

# Calculus 1



# Summer Review 2023

**Each set of problems comes with an answer key to check the completed work.**

## Multiplying/Dividing Fractions and Mixed Numbers

Date \_\_\_\_\_ Period \_\_\_\_\_

**Find each product.**

1)  $-\frac{5}{4} \cdot \frac{1}{3}$

2)  $\frac{8}{7} \cdot \frac{7}{10}$

3)  $\frac{4}{9} \cdot \frac{7}{4}$

4)  $-\frac{2}{3} \cdot \frac{5}{4}$

5)  $-2 \cdot \frac{3}{7}$

6)  $-2\frac{2}{3} \cdot 4\frac{1}{10}$

7)  $-2\frac{1}{5} \cdot -1\frac{3}{4}$

8)  $-1\frac{1}{4} \cdot 9$

9)  $-1\frac{5}{7} \cdot -2\frac{1}{2}$

10)  $-2\frac{3}{8} \cdot 2\frac{1}{2}$

**Find each quotient.**

$$11) \frac{-1}{5} \div \frac{7}{4}$$

$$12) \frac{-1}{2} \div \frac{5}{4}$$

$$13) \frac{-3}{2} \div \frac{-10}{7}$$

$$14) \frac{1}{2} \div \frac{8}{7}$$

$$15) \frac{-9}{5} \div 2$$

$$16) -3\frac{5}{9} \div 3$$

$$17) -2 \div -3\frac{4}{5}$$

$$18) \frac{1}{9} \div -1\frac{1}{3}$$

$$19) 1\frac{6}{7} \div 5\frac{3}{4}$$

$$20) -3\frac{7}{10} \div 2\frac{1}{4}$$

## Multiplying/Dividing Fractions and Mixed Numbers

Find each product.

1)  $-\frac{5}{4} \cdot \frac{1}{3}$

$-\frac{5}{12}$

2)  $\frac{8}{7} \cdot \frac{7}{10}$

$\frac{4}{5}$

3)  $\frac{4}{9} \cdot \frac{7}{4}$

$\frac{7}{9}$

4)  $-\frac{2}{3} \cdot \frac{5}{4}$

$-\frac{5}{6}$

5)  $-2 \cdot \frac{3}{7}$

$-\frac{6}{7}$

6)  $-2\frac{2}{3} \cdot 4\frac{1}{10}$

$-10\frac{14}{15}$

7)  $-2\frac{1}{5} \cdot -1\frac{3}{4}$

$3\frac{17}{20}$

8)  $-1\frac{1}{4} \cdot 9$

$-11\frac{1}{4}$

9)  $-1\frac{5}{7} \cdot -2\frac{1}{2}$

$4\frac{2}{7}$

10)  $-2\frac{3}{8} \cdot 2\frac{1}{2}$

$-5\frac{15}{16}$

Find each quotient.

$$11) \frac{-1}{5} \div \frac{7}{4}$$
$$-\frac{4}{35}$$

$$12) \frac{-1}{2} \div \frac{5}{4}$$
$$-\frac{2}{5}$$

$$13) \frac{-3}{2} \div \frac{-10}{7}$$
$$\frac{21}{20}$$

$$14) \frac{1}{2} \div \frac{8}{7}$$
$$\frac{7}{16}$$

$$15) \frac{-9}{5} \div 2$$
$$-\frac{9}{10}$$

$$16) -3\frac{5}{9} \div 3$$
$$-1\frac{5}{27}$$

$$17) -2 \div -3\frac{4}{5}$$
$$\frac{10}{19}$$

$$18) \frac{1}{9} \div -1\frac{1}{3}$$
$$-\frac{1}{12}$$

$$19) 1\frac{6}{7} \div 5\frac{3}{4}$$
$$\frac{52}{161}$$

$$20) -3\frac{7}{10} \div 2\frac{1}{4}$$
$$-1\frac{29}{45}$$

## Factoring Trinomials (a = 1)

**Factor each completely.**

1)  $b^2 + 8b + 7$

2)  $n^2 - 11n + 10$

3)  $m^2 + m - 90$

4)  $n^2 + 4n - 12$

5)  $n^2 - 10n + 9$

6)  $b^2 + 16b + 64$

7)  $m^2 + 2m - 24$

8)  $x^2 - 4x + 24$

9)  $k^2 - 13k + 40$

10)  $a^2 + 11a + 18$

11)  $n^2 - n - 56$

12)  $n^2 - 5n + 6$

$$13) b^2 - 6b + 8$$

$$14) n^2 + 6n + 8$$



$$15) 2n^2 + 6n - 108$$

$$16) 5n^2 + 10n + 20$$

$$17) 2k^2 + 22k + 60$$

$$18) a^2 - a - 90$$

$$19) p^2 + 11p + 10$$

$$20) 5v^2 - 30v + 40$$

$$21) 2p^2 + 2p - 4$$

$$22) 4v^2 - 4v - 8$$

$$23) x^2 - 15x + 50$$

$$24) v^2 - 7v + 10$$

$$25) p^2 + 3p - 18$$

$$26) 6v^2 + 66v + 60$$

## Factoring Trinomials (a = 1)

Factor each completely.

