

# Salisbury School

## Summer Training Packet

### Grow Your Skills for Success

For Students Entering Precalculus/Honors Precalculus

Name: \_\_\_\_\_

- 1) This packet is an opportunity for you to sharpen your math skills so you that you enter the new school year with confidence.
- 2) Complete all work in the packet or on a separate sheet of paper. Please do the work on your own. This is your practice, and you will only get better if you do them yourself.
- 3) Bring the packet with you on the first day of classes.

**Summer Review Packet for Students Entering Pre-Calculus**

**Radicals:**

To simplify means that 1) no radicand has a perfect square factor and

2) there is no radical in the denominator (rationalize).

Recall the **Product Property**  $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$  and the **Quotient Property**  $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

**Examples:** Simplify  $\sqrt{24} = \sqrt{4} \cdot \sqrt{6}$  find the perfect square factor

$$= 2\sqrt{6} \quad \text{simplify}$$

$$\text{Simplify } \sqrt{\frac{7}{2}} = \frac{\sqrt{7}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \quad \text{multiply numerator \& denominator by } \sqrt{2}$$

$$= \frac{\sqrt{14}}{\sqrt{4}} = \frac{\sqrt{14}}{2} \quad \text{multiply straight across and simplify}$$

If the denominator contains 2 terms, multiply the numerator and denominator by conjugate of the denominator (the conjugate of  $3 + \sqrt{2}$  is  $3 - \sqrt{2}$ )

**Simplify each of the following.**

1.  $\sqrt{32}$

2.  $\sqrt{(2x)^8}$

3.  $\sqrt[3]{-64}$

4.  $\sqrt{49m^2n^8}$

5.  $\sqrt{\frac{11}{9}}$

6.  $\sqrt{60} \cdot \sqrt{105}$

7.  $(\sqrt{5} - \sqrt{6})(\sqrt{5} + \sqrt{2})$

**Rationalize.**

8.  $\frac{1}{\sqrt{2}}$

9a.  $\frac{2}{\sqrt{3}}$

10a.  $\frac{3}{2 - \sqrt{5}}$

**Complex Numbers:**

Form of complex number:  $a + bi$

Where  $a$  is the real part and the  $bi$  is the imaginary part

Always make these substitutions  $\sqrt{-1} = i$  and  $i^2 = -1$

To simplify: pull out the  $\sqrt{-1}$  before performing any operation

Example: $\sqrt{-5} = \sqrt{-1} \cdot \sqrt{5}$ Pull out $\sqrt{-1}$ $= i\sqrt{5}$ Make substitution	Example: $(i\sqrt{5})^2 = i\sqrt{5} \cdot i\sqrt{5}$ $= i^2 \sqrt{25} = (-1)(5) = -5$
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Treat  $i$  like any other variable when  $+$ ,  $-$ ,  $\times$ , or  $\div$  (but always simplify  $i^2 = -1$ )

Example:	$2i(3 + i) = 2(3i) + 2i(i)$ $= 6i + 2i^2$ $= 6i + 2(-1)$ $= -2 + 6i$	Distribute Simplify Substitute Simplify and rewrite in complex form
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Since  $i = \sqrt{-1}$ , no answer can have an 'i' in the denominator. RATIONALIZE!

**Simplify.**

9b.  $\sqrt{-49}$

10b.  $6\sqrt{-12}$

11.  $-6(2 - 8i) + 3(5 + 7i)$

12.  $(3 - 4i)^2$

13.  $(6 - 4i)(6 + 4i)$

**Rationalize.**

14.  $\frac{1 + 6i}{5i}$

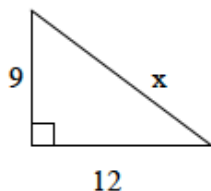
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**Geometry:**

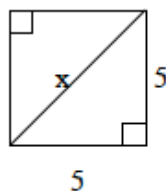
Pythagorean Theorem (right triangles):  $a^2 + b^2 = c^2$

**Find the value of x.**

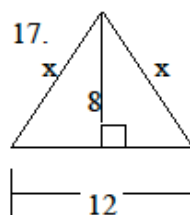
15.



