Summer Assignment

Fractional and Negative Exponents
Simplify, using positive exponents.

1. $3\left(\frac{3}{3-x}\right)\left[\frac{-3}{(3-x)^2}\right]^{-3}$
2. $\frac{\sqrt{9x-81}}{\sqrt[4]{(x-9)^2}}$
3. $\frac{\frac{1}{2}(2x+5)^{-\frac{3}{2}}}{\frac{3}{2}}$
4. $\left(\frac{1}{x^{-2}}+\frac{4}{x^{-1}y^{-1}}+\frac{1}{y^{-2}}\right)^{-\frac{1}{2}}$

Domain
Find the domain of the following functions.

5. $y=\frac{x^2-5x+6}{x^2-3x-18}$
6. $f(x)=\frac{\sqrt{3x-4}}{3x+4}$
7. $g(x)=\sqrt{x^2-5x-14}$
8. $y=\ln(3x+15)$

Asymptotes
For each function, find the equations of both the vertical and horizontal asymptote(s), if they exist.

9. $y=\frac{x+3}{x^2+2}$
10. $f(x)=\frac{x^2-9}{x^3+3x^2-18x}$
11. $g(x)=\frac{3x^3}{x^3-27}$

Complex Fractions
Simplify.

12. $\frac{\frac{2}{x}-\frac{5}{y}}{\frac{5}{x}-\frac{2}{y}}$
13. $\frac{x^{-4}-x^{-1}}{x^{-3}-1}$
14. $\frac{1-\frac{3}{4x}}{x-\frac{9}{16x}}$
15. $\frac{\frac{x^2-y^2}{xy}}{\frac{x+y}{y}}$
16. $\frac{\frac{4}{x-5}+\frac{2}{x+2}}{\frac{2x}{x^2-3x-10}+3}$
17. $\frac{\frac{4x}{x^2-4}+\frac{-2}{x+2}}{x-2}$

Rationalizing:
Remove the sum or difference from the denominator by multiplying the numerator and denominator by the conjugate of the denominator.

18. $\frac{1}{1-\cos x}$
19. $\frac{x}{1-\sqrt{x^2+1}}$
20. $\frac{2}{x+\sqrt{x^2+1}}$

Factoring
Monomial Factoring: Factor out the greatest common factor or as indicated.

1. $3x^4+4x^3-x^2$
2. $2\sqrt{x}+6x^{3}/_{2}$
3. $e^{-x}-xe^{-x}+2x^2e^{-x}$
4. $x^{-1}-2+x=x^{-1}(\quad)$
5. $\frac{x}{2}-6x^2=\frac{x}{2}(\quad)$
6. $\sin x+\tan x=\sin x(\quad)$
7. $\frac{1}{2x^2+4x}=\frac{1}{2x}(\quad)$

Factor completely, or state that the polynomial is prime.

8. $x^2 - 9$
9. $3x^3 - 3x$
10. $5x^3 - 45x$
11. $4x^2 - 4x - 24$
12. $6x^2 - 18x - 60$
13. $2x^4 - 162$
14. $x^3 + 2x^2 - 9x - 18$
15. $2x^2 - 2x - 112$
16. $x^3 - 4x$
17. $x^2 + 64$
18. $x^3 + 2x^2 - 4x - 8$
19. $y^5 - 81y$
20. $20y^4 - 45y^2$
21. $e^{2x} + 2 + e^{-2x}$
22. $x^4 - 7x^2 + 12$
23. $1 - sin^2 x$
24. $x^2 - 12x + 36 - 49y^2$
25. $9b^2x - 16y - 16x + 9b^2y$
26. $x^2y - 16y + 32 - 2x^2$
27. $2x^3 - 8a^2x + 24x^2 + 72x$

Factor and simplify each algebraic expression.