1)  $b^2 + 8b + 7$

$(b + 7)(b + 1)$

2)  $n^2 - 11n + 10$

$(n - 10)(n - 1)$

3)  $m^2 + m - 90$

$(m - 9)(m + 10)$

4)  $n^2 + 4n - 12$

$(n - 2)(n + 6)$

5)  $n^2 - 10n + 9$

$(n - 1)(n - 9)$

6)  $b^2 + 16b + 64$

$(b + 8)^2$

7)  $m^2 + 2m - 24$

$(m + 6)(m - 4)$

8)  $x^2 - 4x + 24$

Not factorable

9)  $k^2 - 13k + 40$

$(k - 5)(k - 8)$

10)  $a^2 + 11a + 18$

$(a + 2)(a + 9)$

11)  $n^2 - n - 56$

$(n + 7)(n - 8)$

12)  $n^2 - 5n + 6$

$(n - 2)(n - 3)$



$$13) b^2 - 6b + 8$$
$$(b - 4)(b - 2)$$

$$14) n^2 + 6n + 8$$
$$(n + 2)(n + 4)$$

$$15) 2n^2 + 6n - 108$$
$$2(n + 9)(n - 6)$$

$$16) 5n^2 + 10n + 20$$
$$5(n^2 + 2n + 4)$$

$$17) 2k^2 + 22k + 60$$
$$2(k + 5)(k + 6)$$

$$18) a^2 - a - 90$$
$$(a - 10)(a + 9)$$

$$19) p^2 + 11p + 10$$
$$(p + 10)(p + 1)$$

$$20) 5v^2 - 30v + 40$$
$$5(v - 2)(v - 4)$$

$$21) 2p^2 + 2p - 4$$
$$2(p - 1)(p + 2)$$

$$22) 4v^2 - 4v - 8$$
$$4(v + 1)(v - 2)$$

$$23) x^2 - 15x + 50$$
$$(x - 10)(x - 5)$$

$$24) v^2 - 7v + 10$$
$$(v - 5)(v - 2)$$

$$25) p^2 + 3p - 18$$
$$(p - 3)(p + 6)$$

$$26) 6v^2 + 66v + 60$$
$$6(v + 10)(v + 1)$$

## Factoring Special Cases

**Factor each completely.**

1)  $16n^2 - 9$

2)  $4m^2 - 25$

3)  $16b^2 - 40b + 25$

4)  $4x^2 - 4x + 1$

5)  $9x^2 - 1$

6)  $n^2 - 25$

7)  $n^4 - 100$

8)  $a^4 - 9$

9)  $k^4 - 36$

10)  $n^4 - 49$

$$11) 98n^2 - 200$$

$$12) 3 + 6b + 3b^2$$

$$13) 400 - 36v^2$$

$$14) 100x^2 + 180x + 81$$

$$15) 10n^2 + 100n + 250$$

$$16) 49n^2 - 56n + 16$$

$$17) 49x^2 - 100$$

$$18) 1 - r^2$$

$$19) 10p^3 - 1960p$$

$$20) 343b^2 - 7b^4$$

$$21) 81v^4 - 900v^2$$

$$22) 200m^4 + 80m^3 + 8m^2$$

## Factoring Special Cases

**Factor each completely.**

1)  $16n^2 - 9$

$(4n + 3)(4n - 3)$

2)  $4m^2 - 25$

$(2m + 5)(2m - 5)$

3)  $16b^2 - 40b + 25$

$(4b - 5)^2$

4)  $4x^2 - 4x + 1$

$(2x - 1)^2$

5)  $9x^2 - 1$

$(3x + 1)(3x - 1)$

6)  $n^2 - 25$

$(n + 5)(n - 5)$

7)  $n^4 - 100$

$(n^2 + 10)(n^2 - 10)$

8)  $a^4 - 9$

$(a^2 + 3)(a^2 - 3)$

9)  $k^4 - 36$

$(k^2 + 6)(k^2 - 6)$

10)  $n^4 - 49$

$(n^2 + 7)(n^2 - 7)$

11)  $98n^2 - 200$

$2(7n + 10)(7n - 10)$

12)  $3 + 6b + 3b^2$

$3(1 + b)^2$

13)  $400 - 36v^2$

$4(10 + 3v)(10 - 3v)$

14)  $100x^2 + 180x + 81$

$(10x + 9)^2$

15)  $10n^2 + 100n + 250$

$10(n + 5)^2$

16)  $49n^2 - 56n + 16$

$(7n - 4)^2$

17)  $49x^2 - 100$

$(7x + 10)(7x - 10)$

18)  $1 - r^2$

$(1 + r)(1 - r)$

19)  $10p^3 - 1960p$

$10p(p + 14)(p - 14)$

20)  $343b^2 - 7b^4$

$7b^2(7 + b)(7 - b)$

21)  $81v^4 - 900v^2$

$9v^2(3v + 10)(3v - 10)$

22)  $200m^4 + 80m^3 + 8m^2$

$8m^2(5m + 1)^2$

## Dividing Polynomials

**Divide.**

1)  $(m^2 - 7m - 11) \div (m - 8)$

2)  $(n^2 - n - 29) \div (n - 6)$

3)  $(n^2 + 10n + 18) \div (n + 5)$

4)  $(k^2 - 7k + 10) \div (k - 1)$

5)  $(n^2 - 3n - 21) \div (n - 7)$

6)  $(a^2 - 28) \div (a - 5)$

7)  $(r^2 + 14r + 38) \div (r + 8)$

8)  $(x^2 + 5x + 3) \div (x + 6)$

9)  $(2x^2 - 17x - 38) \div (2x + 3)$

10)  $(42x^2 - 33) \div (7x + 7)$

11)  $(x^2 - 74) \div (x - 8)$

12)  $(2p^2 + 7p - 39) \div (2p - 7)$

13)  $(n^3 + 7n^2 + 14n + 3) \div (n + 2)$

14)  $(p^3 - 10p^2 + 20p + 26) \div (p - 5)$

15)  $(v^3 - 2v^2 - 14v - 5) \div (v + 3)$

16)  $(x^3 - 13x^2 + 40x + 18) \div (x - 7)$

17)  $(k^3 - 30k - 18 - 4k^2) \div (3 + k)$

18)  $(-5k^2 + k^3 + 8k + 4) \div (-1 + k)$

19)  $(x^3 + 5x^2 - 32x - 7) \div (x - 4)$

20)  $(50k^3 + 10k^2 - 35k - 7) \div (5k - 4)$

## Dividing Polynomials

**Divide.**

1)  $(m^2 - 7m - 11) \div (m - 8)$

$$m + 1 - \frac{3}{m - 8}$$

2)  $(n^2 - n - 29) \div (n - 6)$

$$n + 5 + \frac{1}{n - 6}$$

3)  $(n^2 + 10n + 18) \div (n + 5)$

$$n + 5 - \frac{7}{n + 5}$$

4)  $(k^2 - 7k + 10) \div (k - 1)$

$$k - 6 + \frac{4}{k - 1}$$

5)  $(n^2 - 3n - 21) \div (n - 7)$

$$n + 4 + \frac{7}{n - 7}$$

6)  $(a^2 - 28) \div (a - 5)$

$$a + 5 - \frac{3}{a - 5}$$

7)  $(r^2 + 14r + 38) \div (r + 8)$

$$r + 6 - \frac{10}{r + 8}$$

8)  $(x^2 + 5x + 3) \div (x + 6)$

$$x - 1 + \frac{9}{x + 6}$$

9)  $(2x^2 - 17x - 38) \div (2x + 3)$

$$x - 10 - \frac{8}{2x + 3}$$

10)  $(42x^2 - 33) \div (7x + 7)$

$$6x - 6 + \frac{9}{7x + 7}$$



11)  $(x^2 - 74) \div (x - 8)$

$$x + 8 - \frac{10}{x - 8}$$

12)  $(2p^2 + 7p - 39) \div (2p - 7)$

$$p + 7 + \frac{10}{2p - 7}$$

13)  $(n^3 + 7n^2 + 14n + 3) \div (n + 2)$

$$n^2 + 5n + 4 - \frac{5}{n + 2}$$

14)  $(p^3 - 10p^2 + 20p + 26) \div (p - 5)$

$$p^2 - 5p - 5 + \frac{1}{p - 5}$$

15)  $(v^3 - 2v^2 - 14v - 5) \div (v + 3)$

$$v^2 - 5v + 1 - \frac{8}{v + 3}$$

16)  $(x^3 - 13x^2 + 40x + 18) \div (x - 7)$

$$x^2 - 6x - 2 + \frac{4}{x - 7}$$

17)  $(k^3 - 30k - 18 - 4k^2) \div (3 + k)$

$$k^2 - 7k - 9 + \frac{9}{3 + k}$$

18)  $(-5k^2 + k^3 + 8k + 4) \div (-1 + k)$

$$k^2 - 4k + 4 + \frac{8}{-1 + k}$$

19)  $(x^3 + 5x^2 - 32x - 7) \div (x - 4)$

$$x^2 + 9x + 4 + \frac{9}{x - 4}$$

20)  $(50k^3 + 10k^2 - 35k - 7) \div (5k - 4)$

$$10k^2 + 10k + 1 - \frac{3}{5k - 4}$$

## Solving Rational Equations

Solve each equation. Remember to check for extraneous solutions.

1)  $\frac{1}{6k^2} = \frac{1}{3k^2} - \frac{1}{k}$

2)  $\frac{1}{n^2} + \frac{1}{n} = \frac{1}{2n^2}$

3)  $\frac{1}{6b^2} + \frac{1}{6b} = \frac{1}{b^2}$

4)  $\frac{b+6}{4b^2} + \frac{3}{2b^2} = \frac{b+4}{2b^2}$

5)  $\frac{1}{x} = \frac{6}{5x} + 1$

6)  $\frac{1}{6x^2} = \frac{1}{2x} + \frac{7}{6x^2}$

7)  $\frac{1}{v} + \frac{3v+12}{v^2-5v} = \frac{7v-56}{v^2-5v}$

8)  $\frac{1}{m^2-m} + \frac{1}{m} = \frac{5}{m^2-m}$

9)  $\frac{1}{n-8} - 1 = \frac{7}{n-8}$

10)  $\frac{1}{r-2} + \frac{1}{r^2-7r+10} = \frac{6}{r-2}$

$$11) 1 = \frac{v+2}{v-4} + \frac{7v-42}{v-4}$$

$$12) \frac{r-4}{5r} = \frac{1}{5r} + 1$$

$$13) 1 + \frac{x^2 - 5x - 24}{3x} = \frac{x-6}{3x}$$

$$14) 1 = \frac{1}{x^2 + 2x} + \frac{x-1}{x}$$

$$15) \frac{n+5}{n+8} = 1 + \frac{6}{n+1}$$

$$16) \frac{r+5}{r^2 - 2r} - 1 = \frac{1}{r^2 - 2r}$$

$$17) \frac{1}{x^2 - 5x} = \frac{x+7}{x} - 1$$

$$18) \frac{a-2}{a+3} - 1 = \frac{3}{a+2}$$

$$19) \frac{p+5}{p^2 + p} = \frac{1}{p^2 + p} - \frac{p-6}{p+1}$$

$$20) \frac{5}{n^3 + 5n^2} = \frac{4}{n+5} + \frac{1}{n^2}$$

## Solving Rational Equations

Solve each equation. Remember to check for extraneous solutions.