16.

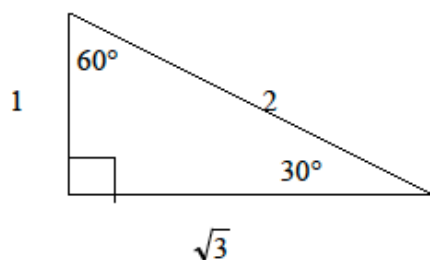


17.

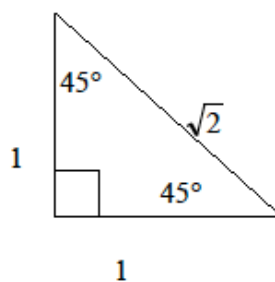


18. A square has perimeter 12 cm. Find the length of the diagonal.

\* In  $30^\circ - 60^\circ - 90^\circ$  triangles, sides are in proportion  $1, \sqrt{3}, 2$ .

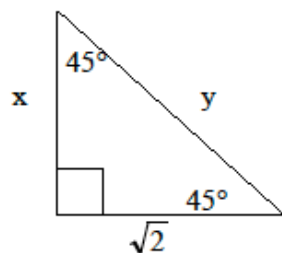


\*In  $45^\circ - 45^\circ - 90^\circ$  triangles, sides are in proportion  $1, 1, \sqrt{2}$ .

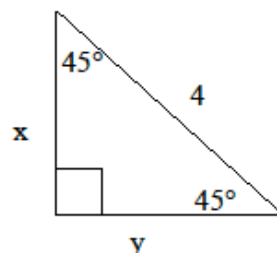


**Solve for x and y.**

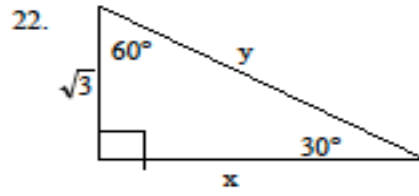
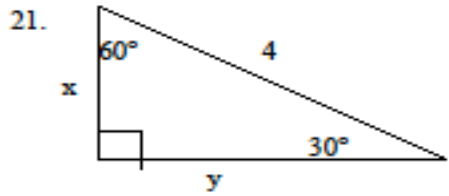
19.



20.



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Equations of Lines:

Slope-intercept form:  $y = mx + b$

Vertical line:  $x = c$  (slope is undefined)

Point-slope form:  $y - y_1 = m(x - x_1)$

Horizontal line:  $y = c$  (slope is zero)

Standard Form:  $Ax + By = C$

Slope:  $m = \frac{y_2 - y_1}{x_2 - x_1}$

23. State the slope and y-intercept of the linear equation:  $5x - 4y = 8$

24. Find the x-intercept and y-intercept of the equation:  $2x - y = 5$

25. Write the equation in standard form:  $y = 7x - 5$

**Write the equation of the line in slope-intercept form with the following conditions:**

26. slope = -5 and passes through the point (-3, -8)

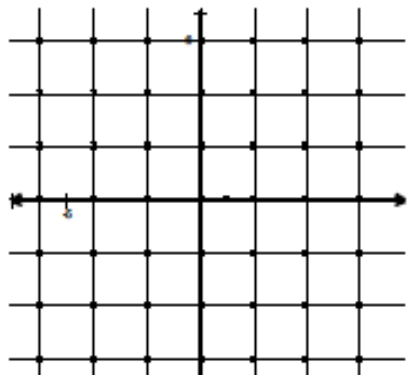
27. passes through the points (4, 3) and (7, -2)

28. x-intercept = 3 and y-intercept = 2

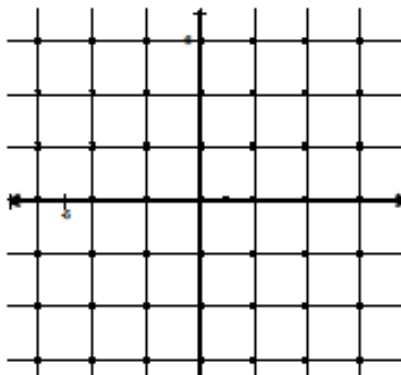
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**Graphing:** Graph each function, inequality, and/or system.

