ALGEBRA PREP FOR GEOMETRY HONORS

Note: The completed packet will be due on the first day of class and will be 50% of the first test grade.

TOPIC LIST

DIAGNOSTIC TEST

Complete and check answers with videos coded on p. 5.

1. Solve for x:

$$\frac{2}{5}(x-3)-2(x+5.2)=-\frac{1}{10}x+4.4$$

2. Solve the system for x and y:

$$\begin{aligned} -14 &= -20y - 7x \\ 10y + 4 &= 2x \end{aligned}$$

3. Solve algebraically:

The number of quarters in a coin purse exceeds the number of dimes by four. If the total amount in the purse is $6.25, find the number of each type of coin.

4. Write the equation of a line passing through the points (5, 3) and (20, -6) in

a) point-slope form, and
b) slope-intercept form.

5. Simplify: $\frac{1}{4}x^2+12x-\frac{x}{2}+(-3-x)+\frac{x^2}{8}$

6. Simplify:

$$6x-\{x+5-(x-4+[9x-5]+2)+2x\}$$

7. Simplify and express the result in standard form: $(x-5)(x^2-6x+2)$

8. Simplify and express in standard form:

a) $(2x-5)(2x+5)$

b) $(x-4)^2$

9. Simplify: $(x+6)^2-3(x-9)^2$

10. Simplify each expression:

a) $\sqrt{250}$

b) $\sqrt{20}\cdot\sqrt{8}$

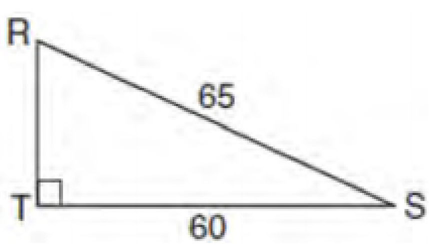
c) $\sqrt{a^2b^2}$

d) $\sqrt{a^2+b^2}$

e) $\sqrt{(a+b)^2}$

11. Simplify each expression:	12. Simplify each expression:
a) $\frac{\sqrt{6}}{\sqrt{2}}$ b) $\frac{\sqrt{6}+\sqrt{3}}{\sqrt{3}}$ c) $\sqrt{\frac{1}{2}}$ d) $\frac{3}{\sqrt{2}}$	a) $5\sqrt{3}+\sqrt{3}$ b) $\sqrt{3}+\sqrt{2}$ c) $\sqrt{16}+3\sqrt{2}$ d) $\frac{1}{2}\sqrt{32}+\sqrt{2}$ e) $\sqrt{16+4}$
13. Solve by factoring: $-\frac{1}{2}x^2-2x+48=0$	14. Solve using the quadratic formula: $x^2+5=8x$

15. Find the length of $\overline{RT}$.



Answer Videos – Click on the QR code to access each video 

Answers to Questions #1 and #2	Answers to Questions #3 and #4
Answers to Questions #5 and #6	Answers to Questions #7 and #8
Answers to Questions #9 and #10	Answers to Questions #11 and #12
Answers to Questions #13-15	

Preliminary Topic A: Operations with Fractions

<table>
<tr>
<td>
Writing Fractions in Lowest Terms

Purpose: to make the numbers easier to work with and understand

Divide numerator and denominator by the same number.

Ex. $\frac{16}{20} = \frac{16 \div 4}{20 \div 4} = \frac{4}{5}$

b) Writing fractions in higher terms:
Purpose: to add or subtract fractions (must be like denominators)

Ex. $\frac{2}{3} = \frac{2 \times 2}{3 \times 2} = \frac{4}{6} \rightarrow \frac{2}{3}\ and\ \frac{4}{6}$ are equivalent fractions.

$\frac{2}{3} = \frac{2 \times 3}{3 \times 3} = \frac{6}{9} \rightarrow \frac{2}{3}\ and\ \frac{6}{9}$ are equivalent fractions.
</td>
<td>
Add/Subtract Fractions

Purpose: to make two fractions into one, to solve equations containing fractions

Denominators must be the same in order to add or subtract.

1. Multiply numerators and denominators by the same number, make the denominators the same. Choose the smallest common denominators.

2. Add/subtract ONLY the numerators.

Ex. $\frac{4}{5} + \frac{2}{3}$ ← These denominators need to be the same. Choose 15 because 5 and 3 go into it.

$\frac{4}{5} + \frac{2}{3} = \frac{4 \cdot 3}{5 \cdot 3} + \frac{2 \cdot 5}{3 \cdot 5} = \frac{12}{\mathbf{15}} + \frac{10}{15} = \frac{22}{15}$

Answer: $\frac{22}{15}$
</td>
</tr>
<tr>
<td>
Multiplying Fractions

1. Multiply numerators.
2. Multiply denominators.
3. Simplify.

Note: You can cross cancel while multiplying instead of #3.

Ex. $\frac{2}{3} \cdot \frac{9}{16} = \frac{2 \times 9}{3 \times 16} = \frac{18}{48} = \frac{18 \div 6}{48 \div 6} = \mathbf{\frac{3}{8}}$
</td>
<td>
Dividing Fractions

1. Flip the second fraction.
2. Multiply the new fractions (see Multiplying)

Ex. $\frac{2}{3} \div \frac{4}{18} = \frac{2}{3} \cdot \frac{18}{4} = \frac{\cancel{2}}{\cancel{3}} \cdot \frac{\cancel{18}}{\cancel{4}} = \frac{1}{1} \cdot \frac{6}{2} = \frac{6}{2} = \mathbf{3}$
</td>
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</table>

Topic #1: Solving Linear Equations in One Variable

Example 1: Basic Procedure (DCM)

Solve: $2x - 6(x - 5) = -3x + 8$

$2x - 6(x - 5) = -3x + 8$ *Distribute coefficients across parentheses.*

$2x - 6x + 30 = -3x + 8$

$-4x + 30 = -3x + 8$ *Combine like terms on each side of the equation.*

$-4x + 30 = -3x + 8$ *Move like terms across the equals sign by adding the opposite.*

$\underline{+4x - 8 \qquad + 4x - 8}$

$\mathbf{22} = \boldsymbol{x}$

Example 2: Solving Proportions

Solve: $\frac{x+10}{3} = \frac{8}{6}$

$\frac{x+10}{3} = \frac{8}{6}$

$6(x + 10) = 24$ *Cross-multiply. Place any binomials in parentheses.*

$6x + 60 = 24$ *Solve.*

$6x = -36$

$\boldsymbol{x = -6}$

Example 3: Equations with Fractional Coefficients

You can make equations easier to solve by multiplying both sides of the equation by the LCD.