1)  $\frac{1}{6k^2} = \frac{1}{3k^2} - \frac{1}{k}$

$\left\{\frac{1}{6}\right\}$

2)  $\frac{1}{n^2} + \frac{1}{n} = \frac{1}{2n^2}$

$\left\{-\frac{1}{2}\right\}$

3)  $\frac{1}{6b^2} + \frac{1}{6b} = \frac{1}{b^2}$

$\{5\}$

4)  $\frac{b+6}{4b^2} + \frac{3}{2b^2} = \frac{b+4}{2b^2}$

$\{4\}$

5)  $\frac{1}{x} = \frac{6}{5x} + 1$

$\left\{-\frac{1}{5}\right\}$

6)  $\frac{1}{6x^2} = \frac{1}{2x} + \frac{7}{6x^2}$

$\{-2\}$

7)  $\frac{1}{v} + \frac{3v+12}{v^2-5v} = \frac{7v-56}{v^2-5v}$

$\{21\}$

8)  $\frac{1}{m^2-m} + \frac{1}{m} = \frac{5}{m^2-m}$

$\{5\}$

9)  $\frac{1}{n-8} - 1 = \frac{7}{n-8}$

$\{2\}$

10)  $\frac{1}{r-2} + \frac{1}{r^2-7r+10} = \frac{6}{r-2}$

$\left\{\frac{26}{5}\right\}$

$$11) 1 = \frac{v+2}{v-4} + \frac{7v-42}{v-4}$$

$$\left\{ \frac{36}{7} \right\}$$

$$12) \frac{r-4}{5r} = \frac{1}{5r} + 1$$

$$\left\{ -\frac{5}{4} \right\}$$

$$13) 1 + \frac{x^2 - 5x - 24}{3x} = \frac{x-6}{3x}$$

$$\{-3, 6\}$$

$$14) 1 = \frac{1}{x^2 + 2x} + \frac{x-1}{x}$$

$$\{-1\}$$

$$15) \frac{n+5}{n+8} = 1 + \frac{6}{n+1}$$

$$\left\{ -\frac{17}{3} \right\}$$

$$16) \frac{r+5}{r^2 - 2r} - 1 = \frac{1}{r^2 - 2r}$$

$$\{4, -1\}$$

$$17) \frac{1}{x^2 - 5x} = \frac{x+7}{x} - 1$$

$$\left\{ \frac{36}{7} \right\}$$

$$18) \frac{a-2}{a+3} - 1 = \frac{3}{a+2}$$

$$\left\{ -\frac{19}{8} \right\}$$

$$19) \frac{p+5}{p^2 + p} = \frac{1}{p^2 + p} - \frac{p-6}{p+1}$$

$$\{4, 1\}$$

$$20) \frac{5}{n^3 + 5n^2} = \frac{4}{n+5} + \frac{1}{n^2}$$

$$\left\{ -\frac{1}{4} \right\}$$

## Rational Expressions

State the excluded values for each.

1)  $\frac{60x^3}{12x}$

2)  $\frac{70v^2}{100v}$

3)  $\frac{m+7}{m^2+4m-21}$

4)  $\frac{n^2+6n+5}{n+1}$

5)  $\frac{35x-35}{25x-40}$

6)  $\frac{-n^2+16n-63}{n^2-2n-35}$

Simplify each and state the excluded values.

7)  $\frac{p+4}{p^2+6p+8}$

8)  $\frac{9}{15a-15}$

9)  $\frac{2a^2+10a}{3a^2+15a}$

10)  $\frac{p^2-3p-10}{p^2+p-2}$

11)  $\frac{x^2+x-6}{x^2+8x+15}$

12)  $\frac{a^2+5a+4}{a^2+9a+20}$

13)  $\frac{x^2 - 2x - 15}{x^2 - 6x + 5}$

14)  $\frac{10x - 6}{10x - 6}$

15)  $\frac{(v-7)(v+8)}{(v+8)(v-10)} \div \frac{1}{v-10}$

16)  $\frac{n+3}{n+2} \div \frac{(n-1)(n+3)}{(n-1)^2}$

17)  $\frac{x+3}{4} \cdot \frac{3(x-6)}{3(x+3)}$

18)  $\frac{x-8}{(x+6)(x-8)} \cdot \frac{4x(x+10)}{x+10}$

19)  $\frac{2b^2 - 12b}{b+5} \div \frac{b-6}{b+5}$

20)  $\frac{1}{n+9} \div \frac{6-n}{3n-18}$

21)  $\frac{28-7b}{b-4} \cdot \frac{1}{b+10}$

22)  $\frac{2}{v^2 - 12v + 27} \cdot \frac{v^2 - 12v + 27}{3}$

23)  $\frac{1}{5p^2} \div \frac{9p-36}{5p^3-35p^2}$

24)  $\frac{8-7x-x^2}{x+8} \cdot \frac{x+5}{9x-9}$

25)  $\frac{x^2-16}{9-x} \cdot \frac{x^2+x-90}{x^2+14x+40}$

26)  $\frac{10x^2-20x}{40x^3-80x^2} \cdot \frac{16x^3+80x^2}{6x+30}$

## Rational Expressions

State the excluded values for each.

1)  $\frac{60x^3}{12x}$

 $\{0\}$ 

2)  $\frac{70v^2}{100v}$

 $\{0\}$ 

3)  $\frac{m+7}{m^2+4m-21}$

 $\{-7, 3\}$ 

4)  $\frac{n^2+6n+5}{n+1}$

 $\{-1\}$ 

5)  $\frac{35x-35}{25x-40}$

 $\left\{\frac{8}{5}\right\}$ 

6)  $\frac{-n^2+16n-63}{n^2-2n-35}$

 $\{-5, 7\}$ 

Simplify each and state the excluded values.

