<u>Algebra 3</u>

Grade Level:	11-12
Prerequisite:	Successful completion of Algebra II
Length:	1 Year
Period(s) Per Day:	1
Credit:	1
Credit Requirement Fulfilled:	Mathematics

<u>Course Description:</u>

The central idea of Algebra III is the study of functions. We will study functions and their inverses as follows: numerically; (as a table of input-output pairs) symbolically; (as a formula describing or modeling real world phenomena) and graphically; (as a plot of inputs vs. outputs.) We will use technology to help break much of the tedious graphing and number crunching so concepts can be focused upon more. Exponential and logarithmic functions will be covered along with sequences and series. If time allows trigonometry will be covered, focusing both on its mathematical behavior and its application to the real world. **This course can be dual credit with MATH 121 within the Montana University System.**

<u>Theme Samples:</u>

1. Equations
2. Graphs
3. Functions
4. Systems
5. Problem Solving
6. Matrices
7. Conics
8. Sequences and Series
9. Probability
10. Trigonometry

<u>Course Objectives and Expectations:</u>

To extend the properties of exponents to rational exponents.
To use properties of rational and irrational numbers.
To reason quantitatively and use units to solve problems.
To perform arithmetic operations with complex numbers.
To represent complex numbers and their operations on the complex plane.
To use complex numbers in polynomial identities and equations.
To know the Fundamental Theorem of Algebra
To perform operations on matrices and use matrices in applications.
To interpret the structure of expressions.
To write expressions in equivalent forms to solve problems.
To perform arithmetic operations on polynomials

To understand the relationship between zeros and factors of polynomials.
To use polynomial identities to solve problems.
To rewrite rational expressions.
To create equations that describe numbers or relationships.
To solve equations and inequalities with one or more variables.
To understand solving equations as a process of reasoning and explain the reasoning.
To represent and solve equations and inequalities graphically.
To understand the concept of a function and use function notation.
To interpret functions that arise in applications in terms of the context.
To analyze functions using different representations.
To use technology to graph functions.
To use technology to identify information from a graph.

Pacing	**Timeline**
Semester 1	
Review	1 weeks
Chapter 1 Equations and Graphs	4 weeks
Chapter 2 Functions	4 weeks
Chapter 3 Polynomial and Rational Functions	4 weeks
Chapter 4 Exponential and Logarithmic Functions	3 weeks
Semester exam	
Semester 2	
Chapter 5 Systems of Equations and Inequalities	3 weeks
Chapter 6 Matrices and Determinants	3 weeks
Chapter 7 Conic Sections	4 weeks
Chapter 8 Sequences and Series	4 weeks
Chapter 9 Counting and Probability	3 weeks
Semester exam	

Resources:
Textbook**:** College Algebra
Authors: James Stewart, Lothar Redlin, Saleem Watson
Company: Cengage Learning
Copyright: 2016
Solution Manuel that is downloaded onto my computer.

Montana Standards for Algebra 3

The Real Number System N-RN

Extend the properties of exponents to rational exponents.

1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define 51/3 to be the cube root of 5 because we want (51/3)3 = 5(1/3)3 to hold, so (51/3)3 must equal 5.*

2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Use properties of rational and irrational numbers.

3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Quantities N-Q

Reason quantitatively and use units to solve problems.

1. Use units as a way to understand problems from a variety of contexts (e.g., science, history, and culture), including those of Montana American Indians, and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

2. Define appropriate quantities for the purpose of descriptive modeling.

3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

The Complex Number System N-CN

Perform arithmetic operations with complex numbers.

1. Know there is a complex number *i* such that *i*2 = –1, and every complex number has the form *a* + *bi* with *a* and *b* real.

2. Use the relation *i*2 = –1 and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

3. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

Represent complex numbers and their operations on the complex plane.

4. (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.

5. (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example, (-1 + √3 i)3 = 8 because (-1 + √3 i) has modulus 2 and argument 120°.*

6. (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

Use complex numbers in polynomial identities and equations.

7. Solve quadratic equations with real coefficients that have complex solutions.

8. (+) Extend polynomial identities to the complex numbers. *For example, rewrite x2 + 4 as (x + 2i)(x – 2i).*

9. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Vector and Matrix Quantities **N-VM**

Perform operations on matrices and use matrices in applications.

6. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
7. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
8. (+) Add, subtract, and multiply matrices of appropriate dimensions.
9. (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
10. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
11. (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
12. (+) Work with 2×2 matrices as a transformation of the plane, and interpret the absolute value of the determinant in terms of area.

Seeing Structure in Expressions **A-SSE**

Interpret the structure of expressions.