Example: $\frac{1}{2}x - \frac{5}{3} = \frac{1}{6}x + \frac{14}{3}$

UGH! Don't want to deal with this?

$6 \cdot \left(\frac{1}{2}x - \frac{5}{3}\right) = \left(\frac{1}{6}x + \frac{14}{3}\right) \cdot 6$

The LCD is 6.
Multiply through by 6.

$\frac{\cancel{6} \cdot 1}{\cancel{2}}x - \frac{\cancel{6} \cdot 5}{\cancel{3}} = \frac{\cancel{6} \cdot 1}{\cancel{6}}x + \frac{\cancel{6} \cdot 14}{\cancel{3}}$

$3x - 10 = x + 28$

$x = 19$

Practice – Lesson 1

Solve each equation.

1) $6(p+5)-6(-7-3p)=-p+7p$

2) $-7m+3m=-5(m+2)+7(m-2)$

3) $-8(6x+1)=7(1-7x)$

4) $-3(x-7)-4x=3(-2-x)+5x$

5) $\frac{7}{2}-\frac{3}{2}b=-\frac{1}{2}b+1$

6) $\frac{7}{3}n-n=-\frac{5}{3}+\frac{1}{2}n$

Answers:

1) $\{-4\}$

2) $\{4\}$

3) $\{15\}$

4) $\{3\}$

5) $\left\{\frac{5}{2}\right\}$

6) $\{-2\}$

Topic #2: Solving Systems of Linear Equations in Two Variables

Basic Principles:

- In order to solve for two unknowns, you must write two equations.

Method 1: Substitution - Use when one variable is solved for or can be solved for easily.

Solve for x and y:

$$3x - y = 7$$
$$2x + 3y = 12$$

$$3x - y = 7$$
$$2x + 3y = 12$$

1. Solve for one variable in one equation.

$$3x - y = 7 \Rightarrow \boldsymbol{y = 3x - 7}$$

2. Substitute the expression for the variable you solved for. Make sure you plug the expression into the other equation, not the one you just rearranged.

$$2x + 3y = 12$$
$$2x + 3(3x - 7) = 12$$

3. Solve for the remaining variable.

$$2x + 9x - 21 = 12$$
$$11x - 21 = 12$$
$$11x = 33$$
$$x = 3$$

4. Plug the answer into either equation and solve for the other variable.

$$3x - y = 7$$
$$3(3) - y = 7$$
$$9 - y = 7$$
$$9 = 7 + y$$
$$y = 2$$

Method 2: Elimination - Use if coefficients prevent you from solving for a variable easily.

Example 2:
$$y + 3x = -4$$
$$2x - 3y = -21$$

$$\begin{aligned} y + 3x &= -4 \\ 2x - 3y &= -21 \end{aligned}$$

1. Align terms vertically.

$$\begin{aligned} y + 3x &= -4 \\ \mathbf{-3y + 2x} &= -21 \end{aligned}$$

2. Multiply one or both equations by a factor that will make one pair of coefficients opposites.

$$\begin{aligned} 3 \cdot (y + 3x &= -4) \\ -3y + 2x &= -21 \end{aligned}$$

3. Add the two equations together. One variable should be eliminated.

$$\begin{aligned} 3y + 9x &= -12 \\ \underline{-3y + 2x} &\underline{= -21} \\ 11x &= -33 \end{aligned}$$

4. Solve the resulting equation.

$$\begin{aligned} 11x &= -33 \\ x &= -3 \end{aligned}$$

5. Plug in the solved variable (into either equation) to solve for the other one.

$$\begin{aligned} y + 3(-3) &= -4 \\ y - 9 &= -4 \\ y &= 5 \end{aligned}$$

Solution: (-3, 5)

Note: If the variables disappear, and the resulting equation is….

TRUE:

The system has infinite solutions.

$$\begin{aligned} y + 3x &= -4 \\ 2y + 6x &= -8 \end{aligned}$$

$$-2(y + 3x = -4) \Rightarrow -2y - 6x = 8$$

$$\underline{2y + 6x = \quad -8 \Longrightarrow 2y + 6x = -8}$$

0 = 0

Since 0 = 0 is true, the system has infinite solutions.

FALSE:

The system has no solutions.

$$\begin{aligned} y + 3x &= -4 \\ 2y + 6x &= -10 \end{aligned}$$

$$-2(y + 3x = -4) \Rightarrow -2y - 6x = 8$$

$$\underline{2y + 6x = -10 \Longrightarrow 2y + 6x = -10}$$

0 = -2

Since 0 ≠2, the system has no solutions.

Practice – Lesson 2

Solve each system of equations.

1) $-10x + 10y = -10$
$10x - 6y = -22$

2) $6x + 4y = 10$
$-3x + 4y = 19$

3) $9x - 3y = 3$
$27x - 9y = 9$

4) $10x - 6y = 14$
$x + y = 3$

5) $9y + 31 - 11x = 0$
$8y - 8x = -40$

6) $2x - \frac{4}{3}y = 8$
$-2x - 10y = 26$

Answers:

1) $(-7, -8)$ 2) $(-1, 4)$ 3) Infinite number of solutions
4) $(2, 1)$ 5) $(-7, -12)$ 6) $(2, -3)$

Topic #3: Solving Word Problems and Writing Expressions

In order to solve word problems, it is helpful to translate verbal phrases into algebraic expressions.

Basic Examples:

	Word Phrases	Expression
+	• add 5 to a number • sum of a number and 5 • 5 more than a number	$n + 5$
−	• subtract 11 from a number • difference of a number and 11 • 11 less than a number	$x - 11$
×	• 3 multiplied by a number • product of 3 and a number	$3m$
÷	• 7 divided into a number • quotient of a number and 7	$\frac{a}{7}$ or $a \div 7$

Tougher Examples

• **twice the sum** of a number and 5	$2(x+5)$
• 10 less than twice the sum of a number and 5	$2(x+5)-10$
• **the square of the sum** of a number and 5 • x+5, quantity squared	$(x+5)^2$
• **the square root of the sum** of x and 5	$\sqrt{x+5}$
• The sum of two numbers is S If one of the numbers is x, express the other number.	$Let\ y = the\ other\ number$ $Then\ \ x + y = S,$ $and\ y = S - x$ ANSWER: $\boldsymbol{S - x}$

Exercises Translate each phrase into an algebraic expression. Let x = that number.

1. the difference between twice the square of the sum of a number and 4, and the square of that number.

2. Two angles that add up to 90 degrees are called complements. Express the sum of twice an angle and three times its complement.

Write a possible algebraic expression for each phrase.

3. $2x^2$

4. $(2x)^2$

5. $\frac{x+5}{x-4}$

Word Problem Example

The perimeter of a rectangle is 40. If its area is 96, find the dimensions of the rectangle.

