## Algebra Skills Useful in Chemistry

This worksheet is designed for any student who may not feel fully confident of their algebra skills as they enter 10th grade. In particular, a topic called "solving literal equations" (by math teachers) is especially useful in Chemistry classes, and if you don't remember how to do this, then working through this worksheet might help. Some of the formulas on this worksheet (marked with a *) are versions of the formulas you'll work with in a Chemistry class, and some are non-Chemistry examples specifically designed to help you understand this idea better. Be sure to read the explanations below before trying the problems!

The algebra that you will use the most in your Chemistry class involves rearranging formulas. So let's start with an example of a very easy formula: The formula for finding the average (which we'll call $A$ ) of $x$ and $y$.

$$
A=\frac{x+y}{2}
$$

This formula is simple enough that you might not even think of it as a formula! But it's a good one to work with at first in order to understand the goal of this worksheet. If you were asked to find the average of 27 and 52 , for example, you might use the formula like this:

$$
A=\frac{27+52}{2}=\frac{79}{2}=38.5
$$

But what if you knew that the average of two numbers was 255.3, and you knew that one of the numbers was 371.8 ? Can you decide what the other number was? You might be able to reason through the problem and figure it out somehow without the formula, but of course the formula should still work, if you use 255.3 for $A$ and 371.8 for $x$ :

$$
255.3=\frac{371.8+y}{2}
$$

The only variable left is $y$, and you can use algebra to solve for it, starting by multiplying both sides by 2 :

$$
\begin{gathered}
2 \cdot(255.3)=\frac{371.8+y}{2} \cdot 2 \\
510.6=371.8+y \\
y=138.8
\end{gathered}
$$

Notice how the 2's cancel on the right, and how we subtracted 371.8 from both sides to get the final answer. But...if you're going to do algebra, which one of the two equations below looks simpler?

$$
255.3=\frac{371.8+y}{2} \quad \text { or } \quad A=\frac{x+y}{2}
$$

Surely the one on the right looks simpler. Here's how you might do the same algebra as above, except using the equation on the right instead of the one on the left:

| $A=\frac{x+y}{2}$ | starting equation |
| :---: | :--- |
| $2 \cdot A=\frac{x+y}{2} \cdot 2$ | multiply both sides by 2 |
| $2 A=x+y$ | cancel the 2's |
| $2 A=x+y$ | subtract x from both sides |
| $-x \quad-x$ |  |
| $y=2 A-x$ | now y is isolated (usually put on the left) |

Now that you have a new formula for y , you can substitute the values you know for $A$ and $x$ :

$$
y=2(255.3)-371.8
$$

The advantages here are: the algebra is easier to follow with letters than with numbers, and that final calculation above can be directly punched up on a calculator in one step. It may be your initial instinct to plug in the numbers and then do the algebra, but you will make a lot fewer mistakes if you first do the algebra, and then plug in the numbers!

Let's do one more example of isolating a variable in a (fictional) formula: let's solve for $a$ in the formula $m=3(a-b)$. Remember, you'll know you're done when $a$ is standing alone on one side of the equation. There are two equally valid methods. Both final answers are the same thing, even if they look different (see if you can convince yourself that this is true!).

Method 1: distribute the 3 first.

$$
m=3(a-b) \quad \text { starting equation }
$$

| $m=3 a-3 b$ | distribute to separate the $a$ from the $b$ |
| :---: | :--- |
| $m=3 a-3 b$ <br> $+3 b$ | add $3 b$ to both sides |
| $m+3 b=3 a$ | cancel the $3 b$ 's |
| $\frac{m+3 b}{3}=\frac{3 a}{3}$ | divide both sides by 3 |
| $a=\frac{m+3 b}{3}$ | cancel 3's, $a$ is now isolated |

Method 2: eliminate the 3 first.

$$
\begin{array}{ll}
m=3(a-b) & \text { starting equation } \\
\frac{m}{3}=\frac{3(a-b)}{3} & \text { divide both sides by } 3
\end{array}
$$

$$
\frac{m}{3}=a-b \quad \text { cancel the 3's }
$$

$$
\begin{array}{rr}
\frac{m}{3}=a-b \\
+b & +b
\end{array} \quad \text { add } b \text { to both sides }
$$

cancel the $b$ 's, $a$ is now isolated

Solve each of the following equations for the indicated variable. Remember that you're finished when that variable is standing alone on one side of the equation. Usually, we put that variable on the left, so that's how the answers will appear. Be sure to check your answers!

1. $\quad A=\frac{x+y+z}{3} \quad$ (for $y$ )
*2. $\quad K=C+273.15 \quad($ for $C)$
*3. $\quad F=1.8 C+32 \quad($ for $C)$
2. $\quad 5 h=2(c+d)($ for $d)$
3. Do \#4 again, but start the other way, as shown in the example on the previous page.
4. $\quad 4 a+10 b=2 c+d \quad($ for $a)$
5. $4 a+10 b=2 c+d($ for $b)$
6. $4 a+10 b=2 c+d($ for $d)$
7. $4 a+10 b=2 c+8 d$ (for $c$ )
8. $a=b-3 c \quad($ for $c)$
(It might be helpful to add $3 c$ to both sides first, so that it's no longer attached to a negative sign!)
*11. $\quad P V=n R T \quad($ for $T)$
*12. $\quad P_{1} V_{1}=P_{2} V_{2}\left(\right.$ for $\left.P_{2}\right)$
(Remember, those are subscripts, not exponents-they're part of the variable. $P_{2}$ for example, just means the 2nd pressure value of possibly multiple values for pressure.)
*13. $\quad q=m c(\Delta T) \quad($ for $c)$
(Note that $\Delta T$ here is a single variable; it means "the change in temperature".)
*14. $\frac{P_{1}}{T_{1}}=\frac{P_{2}}{T_{2}}\left(\right.$ for $\left.P_{2}\right)$

