Geometry
Summer Skills Review

Our Purpose: *Completion of this packet over the summer before beginning Geometry will be of great value to helping students successfully meet the academic challenges awaiting them in Geometry and beyond.*

To Parents/Guardians: *Teachers and administrators of White Plains High School actively encourage parents/guardians to engage in their child's learning. This Summer Review for students entering Geometry has been developed to provide our students with a summer resource and practice to help refresh major topics from previous years.*

By encouraging your child to work on this packet, you will help them enter Geometry prepared with the required skills to reach their full potential. The packet has been designed to provide a review of Algebra skills that are essential for student success in Geometry.

The White Plains High School Mathematics Department highly recommends that students work through this packet during the summer and bring it, with work shown and with any questions, to school on the first day of Geometry.

WPHS
MATH
DEPARTMENT

GEOMETRY – Summer Prep

Basic Skills and Techniques Needed For a Successful Year

Section 1

Unit Conversion

- Convert units of measure using the metric system

Section 2

Area & Perimeter

- Determine the perimeter of basic shapes
- Determine the area of basic shapes
- Find the missing dimension given the area

Section 3

Volume

- Find the volume of a prism, cylinder, sphere and cone

Section 4

Slope

- Find the slope given the coordinates two points
- Find the slope given a graph of two points

Section 5

Equation of a Line

- Write the equation of a line in slope-intercept form
- Write the equation of a line in point-slope form

Section 6

Radicals

- Simplifying radicals
- Adding and Subtracting radicals

Section 7

Pythagorean Theorem

- Using the Pythagorean theorem to find the missing side of a right triangle

Section 8

Factoring

- Factoring by greatest common factor
- Factor a difference of perfect squares
- Factoring a quadratic trinomial

Section 9

Solving Quadratic Equations

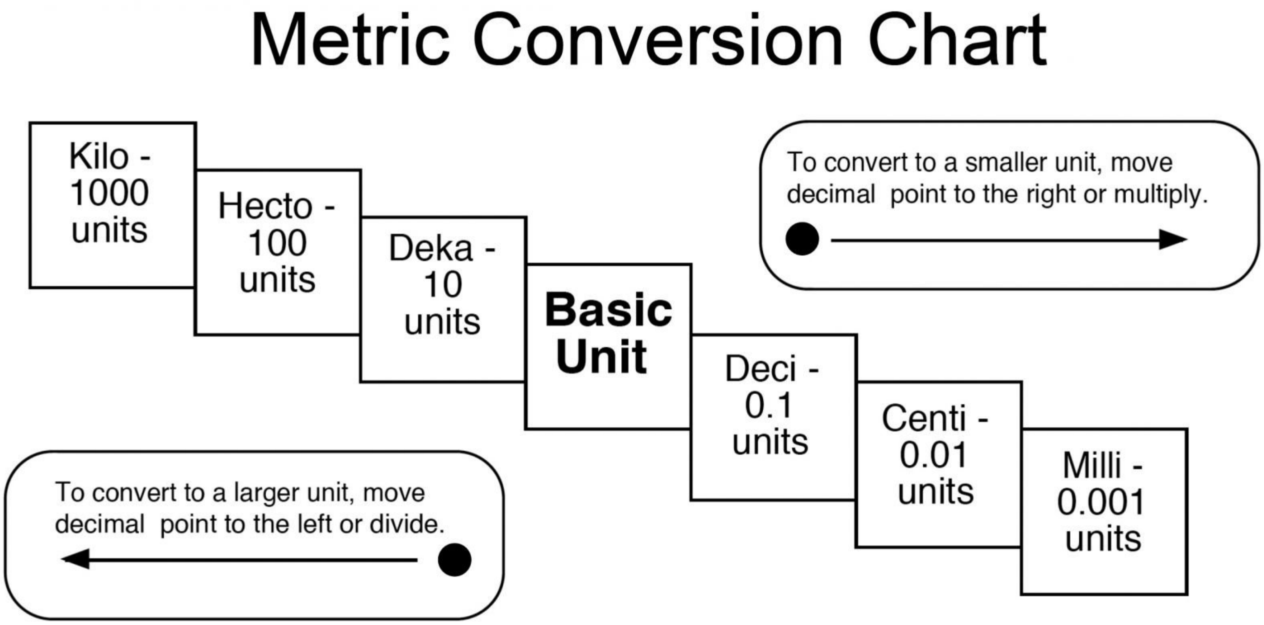
- Solve by factoring
- Solve by using the quadratic formula
- Solve by completing the square