1) Think…the question asks for dimensions. That's length and width, 2 unknowns.

2) Let's assign them each a letter.

   Let L = length

   Let W = width

3) Now let's write our equations.

The perimeter of the rectangle	is	40.
2L + 2W	=	40
The area of the rectangle	is	96.
LW	=	96

4) $2L + 2W = 40$
   $LW = 96$

Use substitution to solve. Solve for one of the variables.

$$2L + 2W = 40 \quad \Rightarrow \quad L = 20 - W$$

Plug $20 - W$ into the other equation:

$LW = 96$

$(20 - W)W = 96$

$20W - W^2 = 96$

$W^2 - 20W + 96 = 0$

$(W - 12)(W - 8) = 0$

$W = 12 \text{ or } W = 8$

Making $L = 20 - 8 = 12 \text{ or } 20 - 12 = 8$

$$\therefore \textit{The dimensions of the rectangle are } 12 \textit{ by } 8.$$

Steps to Solving Word Problems

1. DECIDE HOW MANY UNKNOWNS THERE ARE. YOU WILL WRITE ONE EQUATION PER UNKNOWN.
2. DEFINE YOUR VARIABLES.
3. USE THE CONDITIONS IN THE PROBLEM TO WRITE YOUR EQUATIONS, USING YOUR TRANSLATION SKILLS.
4. SOLVE AND ANSWER THE QUESTION.

Answer Key

1) $2(x + 4)^2 - x^2$
2) $2x + 3(90 - x)$
3) Twice the square of a number
4) The square of twice the number
5) The quotient of the sum of a number and five, and the sum of a number and 4
6) Seth is 5 and Betsy is 12

You try! *Solve algebraically.* Today, Seth is 7 years younger than Betsy. In five years, Seth will be three years younger than Betsy will be in eight years. How old are Seth and Betsy today?

Topic #4: Writing the Equation of a Line

A linear equation is a way of describing a relationship between two sets of numbers, x and y, where the ratio of the change in y to the change in x is the same all the way through (constant).

There are three ways to express this relationship: **standard** form, **slope-intercept form**, and **point-slope form**. We will explore the two latter forms here.

Standard Form	$Ax + By = C$
Point-Slope Form	$y - y_1 = m(x - x_1)$
Slope-Intercept Form	$y = mx + b$

Slope Formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

<table>
<tr><td colspan="2"><u>Example 1</u>: Write the equation of the line that has the given slope and that passes through the given point.</td></tr>
<tr><td><u>Point-Slope Form</u> m = 3, (1,5)
$y - y_1 = m(x - x_1)$
$\mathbf{y - 5 = 3(x - 1)}$</td><td><u>Slope-Intercept Form</u> m = 3, (1,5)
$y = mx + b$
$5 = 3(1) + b$
$b = 2$
$\mathbf{y = 3x + 2}$</td></tr>
</table>

<table>
<tr><td colspan="2"><u>Example 2</u>: Write the equation of the line that passes through the given points.</td></tr>
<tr><td><u>Point-Slope Form</u> (12, -5) and (-4, -1)
Find slope: $\frac{-1-(-5)}{-4-12} = \frac{4}{-16} = -\frac{1}{4}$

Plug in either of the two points and the slope:
$y - (-1) = -\frac{1}{4}(x - (-4))$
$\mathbf{y + 1 = -\frac{1}{4}(x + 4)}$</td><td><u>Slope-Intercept Form</u> (12, -5) and (-4, -1)
Find slope: $\frac{-1-(-5)}{-4-12} = \frac{4}{-16} = -\frac{1}{4}$

Plug in either of the two points and the slope to find the y-intercept:
$y = mx + b$
$-1 = -\frac{1}{4}(-4) + b$
$-1 = 1 + b$
$b = -2$
$\mathbf{y = -\frac{1}{4}x - 2}$</td></tr>
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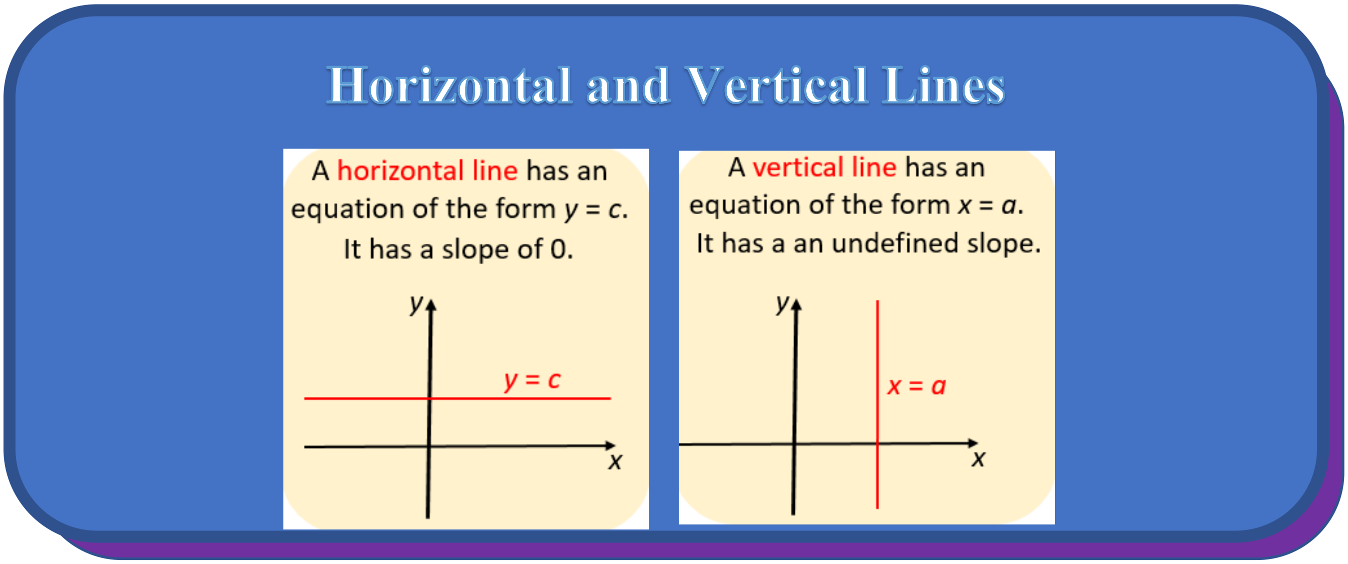


Example 3a: Write the equation of the line that passes through the points (3, -5) and (14, -5).

$$m = \frac{-5-(-5)}{14-3} = \frac{0}{11} = 0 \Rightarrow horizontal\ line, c = -5 \Rightarrow \boldsymbol{y = -5}$$

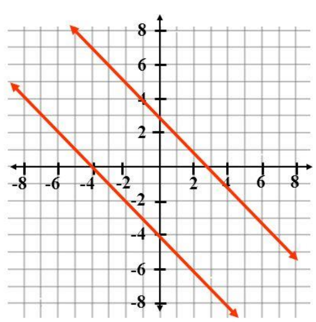
Example 3b: Write the equation of the line that passes through the points (3, -5) and (3, 12).

$$m = \frac{12-(-5)}{3-3} = \frac{17}{0} = undefined \Rightarrow vertical\ line, a = 3 \Rightarrow \boldsymbol{x = 3}$$

Parallel Lines

Parallel lines are lines in the same plane that never intersect.