To do \#14 correctly, you would just have to multiply both sides by $T_{2}$, which leaves $P_{2}$ isolated on the right hand side. But this problem is somewhat harder if you have to solve it for $T_{2}$ instead, because it's in the denominator. The useful idea here is to do what it takes to clear all the denominators first, and then work on isolating $T_{2}$. Here's what that would look like:

$$
\begin{array}{ll}
P_{1} \\
\underline{m} & P_{2} \\
\hline
\end{array} \quad \text { starting equation }
$$

| $T_{1} T_{2} \cdot \frac{P_{1}}{T_{1}}=\frac{P_{2}}{T_{2}} \cdot T_{1} T_{2}$ | multiply both sides by $T_{1} T_{2}$ (because that's <br> what will clear both denominators) |
| ---: | :--- |
| $T_{2} P_{1}=P_{2} T_{1}$ | $T_{1}$ 's cancel on the left, leaving $T_{2}$ and $P_{1}$. <br> $T_{2}$ 's cancel on the right, leaving $P_{2}$ and $T_{1}$. |
| $\frac{T_{2} P_{1}}{P_{1}}=\frac{P_{2} T_{1}}{P_{1}}$ | divide by $P_{1}$ on both sides |

$$
T_{2}=\frac{P_{2} T_{1}}{P_{1}}
$$

cancel the $P_{1}$ 's, $T_{2}$ is now isolated

Now use this idea to try the rest of the problems below.
*15. $\quad d=\frac{m}{V}($ for $V)$
16. $\frac{a b}{c}=\frac{x}{y z} \quad($ for $c)$
17. $\frac{a b}{c}=\frac{x}{y z}($ for $y)$
*18. $\frac{P_{1} V_{1}}{T_{1}}=\frac{P_{2} V_{2}}{T_{2}}\left(\right.$ for $\left.T_{1}\right)$
*19. $\frac{P_{1} V_{1}}{n_{1} T_{1}}=\frac{P_{2} V_{2}}{n_{2} T_{2}}\left(\right.$ for $\left.P_{1}\right)$
*20. $\frac{P_{1} V_{1}}{n_{1} T_{1}}=\frac{P_{2} V_{2}}{n_{2} T_{2}}\left(\right.$ for $\left.V_{2}\right)$
*21. $\frac{P_{1} V_{1}}{n_{1} T_{1}}=\frac{P_{2} V_{2}}{n_{2} T_{2}} \quad\left(\right.$ for $\left.n_{2}\right)$
*22 $\frac{P_{1} V_{1}}{n_{1} T_{1}}=\frac{P_{2} V_{2}}{n_{2} T_{2}}\left(\right.$ for $\left.T_{1}\right)$
23. $\frac{y}{3}=\frac{x}{a+b}($ for $a)$
(This one is trickier: think of $(a+b)$ as a single object and multiply it on both sides.)

## Answers:

1. $y=3 A-x-z$
2. $\quad C=k-273.15$
3. $\quad C=\frac{F-32}{1.8}$
4. $d=\frac{5 h}{2}-c$
(if you started by dividing)
5. $d=\frac{5 h-2 c}{2}$
(if you started by distributing)
6. $a=\frac{2 c+d-10 b}{4}$
7. $b=\frac{2 c+d-4 a}{10}$
8. $d=4 a+10 b-2 c$
9. $c=2 a+5 b-4 d$
(remember that a 2 cancels out of all of those terms!)
10. $c=\frac{b-a}{3}$
11. $T=\frac{P V}{n R}$
12. $\quad P_{2}=\frac{P_{1} V_{1}}{V_{2}}$
13. $\quad c=\frac{q}{m(\Delta T)}$
14. $\quad P_{2}=\frac{P_{1} T_{2}}{T_{1}}$
15. $\quad V=\frac{m}{d}$
(This happens a lot: if there's a single item in the denominator, you can switch it with the single item on the other side of the equals sign, in this case switching $V$ and $d$.)
16. $c=\frac{a b y z}{x}$
17. $y=\frac{c x}{a b z}$
18. $T_{1}=\frac{P_{1} V_{1} T_{2}}{P_{2} V_{2}}$
19. $\quad P_{1}=\frac{P_{2} V_{2} n_{1} T_{1}}{n_{2} T_{2} V_{1}}$
20. $\quad V_{2}=\frac{P_{1} V_{1} n_{2} T_{2}}{n_{1} T_{1} P_{2}}$
21. $n_{2}=\frac{P_{2} V_{2} n_{1} T_{1}}{P_{1} V_{1} T_{2}}$
22. $T_{1}=\frac{P_{1} V_{1} n_{2} T_{2}}{n_{1} P_{2} V_{2}}$
23. $a=\frac{3 x}{y}-b$
or
$a=\frac{3 x-y b}{y}$
(after multiplying by $(a+b)$, there are two different methods, just like \#4 \& \#5 above)