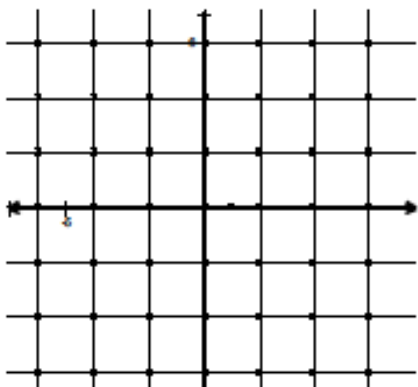
29.  $3x - 4y = 12$



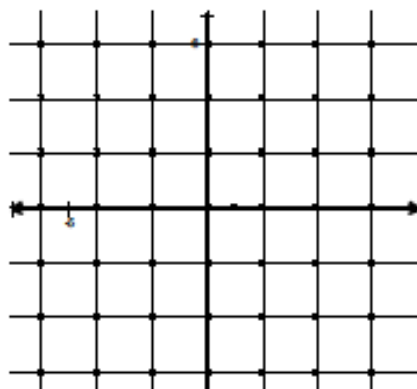
30. 
$$\begin{cases} 2x + y = 4 \\ x - y = 2 \end{cases}$$



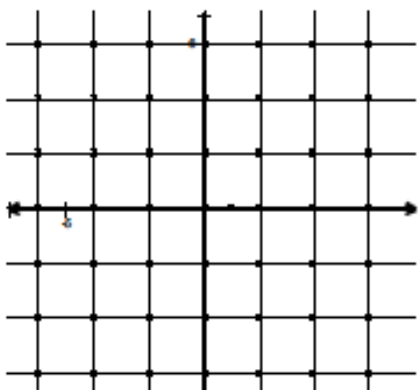
31.  $y < -4x - 2$



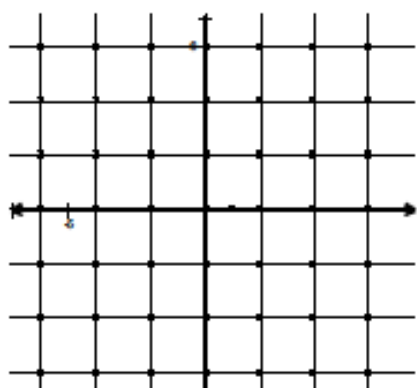
32.  $y + 2 = |x + 1|$



33.  $y > |x| - 1$



34.  $y + 4 = (x - 1)^2$



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**Systems of Equations:**

$$\begin{cases} 3x + y = 6 \\ 2x - 2y = 4 \end{cases}$$

**Substitution:**

Solve 1 equation for 1 variable

Rearrange.

Plug into 2<sup>nd</sup> equation.

Solve for the other variable.

Then plug answer back into an original equation to solve for the 2<sup>nd</sup> variable.

$$y = 6 - 3x$$

Solve 1<sup>st</sup> equation for y

$$2x - 2(6 - 3x) = 4$$

Plug into 2<sup>nd</sup> equation

$$2x - 12 + 6x = 4$$

Distribute

$$8x = 16 \text{ and } x = 2$$

Simplify

**Elimination:**

Find opposite coefficients for 1 variable

Multiply equation(s) by constant(s).

Add equations together (lose 1 variable)

Solve for variable.

$$6x + 2y = 12$$

Multiply 1<sup>st</sup> equation by 2

$$\underline{2x - 2y = 4}$$

coefficients of y are opposite

$$8x = 16$$

Add

$$x = 2$$

Simplify.