Parallel lines have the same slope.



Example 4:

Write the equation of a line that is parallel to the line $y = \frac{2}{3}x + 5$ and passing through (-6,11).

Slope must be $= \frac{2}{3}$:

$$y = mx + b$$
$$11 = \frac{2}{3}(-6) + b$$
$$11 = -4 + b$$
$$b = 15$$
$$\boldsymbol{y = \frac{2}{3}x + 15}$$

Practice – Lesson 4

Write the equation of the line through the given point with the given slope, in a) point-slope form, and b) slope-intercept form.

1) through: $(5, -3)$, slope $= -\frac{1}{5}$

Write the equation of the line through the two given points, in a) point-slope form, and b) slope-intercept form.

2) through: $(1, -3)$ and $(-4, -4)$

Write the slope-intercept form of the equation of the line through the given points.

3) through: $(3, -5)$ and $(3, -4)$

4) through: $(0, -1)$ and $(-4, -1)$

Write the equation of the line described, in point-slope and slope-intercept form.

5) through: $(5, 1)$, parallel to $y = -\frac{4}{7}x + 1$

Answers:

1) a) $y + 3 = -\frac{1}{5}(x - 5)$; b) $y = -\frac{1}{5}x - 2$

2) a) $y + 4 = \frac{1}{5}(x + 4)$; b) $y = \frac{1}{5}x - \frac{16}{5}$

3) $x = 3$

4) $y = -1$

5) a) $y - 1 = -\frac{4}{7}(x - 5)$; b) $y = -\frac{4}{7}x + \frac{27}{7}$

Topic #5: Adding Polynomials

We need to combine algebraic expressions in order to simplify complex algebraic expressions.

Rules:

1. Combine only LIKE TERMS: same variables raised to the same exponents.

   Example: $3x \text{ and } 5x, 6xy^2$ and $-3xy^2$

2. Add the coefficients, but leave the exponents alone.

Examples:

1. Combine like terms: $3x - 6y - \frac{1}{2}x + 6y$

   Solution: $3x - 6y - \frac{1}{2}x + 6y = \left(3 - \frac{1}{2}\right)x + (-6+6)y = \frac{5}{2}x$

   Add only the coefficients of each variable.

   Answer: $\frac{5}{2}x$

2. Find the sum: $3x - 6y + (2x - 8y)$

   Solution: $(3+2)x + (-6-8)y$

   $5x - 14y$

   Answer: $5x - 14y$

Practice – Lesson 5

Simplify each sum.

1) $(3x^2 + 6x - 3) + (5 - 5x^2 - 3x) + (x + 7x^2 + 1)$

2) $\frac{9}{8}n^2 + \frac{5}{3} - 3n - 2n^2 + 2$

3) $2ab - a - 4a^4b - 6a^4b^4 - 2a^4b + 7a + 4a^4b^4 - 3ab - 2a^3 + 6a^4b$

Answers:

1) $5x^2 + 4x + 3$ 2) $-\frac{7}{8}n^2 - 3n + \frac{11}{3}$ 3) $-2a^4b^4 - 2a^3 - ab + 6a$

Topic #6: Subtracting Polynomials/Simplifying Expressions

Rules:

1. Start with the innermost set of parentheses.
2. If there is a minus sign outside of those parentheses:
   a. Change the minus sign to a plus sign.
   b. Change the signs of EVERY term inside the parentheses. Drop the parentheses.
3. Combine all like terms in that parentheses.
4. Repeat, working your way out.

Example 1: Combine like terms: $3x - (2x - 5)$

$3x - \mathbf{2x} + \mathbf{5}$

Change *each term* inside the parentheses to the opposite sign.

$x + 5$

Answer: $x + 5$

Example 2: Subtract $x^2 - 6x - 1$ from $3x^2 - 6x - 2$.

$(3x^2 - 6x - 2) - (x^2 - 6x - 1)$

The expression you are subtracting FROM goes FIRST.

$3x^2 - 6x - 2 - \mathbf{x^2} + \mathbf{6x} + \mathbf{1}$

$2x^2 - 1$

Answer: $2x^2 - 1$

Example 3: Simplify this expression: $\{-2x^2 - [3x - (8x - 3)] + 4x^2\}$

$\{-2x^2 - [3x - (8x - 3)] + 4x^2\}$

$\{-2x^2 - [3x - \mathbf{8x} + \mathbf{3}] + 4x^2\}$

Start with the innermost parentheses. Change all terms inside the parentheses. Combine 3x and – 5x.

$\{-2x^2 - [\mathbf{-5x} + 3] + 4x^2\}$

$\{-2x^2 + \mathbf{5x} - \mathbf{3} + 4x^2\}$

Repeat, working your way to the outside parentheses.

$\{\mathbf{-2x^2} + 5x - 3 + \mathbf{4x^2}\}$

$\mathbf{2x^2} + 5x - 3$

Answer: $\mathbf{2x^2 + 5x - 3}$

<u>Practice – Lesson 6</u>

Simplify each difference.

1) $(1-2p+6p^2)-(4p+p^2-5)$

2) $(2k^2-5+2k)-(3k^2+7k-8)$

3) Subtract $3x^2-6x+9$ from $6x^2+9x-4$.

4) Subtract $\frac{1}{2}x^2+4$ from the sum of $\frac{3}{4}x^2+5x+2$ and $\frac{1}{8}x^2+4x$.

5) Simplify and express the answer in standard form:

$$\{3x^2-[4x+6x^2-(3x+5)+7]+5x^2-10x\}$$

Answers:

1) $5p^2-6p+6$

2) $-k^2-5k+3$

3) $3x^2+15x-13$

4) $\frac{3}{8}x^2-2$

5) $2x^2-11x-2$

Topic #7: Multiplying Polynomials

Rule for multiplying monomials:

1. Multiply the coefficients.
2. Add the exponents.

Example: Simplify: $-3x^5 \cdot -6x^3$

$(-3 \cdot -6)x^{5+3}$

$18x^8$

Multiply the coefficients.
Add the exponents.