Solutions

- Answer keys to each section's practice questions

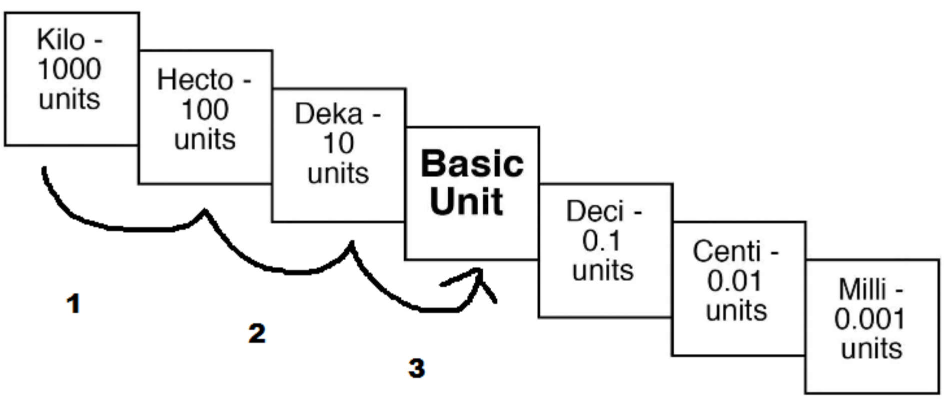
Section 1: Unit Conversion

You will be required to convert basic units. Even though the Regents does provide a reference sheet with conversions, *most* conversions require knowledge of the metric system which is *not* on the reference sheet. Fortunately, it just a matter of moving a decimal place. Please refer to this chart:

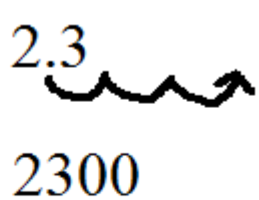


Example: 2.3 kilograms = ________ grams

Here *grams* is the basic unit. To move *from* kilograms *to* the basic unit, I count three boxes to the right:



So I move the decimal place three the right. Any EMPTY spaces will be filled in with ZEROES for place holders.



Practice

1) 30 cm = ___________ mm

2) 5.72 g = ___________ kg

3) 0.097 cm = ___________ km

4) 51 mg = ___________ grams

5) .48 cm = ___________ mm

6) 9038 mm = ___________ km

7) .2 km = ___________ cm

8) .34 mm = ___________ m

9) 346 m = ___________ km

10) .853 g = ___________ mg

Scan me for step-by-step solutions to ODD problems.

Section 2: Area and Perimeter

Definition of Perimeter: The total distance *around* the edge of a figure.
The perimeter of a circle is called the *circumference.*

Definition of Area: The total number of square units contained *within* a figure.

Perimeter is found simply by adding all the edges of a figure or using the circumference formula for a circle.
Area is found by multiplying two dimensions thus the unit of measure is always squared.
Here are the formulas provided by the Regents reference table:

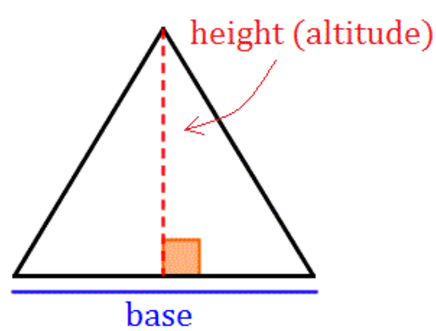
FORMULAS

Triangle	$A = \frac{1}{2}bh$
Parallelogram	$A = bh$
Circle	$A = \pi r^2$
Circle	$C = \pi d$ or $C = 2\pi r$

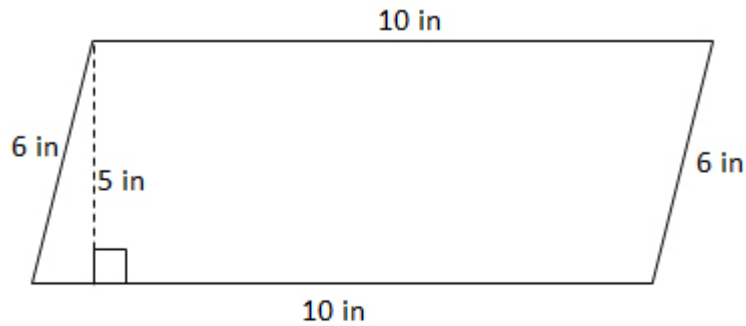
KEY: A = area, C = *circumference,* b = base, h = height, r = radius, d = diameter

NOTE 1: the parallelogram area formula is the same for RECTANGLES and SQUARES.
NOTE 2: If you don't have a pi symbol on your calculator use $\pi = 3.1415$

Remember, the HEIGHT and BASE are always perpendicular to each other (meaning there is a right angle between them!)



Example 1: Find the area and perimeter of the figure below.



To find perimeter add up all the edges:

P = 6 + 6 + 10 + 10 = 32 in

To find the area, first identify the base and height using the right angle:

b = 10 in and h = 5 in

Now identify the shape and apply the correct area formula:

$A=bh$, A = 10 x 5 = 50 in^2

Example 2: The area of a triangle is 44.5 cm^2 and its height is 7.8 cm. Find the length of the base of the triangle rounded to the nearest tenth.