Plug x=2 back into the original equation  $\begin{matrix} 6 + y = 6 \\ y = 0 \end{matrix}$

**Solve each system of equations, using any method.**

35.  $\begin{cases} 2x + y = 4 \\ 3x + 2y = 1 \end{cases}$

36.  $\begin{cases} 2x + y = 4 \\ 3x - y = 14 \end{cases}$

37.  $\begin{cases} 2w - 5z = 13 \\ 6w + 3z = 10 \end{cases}$

**Exponents:**

**Recall the following rules of exponents:**

1.  $a^1 = a$  Any number raised to the power of one equals itself.
2.  $1^a = 1$  One raised to any power is one.
3.  $a^0 = 1$  Any nonzero number raised to the power of zero is one.
4.  $a^m \cdot a^n = a^{m+n}$  When multiplying two powers that have the same base, add the exponents.
5.  $\frac{a^m}{a^n} = a^{m-n}$  When dividing two powers with the same base, subtract the exponents.
6.  $(a^m)^n = a^{mn}$  When a power is raised to another power, multiply the exponents.
7.  $a^{-n} = \frac{1}{a^n}$  and  $\frac{1}{a^{-n}} = a^n$  Any nonzero number raised to a negative power equals its reciprocal raised to the opposite positive power.

Express each of the following in simplest form. Answers should not have any negative exponents.

38.  $5a^0$

39.  $\frac{3c}{c^{-1}}$

40.  $\frac{2ef^{-1}}{e^{-1}}$

41.  $\frac{(n^3 p^{-1})^2}{(np)^{-2}}$

Simplify.

42.  $3m^2 \bullet 2m$

43.  $(a^3)^2$

44.  $(-b^3 c^4)^5$

45.  $4m(3a^2 m)$



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**Polynomials:**

To add/subtract polynomials, combine like terms.

EX:  $8x - 3y + 6 - (6y + 4x - 9)$  *Distribute the negative through the parentheses.*  
 $= 8x - 3y + 6 - 6y - 4x + 9$  *Combine like terms with similar variables.*  
 $= 8x - 4x - 3y - 6y + 6 + 9$   
 $= 4x - 9y + 15$

**Simplify.**

46.  $3x^3 + 9 + 7x^2 - x^3$

47.  $7m - 6 - (2m + 5)$

To multiply two binomials, use FOIL.

EX:  $(3x - 2)(x + 4)$  *Multiply the first, outer, inner, and last terms.*  
 $= 3x^2 + 12x - 2x - 8$  *Combine like terms together.*  
 $= 3x^2 + 10x - 8$

**Multiply.**

48.  $(3a + 1)(a - 2)$

49.  $(s + 3)(s - 3)$

50.  $(c - 5)^2$

51.  $(5x + 7y)(5x - 7y)$

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**Factoring:**

Follow these steps in order to factor polynomials.

**STEP 1:** Look for a GCF in ALL of the terms.

- a) If you have one (other than 1) factor it out.
- b) If you don't have one move on to STEP 2

**STEP 2:** How many terms does the polynomial have?

**2 Terms** a) is it the difference of two squares?  $a^2 - b^2 = (a + b)(a - b)$

**EX:**  $x^2 - 25 = (x + 5)(x - 5)$

b) Is it the sum or difference of two cubes?  $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$   
 $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

**EX:**  $m^3 + 64 = (m + 4)(m^2 - 4m + 16)$   
 $p^3 - 125 = (p - 5)(p^2 + 5p + 25)$

**3 Terms**

**EX:**

$$x^2 + bx + c = (x + \underline{\quad})(x + \underline{\quad})$$

$$x^2 + 7x + 12 = (x + 3)(x + 4)$$

$$x^2 - bx - c = (x - \underline{\quad})(x - \underline{\quad})$$

$$x^2 - 5x + 4 = (x - 1)(x - 4)$$

$$x^2 + bx - c = (x - \underline{\quad})(x + \underline{\quad})$$

$$x^2 + 6x - 16 = (x - 2)(x + 8)$$

$$x^2 - bx - c = (x - \underline{\quad})(x + \underline{\quad})$$

$$x^2 - 2x - 24 = (x - 6)(x + 4)$$

**4 Terms---Factor by Grouping**

- a) Pair up first two terms and last two terms.
- b) Factor out GCF of each pair of numbers.
- c) Factor out front parentheses that the terms have in common.
- d) Put leftover terms in parentheses.