Answer: $18x^8$

Rule for multiplying polynomials:

When multiplying polynomial A with polynomial B, you must multiply each term in A by each term in B.

How you organize this is up to you.

Method 1: Vertical Method

Step 1. Split one expression and multiply each term by the second polynomial.

Multiply $(x+5)(x^2-8x+4)$.

$$\begin{array}{rcrrrr} x(x^2-8x+4) & = & x^3 & -8x^2 & +4x & \\ \underline{5(x^2-8x+4)} & \underline{=} & & \underline{5x^2} & \underline{-40x} & \underline{+20} \\ & & x^3 & -3x^2 & -36x & +20 \end{array}$$

Step 2. As you multiply, line up the terms vertically.

Step 3. Add the like terms.

***Answer:* $\boldsymbol{x^3-3x^2-36x+20}$**

Method 2: Box Method

Multiply: $(x+4)(x-6)$

Step 2. Multiply each pair of terms in the grid.

Step 1. Write the terms of each polynomial across the top and side of each grid.

	x	4
x	x^2	$4x$
-6	$-6x$	-24

$= x^2+4x-6x-24$

$= x^2-2x-24$

Step 3. Add the like terms together.

***Answer:* $\boldsymbol{x^2-2x-24}$**

Practice – Lesson 7

Simplify each expression.

1) $4x^3y^3 \cdot 4x^2y^4$

2) $3x^3y^2 \cdot 3x^4y^3 \cdot 2yx^4$

3) $(3x+5)(8x+3)$

4) $(4x-1)(3x+5)$

5) $(8a+6)(6a^2-8a-2)$

6) $(3n^2+2n-4)(6n^2+2n-3)$

Answers:

1) $16x^5y^7$ 2) $18x^{11}y^6$ 3) $24x^2+49x+15$ 4) $12x^2+17x-5$
5) $48a^3-28a^2-64a-12$ 6) $18n^4+18n^3-29n^2-14n+12$

Topic #8: Squaring Binomials and Multiplying Conjugates

It is helpful to recognize certain patterns when multiplying. It only makes our job easier!

Pattern #1: Multiplying Conjugates Conjugates are a two-term sum and difference pair that contain the same terms. For example: $x+3$ and $x-3$, or $6-\sqrt{2}$ and $6+\sqrt{2}$.	$(a+b)(a-b)=a^2-b^2$
In words: The product of conjugates = first term^2 - second term^2	<u>Examples</u> $(x+3)(x-3)=(x)^2-(3)^2=x^2-9$ $(6+\sqrt{2})(6-\sqrt{2})=(6)^2-(\sqrt{2})^2=36-2=34$ $[(\boldsymbol{x}+\mathbf{4})-2][(\boldsymbol{x}+\mathbf{4})+2]=$ $(\boldsymbol{x}+\mathbf{4})^2-(2)^2=$ $\boldsymbol{x}^{\mathbf{2}}+\mathbf{8}\boldsymbol{x}+\mathbf{16}-4=$ $x^2+8x+12$
Pattern #2: Squaring Binomials Remember that when we square a binomial, we multiply it by itself. So, we expect to end up with four terms, that collapse to three (a trinomial).	$(a+b)^2=a^2+2ab+b^2$
In words: The square of a sum = $(first\ term)^2+\ (second\ term)^2$ $+\ double\ the\ product\ of\ first\ \&\ second$	<u>Examples</u> $(x+3)(x+3)=(x)^2+(3)^2+2(x)(3)=$ x^2+6x+9 $(6-\sqrt{2})(6-\sqrt{2})=(6)^2+(\sqrt{2})^2+2(6)(-\sqrt{2})$ $=36+2-12\sqrt{2}$ $=34-12\sqrt{2}$

Practice – Lesson #8

Find each product.

1) $(b+5)(b-5)$

2) $(6x+8)(6x-8)$

3) $(2-7n)(2+7n)$

4) $\left(\frac{4}{3}b+\frac{8}{5}\right)\left(\frac{4}{3}b-\frac{8}{5}\right)$

5) $(n-3)^2$

6) $(7n-5)^2$

7) $(3m+1)^2$

8) $\left(\frac{3}{2}m+\frac{12}{7}\right)^2$

Answers:

1) b^2-25

2) $36x^2-64$

3) $4-49n^2$

4) $\frac{16}{9}b^2-\frac{64}{25}$

5) n^2-6n+9

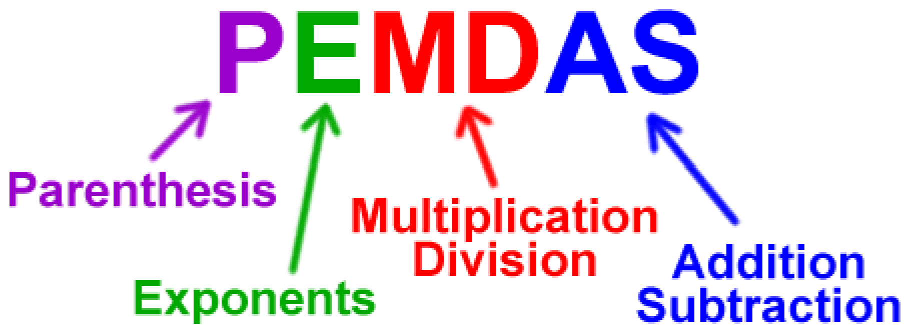
6) $49n^2-70n+25$

7) $9m^2+6m+1$

8) $\frac{9}{4}m^2+\frac{36}{7}m+\frac{144}{49}$

Topic #9: Simplifying Complex Expressions

You may need to simplify expressions with more than one operation when solving an equation. These expressions always follow PEMDAS. It is important to follow the steps in order when simplifying!



Special Notes:

- "Parentheses" only means completing an operation *inside* the parentheses. If there is none to be done, skip this step.
- Multiplication/Division and Addition/Subtraction should be done as they appear from left to right.
- Some expressions have implied parentheses, such as $2|3+5|$ or $\frac{6+4}{2+3}$.
- The most common PEMDAS error in evaluating expressions is multiplying before squaring.

Example: $(\boldsymbol{x}+\mathbf{5})^{\mathbf{2}}-\mathbf{2}(\boldsymbol{x}-\mathbf{3})^{\mathbf{2}}$

No P, evaluate Exponent (square) $\quad (x+5)(x+5)-2(x-3)(x-3)$

$(x^2+10x+25)-2(x^2-6x+9)$

M/D - Multiply 2. $\quad (x^2+10x+25)-(2x^2-2x+18)$

A/S - Subtract. $\quad (x^2+10x+25)\mathbf{-2x^2+12x-18}$

$-x^2+22x+7$

You Try:

Evaluate $(\boldsymbol{x}+\mathbf{8})^{\mathbf{2}}-\mathbf{3}(\boldsymbol{x}-\mathbf{2})^{\mathbf{2}}$

Answer: $\mathbf{-2x^2+28x+52}$

Practice – Lesson 9 Simplify each expression. Express the result in standard form.