Choose the correct area formula and plug in the given information:

$$A = \frac{1}{2}bh, \qquad 44.5 = \frac{1}{2}(b)(7.8)$$

Now solve for the unknown variable:

$$(2) \times 44.5 = (2) x \frac{1}{2}(b)(7.8)$$

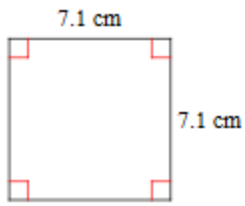
$$\frac{87}{7.8} = \frac{(b)(7.8)}{7.8}$$

$$11.4\,cm = b$$

Practice

Find the area and perimeter (or circumference) of each figure.

1)



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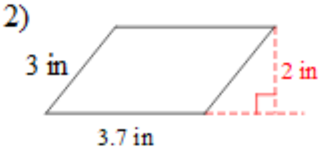

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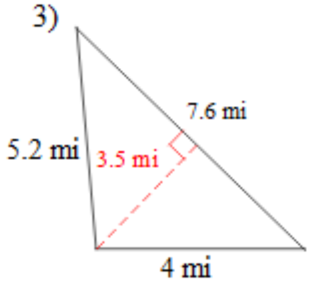

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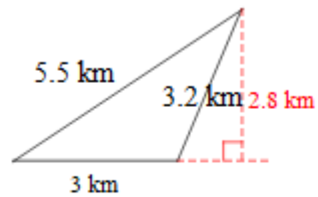

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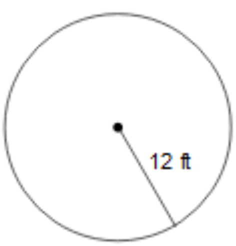

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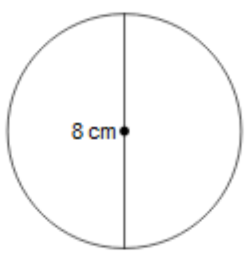
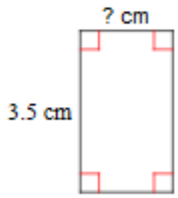


Scan me for step-by-step solutions to ODD problems.

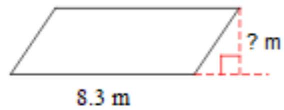
Find the missing side. Round answers to the nearest tenth.

7)



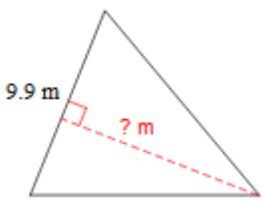
Area = 7 cm^2

8)



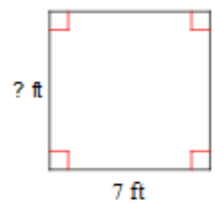
Area = 24.9 m^2

9)



Area = 51.5 m^2

10)



Area = 49 ft^2

11) Find the radius of a circle that has an area of 84.9 mi^2.

Scan me for step-by-step solutions to ODD problems.

Section 3: Volume

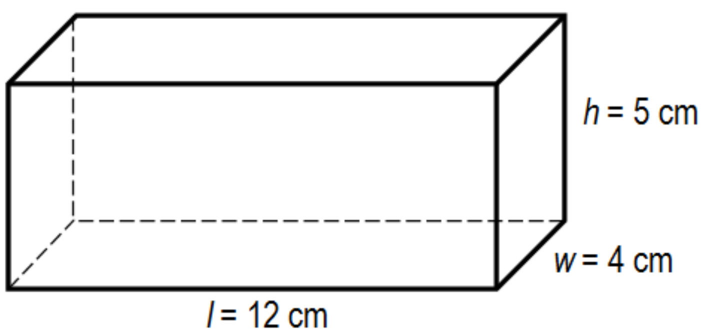
Volume is the amount of space inside a figure. It is found by multiplying three dimensions thus the unit of measure is always cubed. Provided below are the volume formulas from the Common Core reference sheet.

FORMULAS

General Prisms	$V = Bh$
Cylinder	$V = \pi r^2 h$
Sphere	$V = \frac{4}{3}\pi r^3$
Cone	$V = \frac{1}{3}\pi r^2 h$

KEY: V = volume, B = area of the base, r = radius, h = height

Example 1: Find the volume of the figure below.