1. Interpret expressions that represent a quantity in terms of its context.
   a. Interpret parts of an expression, such as terms, factors, and coefficients.
   b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret P(1+r)n as the product of P and a factor not depending on P.*
2. Use the structure of an expression to identify ways to rewrite it. *For example, see x4 – y4 as (x2)2 – (y2)2, thus recognizing it as a difference of squares that can be factored as (x2 – y2)(x2 + y2).*

Write expressions in equivalent forms to solve problems.

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
   a. Factor a quadratic expression to reveal the zeros of the function it defines.
   b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
   c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression 1.15t can be rewritten as (1.151/12)12t ≈ 1.01212t to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*
4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*

Arithmetic with Polynomials and Rational Expressions A-APR

Perform arithmetic operations on polynomials.

1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Understand the relationship between zeros and factors of polynomials.

2. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x – a$ is $p(a)$, so $p(a) = 0$ if and only if $(x – a)$ is a factor of $p(x)$.
3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Use polynomial identities to solve problems.

4. Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity (x2 + y2)2 = (x2 – y2)2 + (2xy)2 can be used to generate Pythagorean triples.*
5. (+) Know and apply the Binomial Theorem for the expansion of $(x + y)n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.1

Rewrite rational expressions.

6. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.
7. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Creating Equations A-CED

Create equations that describe numbers or relationships.

1. Create equations and inequalities in one variable and use them to solve problems from a variety of contexts (e.g., science, history, and culture), including those of Montana American Indians. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*
4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*

Reasoning with Equations and Inequalities A-REI

Understand solving equations as a process of reasoning and explain the reasoning.

1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

Solve equations and inequalities in one variable.

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
4. Solve quadratic equations in one variable.
    a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)2 = q$ that has the same solutions. Derive the quadratic formula from this form.
    b. Solve quadratic equations by inspection (e.g., for $x2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.
5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = –3x$ and the circle $x2 + y2 = 3$.
8. (+) Represent a system of linear equations as a single matrix equation in a vector variable.
9. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3×3 or greater).

Represent and solve equations and inequalities graphically.

10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
11. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
12. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Interpreting Functions F-IF

Understand the concept of a function and use function notation.

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
3. Recognize that sequences are functions, sometimes defined recursively, whose domain Is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1)$ *for* $n \geq 1$.

Interpret functions that arise in applications in terms of the context.

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*.

5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*

6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Analyze functions using different representations.

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

   a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

   b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

   c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

   d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

   e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

   a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

   b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, y = (1.2)t/10, and classify them as representing exponential growth or decay.

9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Building Functions F-BF

Build a function that models a relationship between two quantities.

1. Write a function that describes a relationship between two quantities.

   a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

   b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a* *constant function to a decaying exponential, and relate these functions to the model.*

c. (+) Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*

2. Write arithmetic and geometric sequences both recursively and with an explicit and culture, including those of the Montana American Indian), and translate between the two forms.

Build new functions from existing functions.

3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\, f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

4. Find inverse functions.

a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example, f(x) =2 x3 or f(x) = (x+1)/(x–1) for x ≠ 1.*

b. (+) Verify by composition that one function is the inverse of another.

c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.

d. (+) Produce an invertible function from a non-invertible function by restricting the domain.

5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Linear, Quadratic, and Exponential Models F-LE

Construct and compare linear, quadratic, and exponential models and solve problems.

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.

a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

4. For exponential models, express as a logarithm the solution to $abct = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

Interpret expressions for functions in terms of the situation they model.

5. Interpret the parameters in a linear or exponential function in terms of a context.

Trigonometric Functions F-TF

Extend the domain of trigonometric functions using the unit circle.

1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosines, and tangent for x, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.
4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Similarity, Right Triangles, and Trigonometry G-SRT

Define trigonometric ratios and solve problems involving right triangles.

6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
7. Explain and use the relationship between the sine and cosine of complementary angles.
8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Apply trigonometry to general triangles.

9. (+) Derive the formula $A = 1/2\ ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
10. (+) Prove the Laws of Sines and Cosines and use them to solve problems.
11. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Modeling with Geometry G-MG

Apply geometric concepts in modeling situations.

1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder; modeling a Montana American Indian tipi as a cone).
2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).
3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

Conditional Probability and the Rules of Probability S-CP

Understand independence and conditional probability and use them to interpret data.

1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").
2. Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
3. Understand the conditional probability of A given B as $P(A \text{ and } B)/P(B)$, and interpret independence of A and B as saying that the conditional probability of A given B is the

same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.

4. Construct and interpret two-way frequency tables of data including information from Montana American Indian data sources when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*

5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*

Use the rules of probability to compute probabilities of compound events in a uniform probability model.

6. Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model.

7. Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.

8. (+) Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.

9. (+) Use permutations and combinations to compute probabilities of compound events and solve problems.

Resources*:*

Textbook: College Algebra

Textbook Resources: College Algebra, Teacher Resources

HPS Technology Standards

Montana Common Core State Standards