7)  $\frac{p+4}{p^2+6p+8}$

 $\frac{1}{p+2}; \{-2, -4\}$ 

8)  $\frac{9}{15a-15}$

 $\frac{3}{5(a-1)}; \{1\}$ 

9)  $\frac{2a^2+10a}{3a^2+15a}$

 $\frac{2}{3}; \{0, -5\}$ 

10)  $\frac{p^2-3p-10}{p^2+p-2}$

 $\frac{p-5}{p-1}; \{-2, 1\}$ 

11)  $\frac{x^2+x-6}{x^2+8x+15}$

 $\frac{x-2}{x+5}; \{-3, -5\}$ 

12)  $\frac{a^2+5a+4}{a^2+9a+20}$

 $\frac{a+1}{a+5}; \{-4, -5\}$



13)  $\frac{x^2 - 2x - 15}{x^2 - 6x + 5}$

$\frac{x+3}{x-1}; \{1, 5\}$

14)  $\frac{10x - 6}{10x - 6}$

$1; \left\{\frac{3}{5}\right\}$

15)  $\frac{(v-7)(v+8)}{(v+8)(v-10)} \div \frac{1}{v-10}$

$v-7; \{-8, 10\}$

16)  $\frac{n+3}{n+2} \div \frac{(n-1)(n+3)}{(n-1)^2}$

$\frac{n-1}{n+2}; \{-2, 1, -3\}$

17)  $\frac{x+3}{4} \cdot \frac{3(x-6)}{3(x+3)}$

$\frac{x-6}{4}; \{-3\}$

18)  $\frac{x-8}{(x+6)(x-8)} \cdot \frac{4x(x+10)}{x+10}$

$\frac{4x}{x+6}; \{-6, 8, -10\}$

19)  $\frac{2b^2 - 12b}{b+5} \div \frac{b-6}{b+5}$

$2b; \{-5, 6\}$

20)  $\frac{1}{n+9} \div \frac{6-n}{3n-18}$

$-\frac{3}{n+9}; \{-9, 6\}$

21)  $\frac{28-7b}{b-4} \cdot \frac{1}{b+10}$

$-\frac{7}{b+10}; \{4, -10\}$

22)  $\frac{2}{v^2 - 12v + 27} \cdot \frac{v^2 - 12v + 27}{3}$

$\frac{2}{3}; \{3, 9\}$

23)  $\frac{1}{5p^2} \div \frac{9p-36}{5p^3-35p^2}$

$\frac{p-7}{9(p-4)}; \{0, 7, 4\}$

24)  $\frac{8-7x-x^2}{x+8} \cdot \frac{x+5}{9x-9}$

$-\frac{(x+5)}{9}; \{-8, 1\}$

25)  $\frac{x^2-16}{9-x} \cdot \frac{x^2+x-90}{x^2+14x+40}$

$-(x-4); \{9, -4, -10\}$

26)  $\frac{10x^2-20x}{40x^3-80x^2} \cdot \frac{16x^3+80x^2}{6x+30}$

$\frac{2x}{3}; \{0, 2, -5\}$

## Adding/Subtracting Rational Expressions

**Simplify each expression.**

1)  $\frac{u-v}{8v} + \frac{6u-3v}{8v}$

2)  $\frac{m-3n}{6m^3n} - \frac{m+3n}{6m^3n}$

3)  $\frac{5}{a^2+3a+2} + \frac{5a+1}{a^2+3a+2}$

4)  $\frac{5}{10n^2+16n+6} + \frac{n-6}{10n^2+16n+6}$

5)  $\frac{r+6}{3r-6} + \frac{r+1}{3r-6}$

6)  $\frac{x+2}{2x^2+13x+20} - \frac{x+3}{2x^2+13x+20}$

7)  $\frac{6}{x-1} - \frac{5x}{4}$

8)  $6 - \frac{x+5}{(7x-5)(x+4)}$

9)  $\frac{3}{x+7} + \frac{4}{x-8}$

10)  $\frac{3}{4v^2+4v} - \frac{7}{2}$

11)  $\frac{7}{3} - \frac{8}{12x-8}$

12)  $\frac{5}{n+5} + \frac{4n}{2n+6}$

13)  $\frac{2x}{5x+4} + \frac{6x}{2x+3}$

14)  $\frac{2}{3x^2+12x} + \frac{8}{2x}$

15)  $\frac{7n}{n+1} + \frac{8}{n-7}$

16)  $\frac{2}{n+8} + \frac{4}{n+1}$

17)  $\frac{3}{8} - \frac{3}{3x+4}$

18)  $\frac{3}{b-8} + \frac{7}{b+3}$

19)  $\frac{3}{x+6} + \frac{7}{x-2}$

20)  $\frac{4}{x+1} - \frac{2}{x+2}$

21)  $\frac{5n+5}{5n^2+35n-40} + \frac{7n}{3n}$

22)  $\frac{3}{n-5} + \frac{6}{3n-8}$

23)  $\frac{\frac{25}{4}}{\frac{1}{5} - \frac{4}{25}}$

24)  $\frac{8}{\frac{4}{9} + \frac{16}{9}}$

25)  $\frac{\frac{a}{25} - \frac{a}{5}}{a}$

26)  $\frac{\frac{5}{4}}{\frac{5}{m} - \frac{4}{m}}$

**Critical thinking questions:**

27) Simplify:  $\frac{a}{b} + \frac{c}{d}$

28) Split into a sum of two rational expressions with unlike denominators:

$$\frac{2x+3}{x^2+3x+2}$$

## Adding/Subtracting Rational Expressions

Simplify each expression.

1)  $\frac{u-v}{8v} + \frac{6u-3v}{8v}$

$$\frac{7u-4v}{8v}$$

2)  $\frac{m-3n}{6m^3n} - \frac{m+3n}{6m^3n}$

$$-\frac{1}{m^3}$$

3)  $\frac{5}{a^2+3a+2} + \frac{5a+1}{a^2+3a+2}$

$$\frac{6+5a}{a^2+3a+2}$$

4)  $\frac{5}{10n^2+16n+6} + \frac{n-6}{10n^2+16n+6}$

$$\frac{-1+n}{10n^2+16n+6}$$

5)  $\frac{r+6}{3r-6} + \frac{r+1}{3r-6}$

$$\frac{2r+7}{3r-6}$$

6)  $\frac{x+2}{2x^2+13x+20} - \frac{x+3}{2x^2+13x+20}$

$$-\frac{1}{2x^2+13x+20}$$

7)  $\frac{6}{x-1} - \frac{5x}{4}$

$$\frac{24-5x^2+5x}{4(x-1)}$$

8)  $6 - \frac{x+5}{(7x-5)(x+4)}$

$$\frac{42x^2+137x-125}{(7x-5)(x+4)}$$

9)  $\frac{3}{x+7} + \frac{4}{x-8}$

$$\frac{7x+4}{(x+7)(x-8)}$$

10)  $\frac{3}{4v^2+4v} - \frac{7}{2}$

$$\frac{3-14v^2-14v}{4v(v+1)}$$

11)  $\frac{7}{3} - \frac{8}{12x-8}$

$$\frac{21x-20}{3(3x-2)}$$

12)  $\frac{5}{n+5} + \frac{4n}{2n+6}$

$$\frac{15n+15+2n^2}{(n+3)(n+5)}$$

13)  $\frac{2x}{5x+4} + \frac{6x}{2x+3}$

$$\frac{34x^2+30x}{(5x+4)(2x+3)}$$

14)  $\frac{2}{3x^2+12x} + \frac{8}{2x}$

$$\frac{50+12x}{3x(x+4)}$$

$$15) \frac{7n}{n+1} + \frac{8}{n-7}$$

$$\frac{7n^2 - 41n + 8}{(n+1)(n-7)}$$

$$17) \frac{3}{8} - \frac{3}{3x+4}$$

$$\frac{9x - 12}{8(3x+4)}$$

$$19) \frac{3}{x+6} + \frac{7}{x-2}$$

$$\frac{10x + 36}{(x+6)(x-2)}$$

$$21) \frac{5n+5}{5n^2+35n-40} + \frac{7n}{3n}$$

$$\frac{52n - 53 + 7n^2}{3(n+8)(n-1)}$$

$$23) \frac{\frac{25}{4}}{\frac{1}{5} - \frac{4}{25}}$$

$$\frac{625}{4}$$

$$25) \frac{\frac{a}{25} - \frac{a}{5}}{a}$$

$$-\frac{4}{25}$$

$$16) \frac{2}{n+8} + \frac{4}{n+1}$$

$$\frac{6n + 34}{(n+1)(n+8)}$$

$$18) \frac{3}{b-8} + \frac{7}{b+3}$$

$$\frac{10b - 47}{(b+3)(b-8)}$$

$$20) \frac{4}{x+1} - \frac{2}{x+2}$$

$$\frac{2x + 6}{(x+1)(x+2)}$$

$$22) \frac{3}{n-5} + \frac{6}{3n-8}$$

$$\frac{15n - 54}{(3n-8)(n-5)}$$

$$24) \frac{8}{\frac{4}{9} + \frac{16}{9}}$$

$$\frac{18}{5}$$

$$26) \frac{\frac{5}{4}}{\frac{5}{m} - \frac{4}{m}}$$

$$\frac{5m}{4}$$

### Critical thinking questions:

$$27) \text{Simplify: } \frac{a}{b} + \frac{c}{d}$$

$$\frac{ad + bc}{bd}$$

$$28) \text{Split into a sum of two rational expressions with unlike denominators:}$$

$$\frac{2x+3}{x^2+3x+2}$$

Many solutions. Ex:  $\frac{1}{x+1} + \frac{1}{x+2}$

## Rational Exponent Equations

Solve each equation.

1)  $27 = x^{\frac{3}{2}}$

2)  $m^{\frac{3}{4}} = 27$

3)  $x^{-\frac{3}{2}} = \frac{1}{729}$

4)  $7 = r^{\frac{1}{2}}$

5)  $v^{\frac{5}{4}} = 243$

6)  $n^{\frac{3}{2}} = 125$

7)  $(n - 27)^{\frac{3}{2}} = 64$

8)  $26 = -1 + (27x)^{\frac{3}{4}}$

$$9) 3125 = (-1 - 18p)^{\frac{5}{3}}$$

$$10) 5 = 3 + 4a^{-\frac{1}{6}}$$

$$11) 4b^{-\frac{3}{4}} + 10 = \frac{21}{2}$$

$$12) -x^{\frac{3}{2}} = -27$$

$$13) -54 = 10 - (m - 10)^{\frac{3}{2}}$$

$$14) -5126 = -6 - 5(3x + 22)^{\frac{5}{3}}$$

$$15) 9 + 5\sqrt[3]{2m} = 29$$

$$16) 3646 = 1 + 5(4r + 17)^{\frac{3}{2}}$$

$$17) -646 = -3(65 - n)^{\frac{3}{2}} + 2$$

$$18) -3 + (8 - 2x)^{\frac{5}{4}} = 29$$

## Rational Exponent Equations

Solve each equation.