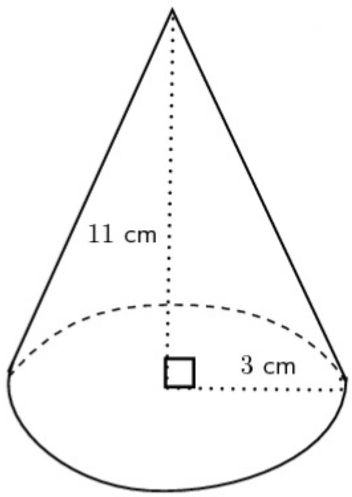
Identify the correct formula to use:

Volume of a prism= (area of the base) x (height) = (length x width) x (height)

Plug in the values and solve:

V = (12 cm x 4 cm) x (5cm) = 240 cm^3

Example 2: Find the volume of the figure below. Round your answer to the nearest hundredth.



Identify the correct formula to use:

Volume of a cone = $\frac{1}{3}\pi r^2 h$

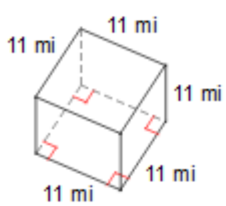
Plug in the values and solve:

V = (1/3) x (π) x $(3)^2$ x (11) = 103.69 cm^3

Practice

Find the volume of each figure. Round your answers to the nearest hundredth.

1)



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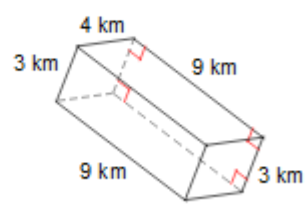

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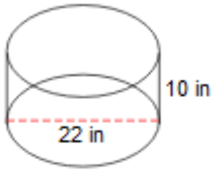

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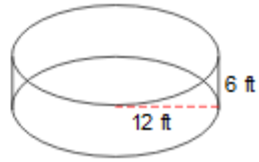

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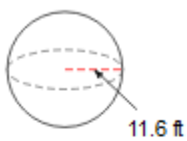

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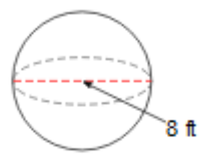

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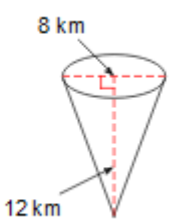

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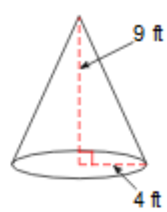


Section 4: The Slope Formula

The slope of a line is a measure of how steep it is. The formula is not provided on the reference sheet, therefore something you must memorize.

FORMULA

Given two points: (x_1, y_1) and (x_2, y_2)

$$Slope = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$$

Example 1: Find the slope of a line that passes through the points (6, 2) and (-4, 8)

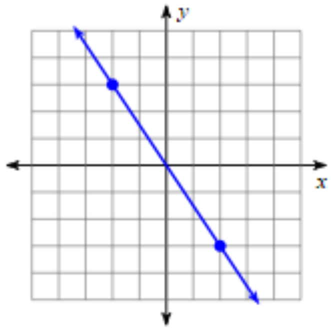
Plug the ordered pairs into the slope formula and then solve:

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - 2}{-4 - 6}$$

$$= \frac{6}{-10}$$

$$= -\frac{3}{5}$$

Example 2: Find the slope of the line in the given graph.



Begin at any point and count the boxes up or down to get to the next point for the *rise*.
Remember up is positive & down is negative!

Then count to the boxes going left or right to reach the next point for the *run*.
Remember right is positive & left is negative!

$\frac{rise}{run} = \frac{6}{-4}$ Now we reduce the fraction: $\frac{6}{-4} = -\frac{3}{2}$

Practice

Find the slope of the line. Simplify all answers.

1) $(-12, 6), (-16, 13)$

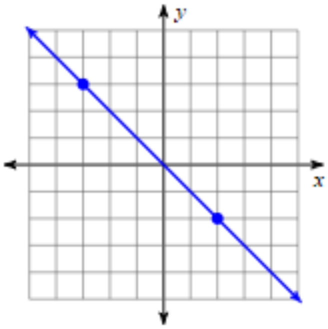
2) $(-8, -18), (-14, -13)$

3) $(9, -8), (6, 5)$

4) $(-1, -10), (2, 18)$

Scan me for step-by-step solutions to ODD problems.

5)



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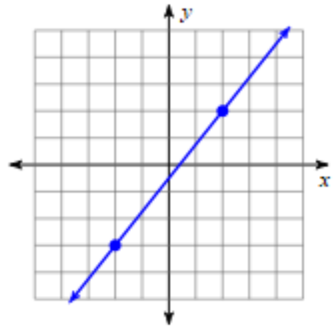

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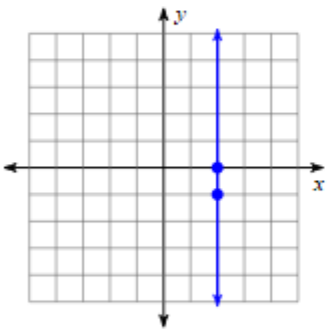