28. $x^{3/2} - x^{1/2}$
29. $4x^{-2/3} + 8x^{1/3}$
30. $(x+3)^{1/2} + (x+3)^{3/2}$
31. $(x+5)^{-1/2} + (x+5)^{-3/2}$
32. $(4x-1)^{1/2} - \frac{1}{3}(4x-1)^{3/2}$

Simplify each of the following:

33. $\frac{3x+9}{6x}$
34. $\frac{x^2}{x^{1/2}}$
35. $y = \frac{x^3-1}{x+1}$
36. $y = \frac{x^6-1}{x+1}$
37. $y = \frac{x^3+1}{x^2-x+1}$
38. $y = \frac{\sqrt{x+7}}{\sqrt{(x+7)^3}}$
39. $y = \frac{3x(x^3+3x)^{-\frac{3}{2}}}{(x^3+3x)^{\frac{1}{2}}}$
40. $y = \frac{1}{2}(x^4+8x)^{-\frac{1}{2}}(4x^3+8)$
41. $y = \frac{\frac{1}{3}3^x}{\sqrt{x}} + 3^x\sqrt{x}$
42. $y = \frac{1}{x^2+x} - \frac{2}{x^2-1}$
43. $y = \frac{(e^x+e^{-x})(e^x+e^{-x})-(e^x-e^{-x})(e^x-e^{-x})}{(e^x-e^{-x})^2}$
44. $\sqrt{x^2+1} - \frac{x^2}{\sqrt{x^2+1}}$
45. $\frac{(x+1)^3(x-2)+3(x+1)^2}{(x+1)^4}$
46. $\frac{x^{1/2}-x^{1/3}}{x^{1/6}}$
47. $\frac{\sqrt{x-1}+(x-1)^{3/2}}{\sqrt{x-1}}$
48. $\frac{1-(\sin x+\cos x)^2}{2\sin x}$

Rewrite each of the following in simplest form.

49. $\frac{(x-1)(x+3)-(x+1)^2}{x+1}$

50. $\frac{\sqrt{x^2+1}-\frac{1}{\sqrt{x^2+1}}}{x^2+1}$

51. $\frac{x^2-5x+6}{x^2-4x+4}$

52. $\frac{1}{x+1}-\frac{1}{x-1}-\frac{2}{x^2-1}$

53. $\frac{x(-2x)}{2\sqrt{1-x^2}}+\sqrt{1-x^2}+\frac{1}{\sqrt{1-x^2}}$

Expanding Logarithmic Expressions

For $A > 0$ *and* $B > 0$, $b > 0, b \neq 1$, *and* p *any real number.*

1. Product Rule $\log_b(AB) = \log_b A + \log_b B$
2. Quotient Rule $\log_b\left(\frac{A}{B}\right) = \log_b A - \log_b B$
3. Power Rule $\log_b A^p = p \log_b A$

Expand using properties:

54. $\log \sqrt[3]{\frac{x}{y}}$

55. $\log \sqrt[5]{\frac{x}{y}}$

56. $\log_b\left(\frac{\sqrt{x}y^3}{z^3}\right)$

57. $\log_b\left(\frac{\sqrt[3]{x}y^4}{z^5}\right)$

58. $\log_5 \sqrt[3]{\frac{x^2y}{25}}$

59. $\log_2 \sqrt[5]{\frac{xy^4}{16}}$

60. $\ln\left[\frac{x^3\sqrt{x^2+1}}{(x+1)^4}\right]$

61. $\ln\left[\frac{x^4\sqrt{x^2+3}}{(x+3)^5}\right]$

62. $\log\left[\frac{10x^2\sqrt{1-x}}{7(x+1)^2}\right]$

63. $\log\left[\frac{100x^3\sqrt[3]{5-x}}{3(x+7)^2}\right]$

Geometry. Trigonometry.

Evaluate each trigonometric function for the value or angle given.

64. $\csc\frac{\pi}{2}$

65. $\cos^{-1}(1)$

66. $\sec\frac{-2\pi}{3}$

67. $\cot\frac{11\pi}{6}$

68. $\arcsin\frac{-\sqrt{2}}{2}$

69. $\arctan(1)$

Determine each of the following:

70. $\sin(\arctan\sqrt{3})$

71. $\cos(\arcsin(-\frac{1}{2})$

72. $\sin(\arccos-\frac{\sqrt{3}}{2})$

73. $\tan(arcsin\ \frac{\sqrt{2}}{2})$