1)  $27 = x^{\frac{3}{2}}$

{9}

2)  $m^{\frac{3}{4}} = 27$

{81}

3)  $x^{-\frac{3}{2}} = \frac{1}{729}$

{81}

4)  $7 = r^{\frac{1}{2}}$

{49}

5)  $v^{\frac{5}{4}} = 243$

{81}

6)  $n^{\frac{3}{2}} = 125$

{25}

7)  $(n - 27)^{\frac{3}{2}} = 64$

{43}

8)  $26 = -1 + (27x)^{\frac{3}{4}}$

{3}



$$9) 3125 = (-1 - 18p)^{\frac{5}{3}}$$

{-7}

$$10) 5 = 3 + 4a^{-\frac{1}{6}}$$

{64}

$$11) 4b^{-\frac{3}{4}} + 10 = \frac{21}{2}$$

{16}

$$12) -x^{\frac{3}{2}} = -27$$

{9}

$$13) -54 = 10 - (m - 10)^{\frac{3}{2}}$$

{26}

$$14) -5126 = -6 - 5(3x + 22)^{\frac{5}{3}}$$

{14}

$$15) 9 + 5\sqrt[3]{2m} = 29$$

{32}

$$16) 3646 = 1 + 5(4r + 17)^{\frac{3}{2}}$$

{16}

$$17) -646 = -3(65 - n)^{\frac{3}{2}} + 2$$

{29}

$$18) -3 + (8 - 2x)^{\frac{5}{4}} = 29$$

{-4}

## Simplifying Rational Exponents

**Simplify.**

1)  $(n^4)^{\frac{3}{2}}$

2)  $(27p^6)^{\frac{5}{3}}$

3)  $(25b^6)^{-1.5}$

4)  $(64m^4)^{\frac{3}{2}}$

5)  $(a^8)^{\frac{3}{2}}$

6)  $(9r^4)^{0.5}$

7)  $(81x^{12})^{1.25}$

8)  $(216r^9)^{\frac{1}{3}}$

**Simplify. Your answer should contain only positive exponents with no fractional exponents in the denominator.**

9)  $2m^2 \cdot 4m^{\frac{3}{2}} \cdot 4m^{-2}$

10)  $3b^{\frac{1}{2}} \cdot b^{\frac{4}{3}}$

11)  $\left(\frac{3}{p^2}\right)^{-2}$

12)  $\left(a^{\frac{1}{2}}\right)^{\frac{3}{2}}$

$$13) \frac{2x^{-\frac{7}{4}}}{4x^{\frac{4}{3}}}$$

$$14) \frac{4x^2}{2x^{\frac{1}{2}}}$$

$$15) \frac{3x^{-\frac{1}{2}} \cdot 3x^{\frac{1}{2}} y^{-\frac{1}{3}}}{3y^{-\frac{7}{4}}}$$

$$16) \frac{3y^{\frac{1}{4}}}{4x^{-\frac{2}{3}} y^{\frac{3}{2}} \cdot 3y^{\frac{1}{2}}}$$

$$17) \left(m \cdot m^{-2} n^{\frac{5}{3}}\right)^2$$

$$18) \left(a^{-1} b^{\frac{1}{3}} \cdot a^{-\frac{4}{3}} b^2\right)^2$$

$$19) \left(\frac{x^{\frac{1}{2}} y^{-2}}{yx^{-\frac{7}{4}}}\right)^4$$

$$20) \frac{(x^3 y^2)^{\frac{3}{2}}}{(x^{-1} y^{-\frac{2}{3}})^{\frac{1}{4}}}$$

$$21) \frac{\left(x^{-\frac{1}{2}} y^2\right)^{-\frac{5}{4}}}{x^2 y^{\frac{1}{2}}}$$

$$22) \frac{\left(x^{-\frac{1}{2}} y^4\right)^{\frac{1}{4}}}{x^{\frac{2}{3}} y^{\frac{3}{2}} \cdot x^{-\frac{3}{2}} y^{\frac{1}{2}}}$$

## Simplifying Rational Exponents

Simplify.

1)  $(n^4)^{\frac{3}{2}}$   
 $n^6$

2)  $(27p^6)^{\frac{5}{3}}$   
 $243p^{10}$

3)  $(25b^6)^{-1.5}$   
 $\frac{1}{125b^9}$

4)  $(64m^4)^{\frac{3}{2}}$   
 $512m^6$

5)  $(a^8)^{\frac{3}{2}}$   
 $a^{12}$

6)  $(9r^4)^{0.5}$   
 $3r^2$

7)  $(81x^{12})^{1.25}$   
 $243x^{15}$

8)  $(216r^9)^{\frac{1}{3}}$   
 $6r^3$

**Simplify. Your answer should contain only positive exponents with no fractional exponents in the denominator.**

9)  $2m^2 \cdot 4m^{\frac{3}{2}} \cdot 4m^{-2}$   
 $32m^{\frac{3}{2}}$

10)  $3b^{\frac{1}{2}} \cdot b^{\frac{4}{3}}$   
 $3b^{\frac{11}{6}}$

11)  $\left(\frac{3}{p^2}\right)^{-2}$   
 $\frac{1}{p^3}$

12)  $\left(\frac{1}{a^2}\right)^{\frac{3}{2}}$   
 $\frac{3}{a^4}$

$$13) \frac{2x^{-\frac{7}{4}}}{4x^{\frac{4}{3}}}$$

$$\frac{x^{\frac{11}{12}}}{2x^4}$$

$$14) \frac{4x^2}{2x^{\frac{1}{2}}}$$

$$2x^{\frac{3}{2}}$$

$$15) \frac{3x^{-\frac{1}{2}} \cdot 3x^{\frac{1}{2}} y^{-\frac{1}{3}}}{3y^{-\frac{7}{4}}}$$

$$3y^{\frac{17}{12}}$$

$$16) \frac{3y^{\frac{1}{4}}}{4x^{-\frac{2}{3}} y^{\frac{3}{2}} \cdot 3y^{\frac{1}{2}}}$$

$$\frac{x^{\frac{2}{3}} y^{\frac{1}{4}}}{4y^2}$$

$$17) \left(m \cdot m^{-2} n^{\frac{5}{3}}\right)^2$$

$$\frac{n^{\frac{10}{3}}}{m^2}$$

$$18) \left(a^{-1} b^{\frac{1}{3}} \cdot a^{-\frac{4}{3}} b^2\right)^2$$

$$\frac{a^{\frac{1}{3}} b^{\frac{14}{3}}}{a^5}$$

$$19) \left(\frac{x^{\frac{1}{2}} y^{-2}}{yx^{\frac{7}{4}}}\right)^4$$

$$\frac{x^9}{y^{12}}$$

$$20) \frac{(x^3 y^2)^{\frac{3}{2}}}{\left(x^{-1} y^{-\frac{2}{3}}\right)^{\frac{1}{4}}}$$

$$y^{\frac{19}{6}} x^4$$

$$21) \frac{\left(x^{-\frac{1}{2}} y^2\right)^{-\frac{5}{4}}}{x^2 y^{\frac{1}{2}}}$$

$$\frac{x^{\frac{5}{8}}}{y^3 x^2}$$

$$22) \frac{\left(x^{-\frac{1}{2}} y^4\right)^{\frac{1}{4}}}{x^{\frac{2}{3}} y^{\frac{3}{2}} \cdot x^{-\frac{3}{2}} y^{\frac{1}{2}}}$$

$$\frac{x^{\frac{17}{24}}}{y}$$

## Radicals and Rational Exponents

Write each expression in radical form.

1)  $7^{\frac{1}{2}}$

2)  $4^{\frac{4}{3}}$

3)  $2^{\frac{5}{3}}$

4)  $7^{\frac{4}{3}}$

5)  $6^{\frac{3}{2}}$

6)  $2^{\frac{1}{6}}$

Write each expression in exponential form.

7)  $(\sqrt{10})^3$

8)  $\sqrt[6]{2}$

9)  $(\sqrt[4]{2})^5$

10)  $(\sqrt[4]{5})^5$

11)  $\sqrt[3]{2}$

12)  $\sqrt[6]{10}$

Write each expression in radical form.

13)  $(5x)^{-\frac{5}{4}}$

14)  $(5x)^{-\frac{1}{2}}$

15)  $(10n)^{\frac{3}{2}}$

16)  $a^{\frac{6}{5}}$

17)  $(6v)^{1.5}$

18)  $m^{-\frac{1}{2}}$

**Write each expression in exponential form.**

19)  $(\sqrt[4]{m})^3$

20)  $(\sqrt[3]{6x})^4$

21)  $\sqrt[4]{v}$

22)  $\sqrt{6p}$

23)  $(\sqrt[3]{3a})^4$

24)  $\frac{1}{(\sqrt{3k})^5}$

**Simplify.**

25)  $9^{\frac{1}{2}}$

26)  $343^{-\frac{4}{3}}$

27)  $1000000^{\frac{1}{6}}$

28)  $36^{\frac{3}{2}}$

29)  $(x^6)^{\frac{1}{2}}$

30)  $(9n^4)^{\frac{1}{2}}$

31)  $(64n^{12})^{-\frac{1}{6}}$

32)  $(81m^6)^{\frac{1}{2}}$

## Radicals and Rational Exponents

Write each expression in radical form.