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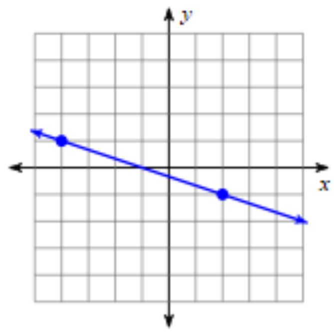
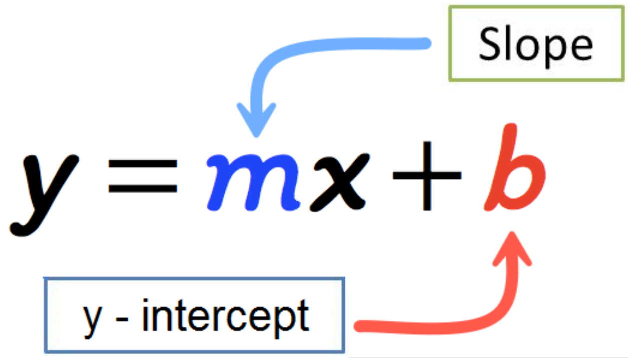


Section 5: The Equation of a Line

The equation of a line comes in many forms. Let's first review the most common form:

Slope-Intercept Form



Example 1: Write the equation of the line given a slope of $-\frac{2}{3}$ and a y-intercept of -6.
Plug the given slope into the *m* and the y-intercept into the *b* of the slope intercept form.

$$y = mx + b \quad \rightarrow \quad y = -\frac{2}{3}x - 6$$

Example 2: Write the equation of the line given a slope of $\frac{1}{2}$ and passes through the point (2, 3)
Plug the given slope into the *m* and the ordered pair into the *x* and *y* respectively.

$$y = mx + b \quad \rightarrow \quad 3 = \frac{1}{2}(2) + b$$

Solve for the y-intercept *b*.

$$3 = \frac{1}{\not{2}}(\not{2}) + b$$

$$3 = 1 + b$$

$$\underline{-1 \quad -1}$$

$$b = 2$$

Now plug *m* and *b* back into the slope-intercept form.

$$y = mx + b \quad \rightarrow \quad y = \frac{1}{2}x + 2$$

Example 3: Write the equation $-2y+8=7x$ in slope-intercept form.

Solve for y by using inverse operations.

$$-2y+8=7x$$
$$\underline{\quad -8 \quad -8}$$
$$\frac{-2y}{-2}=\frac{7x-8}{-2}$$
$$y=-\frac{7}{2}x+4$$

Practice

Write the slope-intercept form of the equation of each line given the slope and y-intercept.

1) Slope $=\frac{7}{5}$, y-intercept $=4$

2) Slope $=2$, y-intercept $=1$

3) Slope $=-1$, y-intercept $=2$

4) Slope $=\frac{1}{2}$, y-intercept $=-4$

Write the slope-intercept form of the equation of the line through the given point with the given slope.

5) through: $(4,-3)$, slope $=-\frac{5}{4}$

6) through: $(2,3)$, slope $=\frac{1}{2}$

7) through: $(2,-3)$, slope $=-\frac{1}{2}$

8) through: $(1,-5)$, slope $=-2$

Scan me for step-by-step solutions to ODD problems.

Write the slope-intercept form of the equation of each line.

9) $2 = -3x + y$

10) $7x = -4y - 16$

11) $20 = -5y + 4x$

12) $-15 + 3x - 5y = 0$

Scan me for step-by-step solutions to ODD problems.

Now let's look at another more versatile form that can be obtained from the slope formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

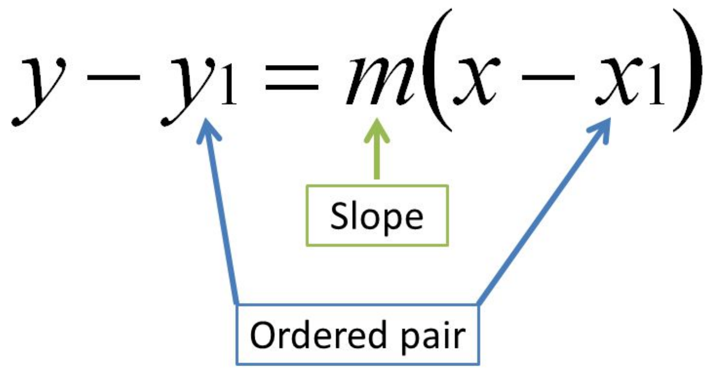
If we multiply both sides by $(x_2 - x_1)$ we get:

$$(x_2 - x_1) \cdot m = \frac{y_2 - y_1}{\cancel{x_2 - x_1}} \cdot \cancel{(x_2 - x_1)}$$

Which becomes:

Point-Slope Form

$$y - y_1 = m(x - x_1)$$



This makes **Example 2** much easier to put into an equation of a line.