$$\begin{aligned} \text{Ex: } x^3 + 3x^2 + 9x + 27 &= (x^3 + 3x^2) + (9x + 27) \\ &= x^2(x + 3) + 9(x + 3) \\ &= (x + 3)(x^2 + 9) \end{aligned}$$

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**Factor completely.**

52.  $z^2 + 4z - 12$

53.  $6 - 5x - x^2$

54.  $2k^2 + 2k - 60$

55.  $-10b^4 - 15b^2$

56.  $9c^2 + 30c + 25$

57.  $9n^2 - 4$

58.  $27z^3 - 8$

59.  $2mn - 2mt + 2sn - 2st$

**To solve quadratic equations, try to factor first and set each factor equal to zero. Solve for your variable. If the quadratic does NOT factor, use the quadratic formula.**

**EX:**  $x^2 - 4x = 21$       *Set equal to zero FIRST.*

$x^2 - 4x - 21 = 0$       *Now factor.*

$(x + 3)(x - 7) = 0$       *Set each factor equal to zero.*

$x + 3 = 0$      $x - 7 = 0$     *Solve for each  $x$ .*

$x = -3$      $x = 7$

**Solve each equation.**

60.  $x^2 - 4x - 12 = 0$

61.  $x^2 + 25 = 10x$

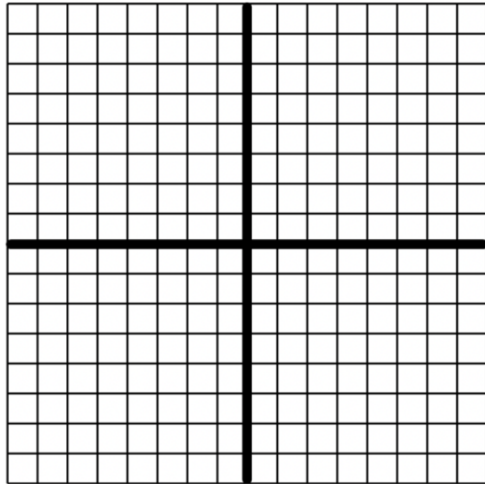
62.  $x^2 - 14x + 40 = 0$

Vertex form:  $y = a(x - h)^2 + k$

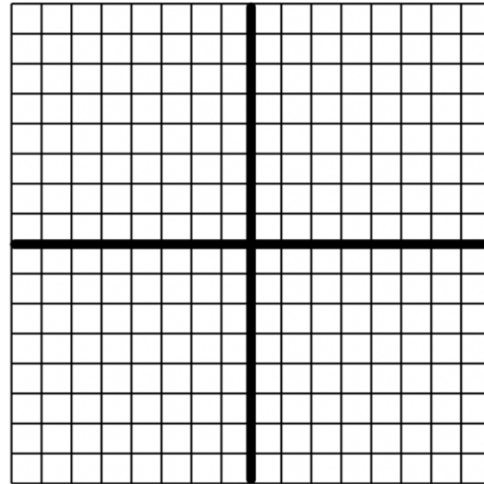
Standard form:  $y = ax^2 - bx + c$

63. Graph each of the following quadratic functions. Be sure to show the vertex and two other points on the graph.

a)  $y = 2(x - 3)^2 + 4$



b)  $y = x^2 - x - 12$



64. Convert to standard form:  $y = \frac{1}{2}(x + 6)^2 - 2$

65. Convert to vertex form by completing the square.  $y = 3x^2 - 6x + 5$

**Synthetic Division**—can ONLY be used when dividing a polynomial by a linear polynomial.

EX:  $\frac{2x^3 + 3x^2 - 6x + 10}{x + 3}$

**Long Division**

$$\frac{2x^3 + 3x^2 - 6x + 10}{x + 3}$$

$$\begin{array}{r} 2x^2 - 3x + 3 + \frac{1}{x+3} \\ x+3 \overline{) 2x^3 + 3x^2 - 6x + 10} \\ \underline{(-)(2x^3 + 6x^2)} \phantom{+ 10} \\ -3x^2 - 6x \phantom{+ 10} \\ \underline{(-)(-3x^2 - 9x)} \phantom{+ 10} \\ 3x + 10 \\ \underline{(-)(3x + 9)} \\ 1 \end{array}$$

**Synthetic Division**

$$\frac{2x^3 + 3x^2 - 6x + 10}{x + 3}$$

$$\begin{array}{r} 2x^3 + 3x^2 - 6x + 10 \\ x + 3 \end{array}$$

$$\begin{array}{rrrrrr} -3 & 2 & 3 & -6 & 10 & \\ & -6 & 9 & -9 & & \\ & 2 & -3 & 3 & 1 & \\ & & & & & 1 \\ & & & & & = 2x - 3x + 3 + \frac{1}{x+3} \end{array}$$