1) $(x-4)^2+(x+8)(x-3)$
2) $(x+9)^2-(x+5)(x-2)$
3) $-3(x-2)^2$
4) $-(x+6)^2+(x+2)\left(\frac{1}{2}x-1\right)$
5) $2(x-4)^2-(2x+7)^2$

Answers:

1) $2x^2-3x-8$
2) $15x+91$
3) $-3x^2+12x-12$
4) $\frac{1}{2}x^2-12x-38$
5) $-2x^2-44x-17$

Topic #10: Simplifying and Multiplying Radicals

$$\sqrt{ab} \underset{\textit{Multiplying}}{\overset{\textit{Simplifying}}{\rightleftarrows}} \sqrt{a}\cdot\sqrt{b}$$

Simplifying Radicals

A radical is in **simplest radical form** when no perfect square factor can divide the radicand. The radicand is the expression under the radical sign.

Example 1: Express $\sqrt{252}$ in simplest radical form.

$\sqrt{252}$

1. Using the rule above, take out the highest perfect square factor of the radicand. $\sqrt{36}\cdot\sqrt{7}$

2. Simplify the perfect square radical. $\boldsymbol{6\sqrt{7}}$

Note: The highest perfect square factor of 252 is 36. However, 9 and 4 also go into it. If you use either of these, you will have to simplify again.

$\sqrt{252}$
$\sqrt{9}\cdot\sqrt{28}$
$3\sqrt{28}$ — Uh-oh! 28 still has a factor of 4 in it. Keep going!
$3\cdot\sqrt{4}\cdot\sqrt{7}$
$3\cdot 2\cdot\sqrt{7}$
$\boldsymbol{6\sqrt{7}}$

Example 2: Simplify $\sqrt{a^2+b^2}$

This radical cannot be simplified because to do so would require breaking up a sum.

Only products can be broken up.

Remember, $\sqrt{a^2+b^2} \neq a+b$!!

Example 3: Simplify $\sqrt{8}\cdot\sqrt{20}$

$\sqrt{8}\cdot\sqrt{20}$

1. Multiply the radicands. $\sqrt{160}$

2. Simplify radical, if needed. (see above) $\sqrt{16}\cdot\sqrt{10}$
$4\sqrt{10}$

Practice – Lesson 10

Simplify.

1) $\sqrt{42}$

2) $\sqrt{112}$

3) $\sqrt{512}$

4) $\sqrt{30}$

5) $5\sqrt{125}$

6) $2\sqrt{200}$

7) $6\sqrt{42}$

8) $5\sqrt{686}$

9) $\sqrt{2} \cdot \sqrt{2}$

10) $\sqrt{5} \cdot \sqrt{20}$

11) $-4\sqrt{6} \cdot 4\sqrt{2}$

12) $5\sqrt{5} \cdot \sqrt{10}$

13) $\sqrt{36+64}$

14) $-3\sqrt{25+100}$

Answers:

1) $\sqrt{42}$
2) $4\sqrt{7}$
3) $16\sqrt{2}$
4) $\sqrt{30}$
5) $25\sqrt{5}$
6) $20\sqrt{2}$
7) $6\sqrt{42}$
8) $35\sqrt{14}$
9) 2
10) 10
11) $-32\sqrt{3}$
12) $25\sqrt{2}$
13) 10
14) $-15\sqrt{5}$

Topic #11: Adding and Subtracting Radicals

Rule:

1. Radicands must be the same to combine radicals.
2. Add or subtract the coefficients.
3. Leave the radicands alone.

Example 1 – Like Radicals: $6\sqrt{3}+4\sqrt{3}$ $6\sqrt{3}+4\sqrt{3}$ $(6+4)\sqrt{3}$ *Add the coefficients only.* $\mathbf{10\sqrt{3}}$	Example 2 – Unlike Radicals: $6\sqrt{5}+2\sqrt{80}$ $6\sqrt{5}+2\sqrt{80}$ $6\sqrt{5}+2\sqrt{\mathbf{16}}\cdot\sqrt{\mathbf{5}}$ *Add the coefficients only.* $6\sqrt{5}+2\cdot\mathbf{4}\cdot\sqrt{\mathbf{5}}$ *Simplify all radicals.* $6\sqrt{5}+\mathbf{8}\sqrt{5}$ *Add the coefficients.* $\mathbf{14\sqrt{5}}$
Example 3 – Unlike Radicals: $\sqrt{3}+4\sqrt{2}$ Since neither of these radicals can be simplified, these radicals cannot be combined. The expression is in simplest radical form.	Example 4 – Square Root of a Sum: $\sqrt{25+100}$ $\sqrt{25+100}$ $\sqrt{125}$ $\sqrt{25}\cdot\sqrt{5}$ $\mathbf{5\sqrt{5}}$ DO NOT SPLIT A SUM!! $\sqrt{25+100}\neq$ $\sqrt{25}+\sqrt{100}\neq$ $5+10\neq$ 15

Practice – Lesson 11

Simplify each expression.

1) $\sqrt{3}+\sqrt{3}+\sqrt{3}$

2) $3\sqrt{5}+2\sqrt{5}$

3) $-\sqrt{2}-3\sqrt{6}-2\sqrt{2}$

4) $3\sqrt{5}+2\sqrt{5}-3\sqrt{2}$

5) $2\sqrt{20}+5\sqrt{20}$

6) $2\sqrt{250}-\sqrt{10}$

7) $2\sqrt{96}+2\sqrt{3456}$

8) $-5\sqrt{18}-7\sqrt{648}$

9) $\sqrt{24+36}$

10) $\sqrt{16+29}-\sqrt{100+80}$

Answers:

1) $3\sqrt{3}$
2) $5\sqrt{5}$
3) $-3\sqrt{2}-3\sqrt{6}$
4) $5\sqrt{5}-3\sqrt{2}$
5) $14\sqrt{5}$
6) $9\sqrt{10}$
7) $56\sqrt{6}$
8) $-141\sqrt{2}$
9) $2\sqrt{15}$
10) $-3\sqrt{5}$