Example 4: Find the equation of the line given a slope of -3 and a point (4,-10).

Plug the slope into *m* and the ordered pair into *x* and *y* respectively.

$$y - y_1 = m(x - x_1) \quad \rightarrow \quad y + 10 = -3(x - 4)$$

Example 5: Find the equation of the line given the points (0, 1) and (3, -3).

First find the slope using the slope formula

$$m = \frac{-3-1}{3-0} = \frac{-4}{3}$$

Now choose *one* of the two given points as the ordered pair and plug in along with the slope.

$$y - y_1 = m(x - x_1) \quad \rightarrow \quad y - 1 = -\frac{4}{3}(x - 0)$$

Practice

Write the point-slope form of the equation of the line through the given point with the given slope.

13) through: $(-3, 1)$, slope $= 0$

14) through: $(3, 3)$, slope $= \frac{8}{3}$

15) through: $(2, -5)$, slope $= -4$

16) through: $(3, 5)$, slope $= \frac{4}{3}$

Write the point-slope form of the equation of the line through the given points.

17) through: $(5, 0)$ and $(0, 1)$

18) through: $(-3, 5)$ and $(5, -3)$

19) through: $(2, -1)$ and $(0, 3)$

20) through: $(2, -4)$ and $(-5, 2)$

Scan me for step-by-step solutions to ODD problems.

Section 6: Radicals

A radical is a number under a root. Usually a square root and the focus of the review will only be with square roots. Example of a radical: $\sqrt{2}$. Here, 2 would be called the radicand.

A simplified radical is when the number under the radical is NOT divisible by a perfect square other than 1.

What are perfect squares again?

$1^2, 2^2, 3^2, 4^2, \dots$, etc. or more commonly written as 1, 4, 9, 16, 25, 36, …, etc. are all perfect squares.

Steps to simplifying a radicals.
Look at the radicand (number under the root) and ask yourself:

1. Is it a perfect square?
   a. If yes, take the square root of it. Done.
   b. If no, move to step 2.
2. Is the number divisible by a perfect square?
   a. If yes, find the factors using the largest perfect square and repeat step 1 with each factor.
   b. If no, then you are done. Radical is in simplest form.

Example 1: Simplify $\sqrt{25}$

Step 1: Yes, it 25 a perfect square. $\sqrt{25} = 5$

Example 2: Simplify $\sqrt{23}$

Step 1: No, 23 is not a perfect square.
Step 2: No, 23 not divisible by a perfect square.

$\sqrt{23}$ is already in simplest form.

Example 3: Simplify $\sqrt{80}$

Step 1: No, 80 is not a perfect square.	$\sqrt{80}$
Step 2: Yes, 80 is divisible by the perfect square 16. Factor:	$\sqrt{16 \cdot 5}$
Repeat Step 1 using "16" : Yes, 16 is a perfect square.	$4\sqrt{5}$
Repeat Step 1 using "5" : No, 5 is not a perfect square.	
Repeat Step 2 using "5" : No, 5 is not divisible by a perfect square.	Done.

Practice

Simplify.

1) $\sqrt{160}$

2) $\sqrt{70}$

3) $\sqrt{50}$

4) $\sqrt{24}$

5) $\sqrt{150}$

6) $\sqrt{256}$

7) $\sqrt{32}$

8) $\sqrt{210}$

9) $\sqrt{490}$

10) $\sqrt{729}$

Scan me for step-by-step solutions to ODD problems.

Steps to simplifying a multiplying radicals.

1. Multiply the coefficients. (numbers outside the radical)
   If there is no visible coefficient, then the coefficient is "1"
2. Multiply the radicals. (numbers inside the radical)
3. Simplify.

Example 4: Multiple $2\sqrt{8} \cdot 5\sqrt{6}$

Step 1: $10 \cdot \sqrt{8} \cdot \sqrt{6}$
Step 2: $10 \cdot \sqrt{48}$
Step 3: $10 \cdot \sqrt{16 \cdot 3}$
$10 \cdot 4\sqrt{3}$
$40\sqrt{3}$

Practice
Multiple. Simplify answers.

11) $\sqrt{6} \cdot -5\sqrt{2}$

12) $\sqrt{9} \cdot -4\sqrt{15}$

13) $4\sqrt{3} \cdot 3\sqrt{12}$

14) $2\sqrt{3} \cdot 3\sqrt{15}$

15) $3\sqrt{10} \cdot -\sqrt{10}$

16) $-3\sqrt{8} \cdot -\sqrt{10}$

Scan me for step-by-step solutions to ODD problems.

Steps to simplifying adding/subtracting radicals.