1)  $7^{\frac{1}{2}}$

$\sqrt{7}$

2)  $4^{\frac{4}{3}}$

$(\sqrt[3]{4})^4$

3)  $2^{\frac{5}{3}}$

$(\sqrt[3]{2})^5$

4)  $7^{\frac{4}{3}}$

$(\sqrt[3]{7})^4$

5)  $6^{\frac{3}{2}}$

$(\sqrt{6})^3$

6)  $2^{\frac{1}{6}}$

$\sqrt[6]{2}$

Write each expression in exponential form.

7)  $(\sqrt{10})^3$

$10^{\frac{3}{2}}$

8)  $\sqrt[6]{2}$

$2^{\frac{1}{6}}$

9)  $(\sqrt[4]{2})^5$

$2^{\frac{5}{4}}$

10)  $(\sqrt[4]{5})^5$

$5^{\frac{5}{4}}$

11)  $\sqrt[3]{2}$

$2^{\frac{1}{3}}$

12)  $\sqrt[6]{10}$

$10^{\frac{1}{6}}$

Write each expression in radical form.

13)  $(5x)^{-\frac{5}{4}}$

$\frac{1}{(\sqrt[4]{5x})^5}$

14)  $(5x)^{-\frac{1}{2}}$

$\frac{1}{\sqrt{5x}}$

15)  $(10n)^{\frac{3}{2}}$

$(\sqrt{10n})^3$

16)  $a^{\frac{6}{5}}$

$(\sqrt[5]{a})^6$



$$17) (6v)^{1.5}$$
$$(\sqrt{6v})^3$$

$$18) m^{-\frac{1}{2}}$$
$$\frac{1}{\sqrt{m}}$$

Write each expression in exponential form.

$$19) (\sqrt[4]{m})^3$$
$$m^{\frac{3}{4}}$$

$$20) (\sqrt[3]{6x})^4$$
$$(6x)^{\frac{4}{3}}$$

$$21) \sqrt[4]{v}$$
$$v^{\frac{1}{4}}$$

$$22) \sqrt{6p}$$
$$(6p)^{\frac{1}{2}}$$

$$23) (\sqrt[3]{3a})^4$$
$$(3a)^{\frac{4}{3}}$$

$$24) \frac{1}{(\sqrt{3k})^5}$$
$$(3k)^{-\frac{5}{2}}$$

Simplify.

$$25) 9^{\frac{1}{2}}$$
$$3$$

$$26) 343^{-\frac{4}{3}}$$
$$\frac{1}{2401}$$

$$27) 1000000^{\frac{1}{6}}$$
$$10$$

$$28) 36^{\frac{3}{2}}$$
$$216$$

$$29) (x^6)^{\frac{1}{2}}$$
$$x^3$$

$$30) (9n^4)^{\frac{1}{2}}$$
$$3n^2$$

$$31) (64n^{12})^{-\frac{1}{6}}$$
$$\frac{1}{2n^2}$$

$$32) (81m^6)^{\frac{1}{2}}$$
$$9m^3$$

## Simplifying Radicals

**Simplify. Use absolute value signs when necessary.**

1)  $\sqrt{24}$

2)  $\sqrt[3]{1000}$

3)  $\sqrt[3]{-162}$

4)  $\sqrt{512}$

5)  $\sqrt[4]{128n^8}$

6)  $\sqrt{98k}$

7)  $\sqrt[5]{224r^7}$

8)  $\sqrt[3]{24m^3}$

9)  $\sqrt{392x^2}$

10)  $\sqrt{512x^2}$

11)  $\sqrt[4]{405x^3y^2}$

12)  $\sqrt[3]{-16a^3b^8}$

13)  $\sqrt[4]{128x^7y^7}$

14)  $\sqrt[3]{16xy}$

15)  $\sqrt[6]{448x^7y^7}$

16)  $\sqrt[3]{56x^5y}$

**Critical thinking questions:**

17) What simplifies into  $2mn^2\sqrt[3]{5mn^2}$  ?

18) Simplify  $\sqrt[n]{3 \cdot 2^n \cdot x^{2n} y^{n+3}}$

## Simplifying Radicals

Simplify. Use absolute value signs when necessary.

1)  $\sqrt{24}$

$2\sqrt{6}$

2)  $\sqrt[3]{1000}$

$10$

3)  $\sqrt[3]{-162}$

$-3\sqrt[3]{6}$

4)  $\sqrt{512}$

$16\sqrt{2}$

5)  $\sqrt[4]{128n^8}$

$2n^2\sqrt[4]{8}$

6)  $\sqrt{98k}$

$7\sqrt{2k}$

7)  $\sqrt[5]{224r^7}$

$2r\sqrt[5]{7r^2}$

8)  $\sqrt[3]{24m^3}$

$2m\sqrt[3]{3}$

9)  $\sqrt{392x^2}$

$14|x|\sqrt{2}$

10)  $\sqrt{512x^2}$

$16|x|\sqrt{2}$

11)  $\sqrt[4]{405x^3y^2}$

$3\sqrt[4]{5x^3y^2}$

12)  $\sqrt[3]{-16a^3b^8}$

$-2ab^2\sqrt[3]{2b^2}$

13)  $\sqrt[4]{128x^7y^7}$

$2|x| \cdot |y|\sqrt[4]{8x^3y^3}$

14)  $\sqrt[3]{16xy}$

$2\sqrt[3]{2xy}$

15)  $\sqrt[6]{448x^7y^7}$

$2|x| \cdot |y|\sqrt[6]{7xy}$

16)  $\sqrt[3]{56x^5y}$

$2x\sqrt[3]{7x^2y}$

Critical thinking questions:

17) What simplifies into  $2mn^2\sqrt[3]{5mn^2}$  ?

$\sqrt[3]{40m^4n^8}$

18) Simplify  $\sqrt[n]{3 \cdot 2^n \cdot x^{2n} y^{n+3}}$

$2x^2y\sqrt[n]{3y^3}$

## Exponential Equations Not Requiring Logarithms

**Solve each equation.**

1)  $5^{3n} = 125$

2)  $2^{2n} = 16$

3)  $5^{3r} = 5^{-2r}$

4)  $3^{-2k} = 81$

5)  $2^{-3x} = 2^{x-1}$

6)  $6^{3m} = 36$

7)  $10^{3x} = 10000$

8)  $4^{r+1} = 1$

9)  $\left(\frac{1}{8}\right)^{3x} \cdot 64^{2x+1} = 4$

10)  $32^{2x} = 8$

11)  $6^{-3v-2} = 36$

12)  $243^x = 81$

$$13) 2^{-2n} \cdot 2^{n+1} = 2^{-2n}$$

$$14) \left(\frac{1}{16}\right)^{2a} \cdot 16^{-2a-3} = 64^{2a}$$

$$15) 3^{2n} = 9$$

$$16) \left(\frac{1}{6}\right)^n = 36$$

$$17) \frac{216^{2a}}{36^{-a}} = 216$$

$$18) 36^{3n-2} \cdot \frac{1}{216} = 36^{3n-2}$$

$$19) \frac{16}{\left(\frac{1}{16}\right)^x} = \left(\frac{1}{4}\right)^{2-2x}$$

$$20) 25 \cdot 125^{-y} = 625$$

$$21) 27^{3x} \cdot \left(\frac{1}{9}\right)^{-x} = 243^{-x-3}$$

$$22) \left(\frac{1}{6}\right)^{3a} \cdot 36^{-3a} = \frac{1}{36}$$

$$23) 125^{3n} \cdot 625^{-n} = 625^{-2n-1}$$

$$24) \frac{125}{25^{-3m}} = 25^{-2m-2}$$

## Exponential Equations Not Requiring Logarithms

Solve each equation.