Divide each polynomial using long division OR synthetic division.

66.  $\frac{c^3 - 3c^2 + 18c - 16}{c^2 + 3c - 2}$

67.  $\frac{x^4 - 2x^2 - x + 2}{x + 2}$

To evaluate a function for the given value, simply plug the value into the function for x.

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**Evaluate each function for the given value.**

68.  $f(x) = x^2 - 6x + 2$

$f(3) = \underline{\hspace{2cm}}$

69.  $g(x) = 6x - 7$

$g(x+h) = \underline{\hspace{2cm}}$

70.  $f(x) = 3x^2 - 4$

$5[f(x+2)] = \underline{\hspace{2cm}}$

**Composition and Inverses of Functions:**

**Recall:**  $(f \circ g)(x) = f(g(x))$  OR  $f[g(x)]$  read “**f of g of x**” means to plug the inside function in for x in the outside function.

**Example:** Given  $f(x) = 2x^2 + 1$  and  $g(x) = x - 4$  find  $f(g(x))$ .

$$\begin{aligned} f(g(x)) &= f(x-4) \\ &= 2(x-4)^2 + 1 \\ &= 2(x^2 - 8x + 16) + 1 \\ &= 2x^2 - 16x + 32 + 1 \\ f(g(x)) &= 2x^2 - 16x + 33 \end{aligned}$$

**Suppose**  $f(x) = 2x$ ,  $g(x) = 3x - 2$ , and  $h(x) = x^2 - 4$ . **Find the following:**

71.  $f[g(2)] = \underline{\hspace{2cm}}$

72.  $f[g(x)] = \underline{\hspace{2cm}}$

73.  $f[h(3)] = \underline{\hspace{2cm}}$

74.  $g[f(x)] = \underline{\hspace{2cm}}$

To find the inverse of a function, simply switch the x and the y and solve for the new “y” value.

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<b>Example:</b>	$f(x) = \sqrt[3]{x+1}$	Rewrite $f(x)$ as $y$
	$y = \sqrt[3]{x+1}$	Switch $x$ and $y$
	$x = \sqrt[3]{y+1}$	Solve for your new $y$
	$(x)^3 = (\sqrt[3]{y+1})^3$	Cube both sides
	$x^3 = y+1$	Simplify
	$y = x^3 - 1$	Solve for $y$
	$f^{-1}(x) = x^3 - 1$	Rewrite in inverse notation

**Find the inverse,  $f^{-1}(x)$ , if possible.**

75.  $f(x) = 5x + 2$

76.  $f(x) = \frac{1}{2}x - \frac{1}{3}$

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***Multiplying and Dividing:*** Factor numerator and denominator completely. Cancel any common factors in the top and bottom. If dividing, change divide to multiply and flip the second fraction.