1. If the radicands are the same, you can add or subtract the coefficients.

   Example: $3\sqrt{5} + \sqrt{5} + \sqrt{3} = 4\sqrt{5}+\sqrt{3}$

2. If the radicands are different, simplify both radical to see you can add/subtract them.

   Example: $3\sqrt{20} + 6\sqrt{5} - \sqrt{15}$

   Simplify since none of the radicands are the same, simplify where possible.

$$3\sqrt{20} + 6\sqrt{5} - \sqrt{15}$$
$$3\sqrt{4 \cdot 5} + 6\sqrt{5} - \sqrt{15}$$
$$6\sqrt{5} + 6\sqrt{5} - \sqrt{15}$$
$$12\sqrt{5} - \sqrt{15}$$

Practice

Add or Subtract. Simplify answers.

17) $\sqrt{6} + \sqrt{6} + \sqrt{6}$

18) $\sqrt{5} + \sqrt{3} + \sqrt{3}$

19) $2\sqrt{5} + 3\sqrt{5} + 3\sqrt{6}$

20) $-2\sqrt{3} + 2\sqrt{3} - 3\sqrt{5}$

21) $\sqrt{8} + \sqrt{2}$

22) $\sqrt{6} + \sqrt{6}$

23) $3\sqrt{8} + 2\sqrt{8}$

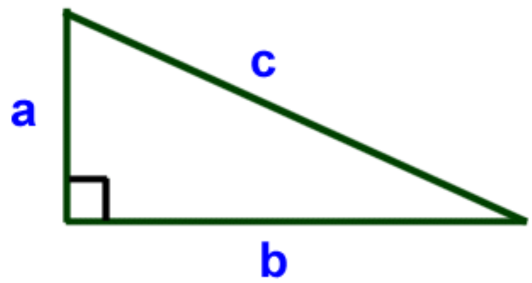
24) $-2\sqrt{24} - \sqrt{6}$

Scan me for step-by-step solutions to ODD problems.

Section 7: Pythagorean Theorem

The Pythagorean Theorem deals with find the lengths of the sides of a RIGHT triangle.

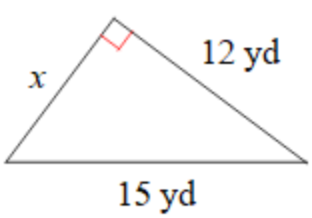
It is written as:



$$a^2 + b^2 = c^2$$

where *a* and *b* are the legs of the triangle
and *c* is always the hypotenuse.

Example: Find the missing side.

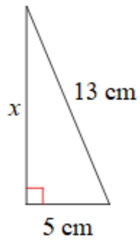


$$a^2 + b^2 = c^2$$
$$x^2 + 12^2 = 15^2$$
$$x^2 + 144 = 225$$
$$-144 \quad -144$$
$$x^2 = 81$$
$$\sqrt{x^2} = \sqrt{81}$$
$$x = 9$$

Practice

Find the missing side of each triangle.

1)



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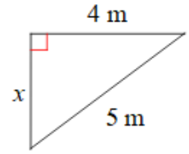

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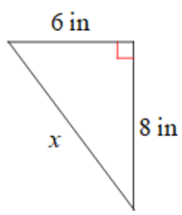
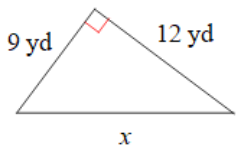
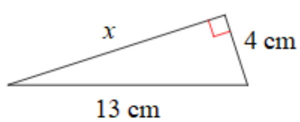


4)



Find the missing side of each triangle. Leave your answers in simplest radical form.

5)



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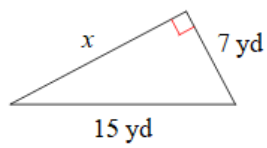
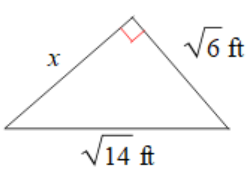
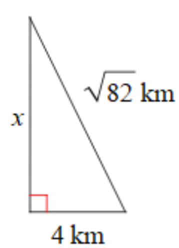


7)



8)



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Section 8: Factoring

Factoring is the process of "breaking" a number into numbers that can be multiplied back into the original number.

Example: 12 factors into $2 \cdot 2 \cdot 3$

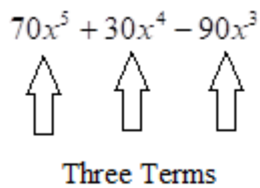
First we will review Factoring by GCF.

1. Examine each term and find the greatest common factor.
2. Take out the GCF outside a set of parenthesis
3. Divided each term by the GCF and write the answer inside the parenthesis.

Example 1

Factor $70x^5 + 30x^4 - 90x^3$

Step 1: Look at each term and figure out what factor is in common



The common factor is $10x^3$ because: 10 is the largest number that goes into 70, 30, and -90.
x^3 is the most number of *x's* common among $x^5, x^4, and\ x^3$.