Topic #12: Dividing Radicals and Rationalizing Denominators

Dividing Radicals *The quotient of two square roots is equal to the square root of the quotient.*	$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$
Rationalizing Denominators *If a denominator is irrational, and the radicals cannot be divided,* rationalize *the denominator by multiplying numerator and denominator by the denominator's radical.*	$\frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{a}}{\sqrt{b}} \bullet \frac{\sqrt{b}}{\sqrt{b}} = \frac{\sqrt{ab}}{b}$ $\frac{a}{\sqrt{b}} = \frac{a}{\sqrt{b}} \bullet \frac{\sqrt{b}}{\sqrt{b}} = \frac{a\sqrt{b}}{b}$

<u>Example 1 – Dividing Radicals</u>

Simplify: $\frac{\sqrt{48}}{\sqrt{3}}$

$\frac{\sqrt{48}}{\sqrt{3}}$

$\sqrt{\frac{48}{3}}$ *Divide radicands.*

$\sqrt{16}$ *Simplify.*

4

<u>Example 2 – Simplifying Fractional Radicands</u>

Simplify: $\frac{5}{\sqrt{8}}$

$\frac{5 \cdot \sqrt{8}}{\sqrt{8} \cdot \sqrt{8}}$ *Multiply numerator and denominator by* $\sqrt{8}$.

$\frac{5\sqrt{8}}{\sqrt{64}}$

$\frac{5 \cdot \mathbf{2}\sqrt{\mathbf{2}}}{8}$ *Simplify radicals.*

$\frac{10\sqrt{2}}{8}$

$\frac{\mathbf{5\sqrt{2}}}{\mathbf{4}}$ *Simplify the fraction.*

<u>Example 3 – Fractional Radicands</u>

The same can be done inside the radical.

$$\sqrt{\frac{3}{8}} = \sqrt{\frac{3 \cdot 2}{8 \cdot 2}} = \sqrt{\frac{6}{16}} = \frac{\sqrt{6}}{\sqrt{16}} = \frac{\mathbf{\sqrt{6}}}{\mathbf{4}}$$

NOTE: You can multiply top and bottom by any radical that would make the denominator a perfect square. So, $\sqrt{2}$ will also work:

Example: $\frac{5}{\sqrt{8}} = \frac{5 \cdot \sqrt{2}}{\sqrt{8} \cdot \sqrt{2}} = \frac{5\sqrt{2}}{\sqrt{16}} = \frac{\mathbf{5\sqrt{2}}}{\mathbf{4}}$

Lesson 12 – Practice

Simplify each expression.

1) $\dfrac{\sqrt{360}}{\sqrt{3}}$

2) $\dfrac{\sqrt{90}}{\sqrt{10}}$

3) $\dfrac{3\sqrt{12}+3\sqrt{24}}{\sqrt{4}}$

4) $\dfrac{14\sqrt{200}+7\sqrt{60}}{7\sqrt{10}}$

5) $\dfrac{3}{\sqrt{3}}$

6) $\dfrac{4}{\sqrt{2}}$

7) $\dfrac{\sqrt{5}}{\sqrt{2}}$

8) $\sqrt{\dfrac{7}{12}}$

Answers:

1) $2\sqrt{30}$
2) 3
3) $3\sqrt{3}+3\sqrt{6}$
4) $4\sqrt{5}+\sqrt{6}$
5) $\sqrt{3}$
6) $2\sqrt{2}$
7) $\dfrac{\sqrt{10}}{2}$
8) $\dfrac{\sqrt{21}}{6}$

Topic #13: Solving Quadratic Equations by Factoring

Factoring is a quick and efficient method to solve some quadratic equations of the form $ax^2 + bx + c = 0$.

<u>Example:</u> **Solve $\boldsymbol{x^2 = 6x + 7}$**

1. Write the equation in standard form: $ax^2 + bx + c = 0$

$$x^2 = 6x + 7$$
$$x^2 - 6x - 7 = 0$$

2. Factor the equation.

$$(x - 7)(x + 1) = 0$$

3. Set each linear factor equal to zero.

$$x - 7 = 0 \; or \; x + 1 = 0$$

4. Solve each linear equation.

$$x = 7 \; or \; x = -1$$

<u>Pro Tips:</u>

• If a quadratic contains any fractional coefficients, multiply through by the least common denominator. This will eliminate the fraction(s).	Example: $\frac{1}{2}x^2 - 2x - 16 = 0$ $2 \cdot (\frac{1}{2}x^2 - 2x - 16 = 0) \cdot 2$ $x^2 - 4x - 32 = 0$ $(x - 8)(x + 4) = 0$ $x - 8 = 0 \; or \; x + 4 = 0$ $x = 8 \; or \; x = -4$
• Move the terms to the side where the lead coefficient is positive.	Example: $x^2 = 6x + 8$ $\underline{-6x - 8 \;\; - \;\; 6x - 8}$ $x^2 - 6x - 8 = 0$ $(x - 8)(x + 2) = 0$ $x - 8 = 0 \; or \; x + 2 = 0$ $x = 8 \; or \; x = -2$
• Not sure whether factoring will work? Find the value of $a, b, and\ c$ and plug it into the expression $b^2 - 4ac$. If the answer is a square number, the expression is factorable.	Example: Is the expression $x^2 - 3x - 28$ factorable? $a = 1$ $b = -3$ $c = -28$ Plug into $b^2 - 4ac$: $(-3)^2 - 4(1)(-28)$ $9 + 112$ 121 Since 121 is a perfect square, the expression is factorable.

- Not sure which factoring method to use? Try these methods, in order:

1. Does it have a **common factor**? Take out the GCF. If it's a number, divide both sides by that number. If it contains a variable, set that variable equal to zero.	$x^2 - 4x = 0$ $x(x-4) = 0$ $x = 0 \text{ or } x - 4 = 0$ $x = 0 \text{ or } x = 4$
2. Is it a **trinomial with leading coefficient 1** (and no GCF)? Factor into two binomials.	$x^2 - 4x - 60 = 0$ $(x-10)(x+6) = 0$ $x - 10 = 0 \text{ or } x + 6 = 0$ $x = 10 \text{ or } x = -6$
3. Is it a **trinomial with leading coefficient > 1** (and no GCF)? Factor according to preferred method (see below).	$2x^2 + 5x + 2 = 0$ $(2x+1)(x+2) = 0$ $2x + 1 = 0 \text{ or } x + 2 = 0$ $x = -\frac{1}{2} \text{ or } x = -2$
4. Does it have **two perfect square terms with a minus sign?** Follow the Difference of Two Squares pattern (DOTS): $a^2 - b^2 = (a+b)(a-b)$	$x^2 - 64 = 0$ $(x-8)(x+8) = 0$ $x - 8 = 0 \text{ or } x + 8 = 0$ $x = 8 \text{ or } x = -8$

Methods for Factoring $ax^2 + bx + c = 0$:

a. "Borrow and Payback"	***b. Factoring by Grouping***
$2x^2 + 5x + 2 = 0$ $x^2 + 5x + 4 = 0$ Multiply lead coefficient by c. $(x+1)(x+4) = 0$ Factor. $(x+1)\left(x + \frac{4}{2}\right) = 0$ Divide one constant by lead coefficient. You may have to split it into factors. $(\mathbf{2}x+1)(x+2) = 0$ The number you divided by in the last step becomes the lead coefficient in the *opposite* bracket.	$2x^2 + 5x + 2 = 0$ $2x^2 + \mathbf{1x} + \mathbf{4x} + 2 = 0$ Split up the b-term into a sum. Choose sum wisely. $x(2x+1) + 2(2x+1) = 0$ Take out GCF from first two and last two terms. The remaining factor should be the same in both groups. $(2x+1)(x+2) = 0$ Factor out the binomial GCF.

