Understand and apply the Pythagorean Theorem. - Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. (CCSS.MATH.CONTENT.8.G.B.7) - Apply the Pythagorean Theorem to find the distance between two points in a coordinate system. (CCSS.MATH.CONTENT.8.G.B.8) - Mathematical Practices - Look for and make use of structure. (CCSS.MATH.MP.7) Madison Public Schools Profile of a Graduate - Analyzing: Examining information/data/evidence from multiple sources to identify possible underlying assumptions, patterns, and relationships in order to make inferences. (POG.1.2)	<i>Students will know...</i> K1 Vocabulary: exponent, base (of an exponent), reciprocal, scientific notation, square root, irrational number, rational number, Pythagorean Theorem, hypotenuse, legs (of a right triangle), cube root, repeating decimal, terminating decimal K2 The process for multiplying powers of 10 ($10^n \cdot 10^m = 10^{n+m}$) K3 The process for dividing powers of 10 ($\frac{10^n}{10^m} = 10^{n-m}$) K4 The process for raising a power of 10 to another power ($(10^n)^m = 10^{n \cdot m}$) K5 Any number to the zero power is 1. K6 How exponent rules extend to expressions involving negative exponents ($10^{-n} = \frac{1}{10^n}$) K7 That the same exponent rules for powers of 10 apply to non-zero bases other than 10. K8 That an irrational number is a number that is not rational and that the square root of 2 is an example of an irrational number. K9 That a rational number is a number that can be written as a fraction or its opposite, and that a rational number can be represented with a decimal expansion that "repeats" or "terminates." K10 That $-\sqrt{a}$ represents the opposite of $\sqrt{a}$. K11 That the Pythagorean Theorem is $a^2 + b^2 = c^2$, where a and b are the lengths of the legs and c is the length of the hypotenuse of a right triangle. K12 Which side is the hypotenuse of a right triangle and which sides are the legs. K13 That if the side lengths of a right triangle satisfy the Pythagorean Theorem then the triangle must be a right triangle. K14 What a cube root is and what a square root is. K15 That a repeating decimal can be written as a fraction with a denominator of a multiple of 9.	<i>Students will be skilled at...</i> S1 Using exponents to describe repeated multiplication. S2 Multiplying and dividing powers of 10. S3 Raising a power of 10 to another power. S4 Using the exponent rules with negative exponents. S5 Using the exponent rules for bases other than 10. S6 Writing very large or very small numbers as an expression using a power of 10. S7 Multiplying, dividing, adding, and subtracting numbers in scientific notation. S8 Identifying numbers as either rational or irrational. S9 Finding decimal approximations for square roots and cube roots and placing them on a number line. S10 Finding a solution to an equation of the form $x^2 = a$ and graphing it on a number line. S11 Calculating an unknown side length of a right triangle using the Pythagorean Theorem. S12 Knowing if a triangle is a right triangle when given the lengths of its sides. S13 Using the Pythagorean Theorem to solve real-world problems. S14 Finding the distance between two points in the coordinate plane. S15 Coordinating representations of a cube root, including cube root notation, decimal representation, the side length of a cube of given volume, and a point on the number line. S16 Representing rational numbers as equivalent decimals and fractions. S17 Changing an expression with a negative exponent into an equivalent expression with a positive exponent. S18 Using the appropriate rule for multiplying terms that have different bases but the same exponent.