1)  $5^{3n} = 125$

 $\left\{ \frac{1}{3} \right\}$ 

2)  $2^{2n} = 16$

 $\{2\}$ 

3)  $5^{3r} = 5^{-2r}$

 $\{0\}$ 

4)  $3^{-2k} = 81$

 $\{-2\}$ 

5)  $2^{-3x} = 2^{x-1}$

 $\left\{ \frac{1}{4} \right\}$ 

6)  $6^{3m} = 36$

 $\left\{ \frac{2}{3} \right\}$ 

7)  $10^{3x} = 10000$

 $\left\{ \frac{4}{3} \right\}$ 

8)  $4^{r+1} = 1$

 $\{-1\}$ 

9)  $\left(\frac{1}{8}\right)^{3x} \cdot 64^{2x+1} = 4$

 $\left\{ -\frac{4}{3} \right\}$ 

10)  $32^{2x} = 8$

 $\left\{ \frac{3}{10} \right\}$ 

11)  $6^{-3v-2} = 36$

 $\left\{ -\frac{4}{3} \right\}$ 

12)  $243^x = 81$

 $\left\{ \frac{4}{5} \right\}$

13)  $2^{-2n} \cdot 2^{n+1} = 2^{-2n}$

 $\left\{ -1 \right\}$ 

14)  $\left(\frac{1}{16}\right)^{2a} \cdot 16^{-2a-3} = 64^{2a}$

 $\left\{ -\frac{3}{7} \right\}$ 

15)  $3^{2n} = 9$

 $\{1\}$ 

16)  $\left(\frac{1}{6}\right)^n = 36$

 $\{-2\}$ 

17)  $\frac{216^{2a}}{36^{-a}} = 216$

 $\left\{ \frac{3}{8} \right\}$ 

18)  $36^{3n-2} \cdot \frac{1}{216} = 36^{3n-2}$

No solution.

19)  $\frac{16}{\left(\frac{1}{16}\right)^x} = \left(\frac{1}{4}\right)^{2-2x}$

No solution.

20)  $25 \cdot 125^{-y} = 625$

 $\left\{ -\frac{2}{3} \right\}$ 

21)  $27^{3x} \cdot \left(\frac{1}{9}\right)^{-x} = 243^{-x-3}$

 $\left\{ -\frac{15}{16} \right\}$ 

22)  $\left(\frac{1}{6}\right)^{3a} \cdot 36^{-3a} = \frac{1}{36}$

 $\left\{ \frac{2}{9} \right\}$ 

23)  $125^{3n} \cdot 625^{-n} = 625^{-2n-1}$

 $\left\{ -\frac{4}{13} \right\}$ 

24)  $\frac{125}{25^{-3m}} = 25^{-2m-2}$

 $\left\{ -\frac{7}{10} \right\}$

## Rational Equations

Solve each equation. Remember to check for extraneous solutions.

1)  $\frac{1}{4x} - \frac{1}{4} = \frac{1}{2}$

2)  $\frac{5v-5}{v} - \frac{5v+15}{v} = 1$

3)  $\frac{5a+20}{6a} + \frac{1}{a} = \frac{3}{2a}$

4)  $\frac{2}{m^2} = \frac{1}{m} + \frac{1}{m^2}$

5)  $1 + \frac{4}{r-2} = \frac{5}{r-2}$

6)  $\frac{n-1}{2n} = 1 + \frac{1}{2n}$

7)  $\frac{5}{k} = \frac{1}{k} - 1$

8)  $\frac{5}{b} = \frac{1}{b} + 4$



$$9) \frac{4}{n+1} - \frac{1}{n^2+7n+6} = \frac{3}{n^2+7n+6}$$

$$10) \frac{1}{r-1} + 4 = \frac{2}{r-1}$$

$$11) \frac{3v^2+21v+18}{v^2+2v} = \frac{1}{v} - \frac{1}{v^2+2v}$$

$$12) \frac{v^2-5v+4}{v^2-6v} = \frac{1}{v^2-6v} + \frac{1}{v}$$

$$13) \frac{b+1}{b^2-2b} + \frac{6b^2-24b+18}{b^2-2b} = \frac{2b-4}{b}$$

$$14) \frac{5}{b^2+b} = \frac{b+4}{b} + \frac{1}{b^2+b}$$

$$15) \frac{1}{n^2+11n+30} = \frac{1}{n+5} - 4$$

$$16) \frac{n}{n+5} = \frac{n+4}{2n+10} + \frac{n^2+6n+8}{2n^2+10n}$$

## Rational Equations

Solve each equation. Remember to check for extraneous solutions.

1)  $\frac{1}{4x} - \frac{1}{4} = \frac{1}{2}$

$$\left\{ \frac{1}{3} \right\}$$

2)  $\frac{5v-5}{v} - \frac{5v+15}{v} = 1$

$$\{-20\}$$

3)  $\frac{5a+20}{6a} + \frac{1}{a} = \frac{3}{2a}$

$$\left\{ -\frac{17}{5} \right\}$$

4)  $\frac{2}{m^2} = \frac{1}{m} + \frac{1}{m^2}$

$$\{1\}$$

5)  $1 + \frac{4}{r-2} = \frac{5}{r-2}$

$$\{3\}$$

6)  $\frac{n-1}{2n} = 1 + \frac{1}{2n}$

$$\{-2\}$$

7)  $\frac{5}{k} = \frac{1}{k} - 1$

$$\{-4\}$$

8)  $\frac{5}{b} = \frac{1}{b} + 4$

$$\{1\}$$

$$9) \frac{4}{n+1} - \frac{1}{n^2+7n+6} = \frac{3}{n^2+7n+6}$$

$\{-5\}$

$$10) \frac{1}{r-1} + 4 = \frac{2}{r-1}$$

$\left\{\frac{5}{4}\right\}$

$$11) \frac{3v^2+21v+18}{v^2+2v} = \frac{1}{v} - \frac{1}{v^2+2v}$$

$\left\{-1, -\frac{17}{3}\right\}$

$$12) \frac{v^2-5v+4}{v^2-6v} = \frac{1}{v^2-6v} + \frac{1}{v}$$

$\{3\}$

$$13) \frac{b+1}{b^2-2b} + \frac{6b^2-24b+18}{b^2-2b} = \frac{2b-4}{b}$$

$\left\{1, \frac{11}{4}\right\}$

$$14) \frac{5}{b^2+b} = \frac{b+4}{b} + \frac{1}{b^2+b}$$

$\{-5\}$

$$15) \frac{1}{n^2+11n+30} = \frac{1}{n+5} - 4$$

$\left\{-\frac{23}{4}\right\}$

$$16) \frac{n}{n+5} = \frac{n+4}{2n+10} + \frac{n^2+6n+8}{2n^2+10n}$$

$\left\{-\frac{4}{5}\right\}$

## Dividing Polynomials

Determine if  $d(x)$  is a factor of  $f(x)$ .

1)  $f(x) = 3x^4 + 7x^3 + 3x^2 - x - 4$   
 $d(x) = x + 2$

2)  $f(x) = 2x^3 - x^2 - 6x - 1$   
 $d(x) = x + 1$

3)  $f(x) = 4x^3 - 2x^2 + x - 3$   
 $d(x) = x - 1$

4)  $f(x) = 3x^4 - 3x^3 - 9x^2 + 5x - 2$   
 $d(x) = x - 2$

Divide. Write your answer in fraction form.

5)  $(2x^5 - 15x^3 - 9x^2 + 11x + 12) \div (x + 2)$

6)  $(x^4 - x^3 - 19x^2 - 3x - 19) \div (x - 5)$

7)  $(10x^4 - 4x^3 + 14x^2 - 14x - 16) \div (2x - 2)$

8)  $(9x^5 - 9x^4 - x^3 - 12x^2 + x - 11) \div (3x - 5)$

$$9) (16x^4 + 4x^3 + 2x^2 - 21x + 7) \div (4x - 1)$$

$$10) (6x^5 + 21x^4 - 14x^3 - 8x^2 + x - 6) \div (x + 4)$$

$$11) (5x^6 - 16x^5 - 11x^4 + 22x^3 + 14x^2 - 4x + 9) \div (x^2 - 4x + 2)$$

$$12) (3x^4 + 15x^3 + 3x^2 - 12x + 4) \div (x^2 + 4x - 2)$$

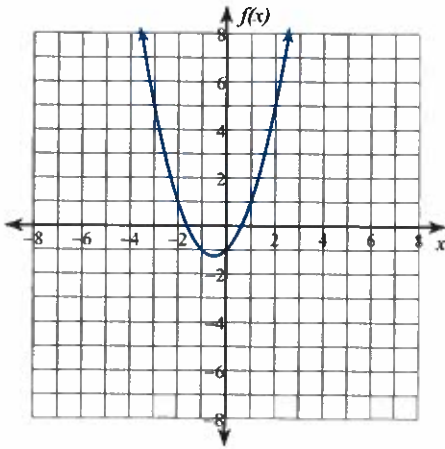
$$13) (4x^6 - 3x^5 + 15x^4 - 12x^3 - 4x^2 + 10) \div (4x^3 - 3x^2 - x)$$