**EX:**  $\frac{x^2 + 10x + 21}{5 - 4x - x^2} \bullet \frac{x^2 + 2x - 15}{x^3 + 4x^2 - 21x}$

*Factor everything completely.*

$$= \frac{(x+7)(x+3)}{(5+x)(1-x)} \bullet \frac{(x+5)(x-3)}{x(x-3)(x+7)}$$

*Cancel out common factors in the top and bottom.*

$$= \frac{(x+3)}{x(1-x)}$$

*Simplify.*

77.  $\frac{5z^3 + z^2 - z}{3z}$

78.  $\frac{m^2 - 25}{m^2 + 5m}$

79.  $\frac{10r^5}{21s^2} \bullet \frac{3s}{5r^3}$

80.  $\frac{a^2 - 5a + 6}{a + 4} \bullet \frac{3a + 12}{a - 2}$

81.  $\frac{6d - 9}{5d + 1} \div \frac{6 - 13d + 6d^2}{15d^2 - 7d - 2}$

**Addition and Subtraction**



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First find the least common denominator. Write each fraction with that LCD. Add/subtract numerators as indicated and leave the denominators as they are.

EX:  $\frac{3x+1}{x^2+2x} + \frac{5x-4}{2x+4}$  *Factor denominator completely.*

$\frac{3x+1}{x(x+2)} + \frac{5x-4}{2(x+2)}$  *Find LCD, which is  $(2x)(x+2)$*

$\frac{2(3x+1)}{2x(x+2)} + \frac{x(5x-4)}{2x(x+2)}$  *Rewrite each fraction with the LCD in the denominator.*

$\frac{6x+2+5x^2-4x}{2x(x+2)}$  *Write as one fraction.*

$\frac{5x^2+2x+2}{2x(x+2)}$  *Combine like terms.*

82.  $\frac{2x}{5} - \frac{x}{3}$

83.  $\frac{b-a}{a^2b} + \frac{a+b}{ab^2}$

84.  $\frac{2-a^2}{a^2+a} + \frac{3a+4}{3a+3}$

85. Given:  $x = \frac{2}{3}$  and  $y = \frac{1}{9}$  find and simplify each of the following:

a)  $x + y$

b)  $x - y$

c)  $\frac{x}{y}$

d)  $2x - 3y$

86.  $\frac{1 - \frac{1}{2}}{2 + \frac{1}{4}}$

87.  $\frac{1 + \frac{1}{x}}{x+1}$

**Solving Rational Equations:**

Multiply each term by the LCD of all the fractions. This should eliminate all of our fractions. Then solve the equation as usual.

$$\frac{5}{x+2} + \frac{1}{x} = \frac{5}{x}$$

*Find LCD first  $x(x+2)$*

$$x(x+2)\frac{5}{x+2} + x(x+2)\frac{1}{x} = \frac{5}{x}x(x+2)$$

*Multiply each term by the LCD.*

$$5x + 1(x+2) = 5(x+2)$$

*Simplify and solve.*

$$5x + x + 2 = 5x + 10$$

$$6x + 2 = 5x + 10$$

$$x = 8 \quad \leftarrow \text{Check your answer! Sometimes they do not check!}$$

Check:  $\frac{5}{8+2} + \frac{1}{8} = \frac{5}{8}$

$$\frac{5}{10} + \frac{1}{8} = \frac{5}{8}$$

$$\frac{5}{8} = \frac{5}{8}$$

***Solve each equation. Check your solutions.***

88.  $\frac{12}{x} + \frac{3}{4} = \frac{3}{2}$

89.  $\frac{x+10}{x^2-2} = \frac{4}{x}$

90.  $\frac{5}{x-5} = \frac{x}{x-5} - 1$