Practice – Lesson 13

Solve each equation by factoring.

1) $n^2 - 5n + 4 = 0$

2) $n^2 - 3n + 7 = 5$

3) $5n^2 - 80 = 0$

4) $6n^2 - 12n + 37 = 5 + 5n^2$

5) $7x^2 + 50x = -48$

6) $\frac{1}{2}x^2 - \frac{16}{5}x + \frac{6}{5} = 0$

Answers:

1) $\{4, 1\}$

2) $\{2, 1\}$

3) $\{-4, 4\}$

4) $\{4, 8\}$

5) $\left\{-\frac{8}{7}, -6\right\}$

6) $\{\frac{2}{5}, 6\}$

Topic #14: Solving Quadratic Equations with the Quadratic Formula

You can solve ANY quadratic equation of the form $ax^2 + bx + c = 0$ using the quadratic formula. This method is especially helpful if the equation does not factor.

The Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 1: Find the roots of $x^2 - 4x = 2$.

$$x^2 - 4x = 2$$

1. Write the equation in standard form. $x^2 - 4x - 2 = 0$
2. Find the value of a, b, and c. $a = 1$, $b = -4$, $c = -2$
3. Plug them into the quadratic formula. $$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-2)}}{2(1)}$$
4. Evaluate the radicand (under the radical sign). $$x = \frac{-(-4) \pm \sqrt{24}}{2(1)}$$
5. Simplify the radical, if necessary. $$x = \frac{4 \pm 2\sqrt{6}}{2}$$
6. Divide. $$x = 2 \pm \sqrt{6}$$

Solution set: $\{2 + \sqrt{6}, 2 - \sqrt{6}\}$

- Note: If the radicand (number under the radical sign) comes out a negative, the solutions to the quadratic equation are not real numbers.

You try! Solve using the quadratic formula.

1. $x^2 - 5x = 50$
2. $x^2 - 2x = 0$
3. $x^2 = 12$
4. $-x^2 + 6x = 10$

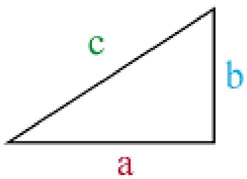
Answers:

1. {10, -5}
2. {0, 2}
3. $\{\pm 2\sqrt{3}\}$
4. No real solutions

Topic #15: The Pythagorean Theorem

The Pythagorean Theorem is used to find the length of a missing side of a right triangle.

The Pythagorean Theorem

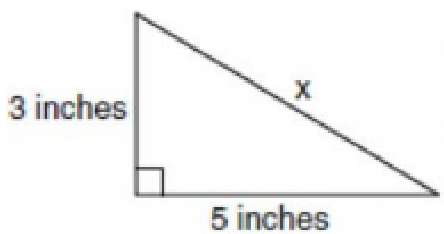


$a^2 + b^2 = c^2$

Things to Remember:

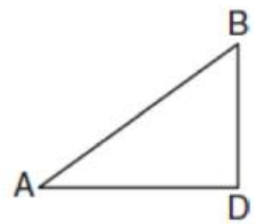
- Only used for right triangles
- *c* must be the length of the hypotenuse (side opposite the right angle)
- Can only find side lengths, never angles

Example 1: Find the length of the missing side.



Example 2:

In the diagram below of $\triangle ADB$, $m\angle BDA = 90$, $AD = 5\sqrt{2}$, and $AB = 2\sqrt{15}$.



What is the length of $\overline{BD}$?

Pythagorean Triples

Pythagorean triples are positive integers that fit the Pythagorean Theorem. It is helpful to memorize these because they help you solve problems faster. Below is a list of common Pythagorean Triples.

(NOTE: Multiples of these triples also work. For example, (3, 4, 5) x 2 = 6, 8, 10 is also a triple.

These are the next most common.

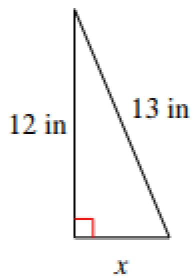
Pythagorean Triples with c < 100

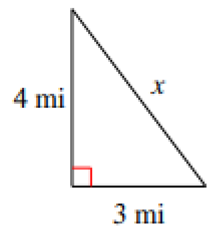
These triples are the most common.

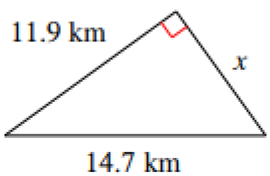
(3, 4, 5)	(5, 12, 13)	(8, 15, 17)	(7, 24, 25)
(20, 21, 29)	(12, 35, 37)	(9, 40, 41)	(28, 45, 53)
(11, 60, 61)	(16, 63, 65)	(33, 56, 65)	(48, 55, 73)
(13, 84, 85)	(36, 77, 85)	(39, 80, 89)	(65, 72, 97)

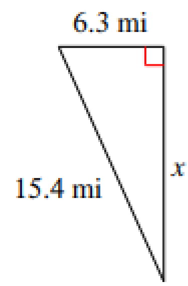
Practice – Lesson 15

Find the missing side of each triangle. Round your answers to the nearest tenth if necessary.

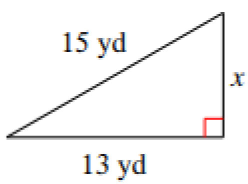
1) 

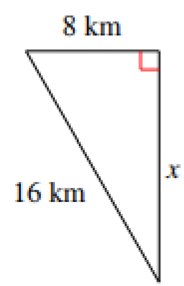
2) 

3) 

4) 

Find the missing side of each triangle. Leave your answers in simplest radical form.

5) 

6) 

Answers

1) 5
2) 5
3) 8.6
4) 14.1
5) $2\sqrt{14}$
6) $8\sqrt{3}$