$$14) (x^4 - 17x^2 + 4x - 2) \div (x^2 + 4x - 1)$$

## Average Rates of Change

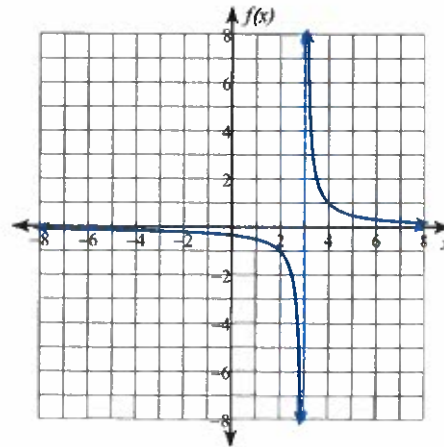
For each problem, find the average rate of change of the function over the given interval.

1)  $f(x) = x^2 + x - 1$ ;  $[-2, -\frac{7}{4}]$



$$-\frac{11}{4}$$

2)  $f(x) = \frac{1}{x-3}$ ;  $[-2, -\frac{3}{2}]$



$$-\frac{2}{45}$$

3)  $f(x) = x^2 - x - 1$ ;  $[3, \frac{13}{4}]$

$$\frac{21}{4}$$

4)  $f(x) = -x^2 - x + 1$ ;  $[2, \frac{7}{3}]$

$$-\frac{16}{3}$$

5)  $f(x) = 2x^2 - 2x - 1$ ;  $[1, \frac{3}{2}]$

$$3$$

6)  $f(x) = \frac{1}{x-1}$ ;  $[-4, -\frac{11}{3}]$

$$-\frac{3}{70}$$

7)  $f(x) = \frac{1}{x+3}$ ;  $[-1, -\frac{2}{3}]$

$$-\frac{3}{14}$$

8)  $f(x) = -\frac{1}{x-1}$ ;  $[-4, -\frac{7}{2}]$

$$\frac{2}{45}$$

- 9) The police have accused a driver of breaking the speed limit of 60 miles per hour. As proof, they provide two photographs. One photo shows the driver's car passing a toll booth at exactly 6 PM. The second photo shows the driver's car passing another toll booth 31 miles down the highway at exactly 6:30 PM. Does the photo evidence prove that the driver broke the speed limit during this time?

Yes. The average rate of change is 62 mph, so the driver must have been breaking the speed limit some of the time.

## Power Functions

Consider each power function. Determine the power and constant of variation.

1)  $f(x) = 6x^{\frac{3}{7}}$

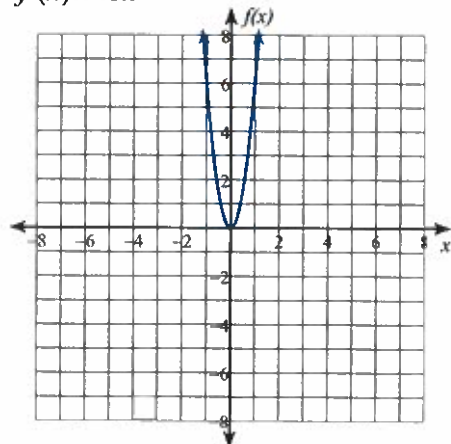
2)  $f(x) = 5x^{-6}$

3)  $f(x) = 8x^{-\frac{1}{3}}$

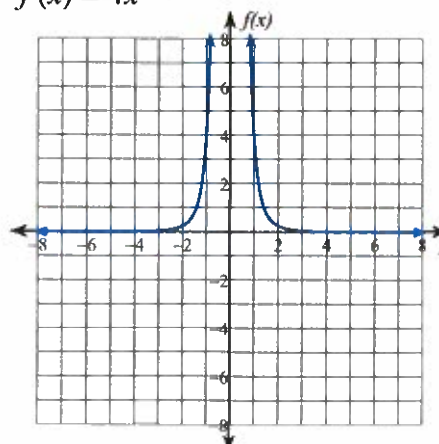
4)  $f(x) = 2x^{\frac{5}{7}}$

Consider each power function. Determine the domain and range, intercepts, end behavior, continuity, and regions of increase and decrease.

5)  $f(x) = 6x^2$



6)  $f(x) = 4x^{-4}$



7)  $f(x) = 5x^{-4}$

8)  $f(x) = 8x^{-\frac{4}{3}}$

## Power Functions

Consider each power function. Determine the power and constant of variation.

1)  $f(x) = 6x^{\frac{3}{7}}$

Power:  $\frac{3}{7}$  Constant: 6

2)  $f(x) = 5x^{-6}$

Power: -6 Constant: 5

3)  $f(x) = 8x^{-\frac{1}{3}}$

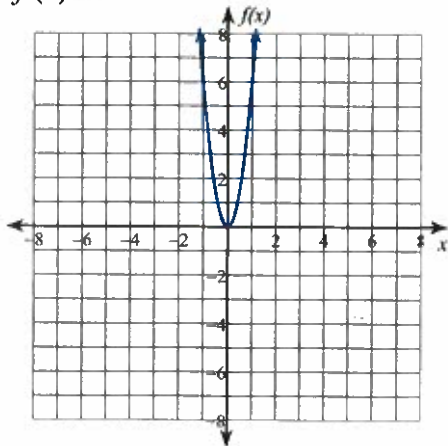
Power:  $-\frac{1}{3}$  Constant: 8

4)  $f(x) = 2x^{\frac{5}{7}}$

Power:  $\frac{5}{7}$  Constant: 2

Consider each power function. Determine the domain and range, intercepts, end behavior, continuity, and regions of increase and decrease.

5)  $f(x) = 6x^2$



Domain:  $(-\infty, \infty)$

Range:  $[0, \infty)$

x-intercept: 0 y-intercept: 0

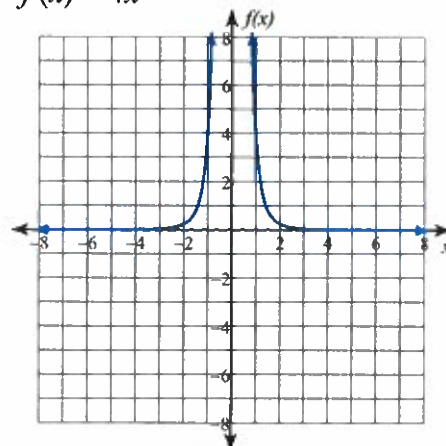
$\lim_{x \rightarrow -\infty} f(x) = \infty$   $\lim_{x \rightarrow \infty} f(x) = \infty$

Continuous on  $(-\infty, \infty)$

Increasing:  $(0, \infty)$

Decreasing:  $(-\infty, 0)$

6)  $f(x) = 4x^{-4}$



Domain:  $(-\infty, 0) \cup (0, \infty)$

Range:  $(0, \infty)$

No intercepts

$\lim_{x \rightarrow -\infty} f(x) = 0$   $\lim_{x \rightarrow \infty} f(x) = 0$

Infinite discontinuity at  $x = 0$

Increasing:  $(-\infty, 0)$

Decreasing:  $(0, \infty)$

7)  $f(x) = 5x^{-4}$

Domain:  $(-\infty, 0) \cup (0, \infty)$

Range:  $(0, \infty)$

No intercepts

$\lim_{x \rightarrow -\infty} f(x) = 0$   $\lim_{x \rightarrow \infty} f(x) = 0$

Infinite discontinuity at  $x = 0$

Increasing:  $(-\infty, 0)$

Decreasing:  $(0, \infty)$

8)  $f(x) = 8x^{-\frac{4}{3}}$

Domain:  $(-\infty, 0) \cup (0, \infty)$

Range:  $(0, \infty)$

No intercepts

$\lim_{x \rightarrow -\infty} f(x) = 0$   $\lim_{x \rightarrow \infty} f(x) = 0$

Infinite discontinuity at  $x = 0$

Increasing:  $(-\infty, 0)$

Decreasing:  $(0, \infty)$