Step 2: Write the GCF outside the parenthesis:

$$10x^3(\qquad\qquad\qquad)$$

Step 3: Divide each term by the GCF:

$$\frac{70x^5}{10x^3} + \frac{30x^4}{10x^3} - \frac{90x^3}{10x^3}$$

Write the answer inside the parenthesis:

Answer: $10x^3(7x^2 + 3x - 9)$

Practice

Factor.

1) $40x^5 + 70x^4 + 70x^3$

2) $18v^2 + 15v + 6$

3) $30 - 21x + 15x^7$

4) $-90n^4 + 27n + 54$

5) $70x^8 + 80x^5 - 50x^4$

6) $12x^3 + 15x - 3$

Scan me for step-by-step solutions to ODD problems.

Secondly, we will review factoring the difference of perfect square binomials:

1. Factor using GCF (see example 1) if possible, then:
2. Check that it has two terms and look for a subtraction operation.
   Check that both terms are perfect squares. If it is a variable, the power must be an even number.
   If step 2 is true, proceed to step 3. Otherwise stop.
3. Factor using the format:

$$a^2 - b^2 = (a - b)(a + b)$$

Example 2

Factor $4x^6 - 13$

Step 1: No GCF.

Step 2: Has two terms and subtraction operation.
4 is a perfect square, but 13 is not. Cannot be factored anymore.

Example 3
Factor $3x^6 - 12$

Step 1: Has a GCF of 3. $3(x^6 - 4)$

Step 2: x has an even power and 4 is a perfect square. Proceed to step 3.

Step 3: Take the square root of each term and put into the format:

$$x^6 - 4 = (x^3 - 2)(x^3 + 2)$$

Don't forget the GCF in front, so the answer is $3(x^3 - 2)(x^3 + 2)$

Practice.
Factor:

7) $49p^2 - 9$

8) $4v^2 - 81$

9) $64b^2 - 1$

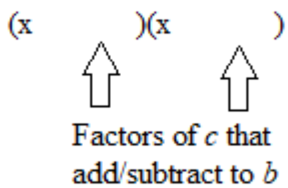
10) $27v^2 - 12$

Scan me for step-by-step solutions to ODD problems.

Finally, we will review factoring quadratic trinomials in the form:

$ax^2 + bx + c$, where $a=1$ *and* b & c are constants.

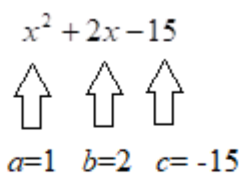
1. Examine the constant *c*. List all pairs of factors that multiple to *c*.
2. Find the pair that will add up to the constant *b*.
3. Write the factors into the format using negatives as subtraction and positives as addition:



Example 4

Factor $x^2 + 2x - 15$

Step 1: Here



So c = -15. The factors that multiple to -15 are:

$-1 \cdot 15$

$-3 \cdot 5$

$-15 \cdot 1$

$-5 \cdot 3$

Step 2: Since b=2, the pair that add up to 2 is -3 *and* 5

Step 3: Answer: $(x-3)(x+5)$

Practice

Factor.

11) $p^2 + 11p + 18$

12) $x^2 - 9x + 14$

13) $n^2 - 6n - 27$

14) $k^2 - 4k - 21$

15) $v^2 - 12v + 20$

16) $m^2 + 2m - 80$

Scan me for step-by-step solutions to ODD problems.

Section 9: Solving Quadratic Equations

What we did in the last section was factor quadratic expressions. We will now solve quadratic equations which will be very similar to what we just did with a few additional steps.

Solving a quadratic trinomial:

1. Write the quadratic in the form: $ax^2 + bx + c = 0$
2. Examine the constant *c*. List all pairs of factors that multiple to *c*.
3. Find the pair that will add up to the constant *b*.
4. Write the factors into the format using negatives as subtraction and positives as addition:

(x   )(x   ) =0

⇧   ⇧

Factors of *c* that
add/subtract to *b*

Set each factor = 0. Solve for x.

Example 1

Solve $4x^2 - 10x - 4 = -7x + 3x^2$.

Step 1: Move all the terms to one side so that the equation is equal to zero.

$$\begin{array}{r} 4x^2 - 10x - 4 = -7x + 3x^2 \\ +3x^2 + 7x \qquad +7x - 3x^2 \\ \hline x^2 - 3x - 4 = 0 \end{array}$$

Step 2: Here $x^2 - 3x - 4 = 0$

⇧ ⇧ ⇧

a=1 *b*= -3 *c*= -4

So, *c* = -4 . The factors that multiple to -4 are:

$-1 \cdot 4$

$-2 \cdot 2$

$-4 \cdot 1$

Step 3: Since *b*= -3, the pair that add up to -3 is $-4 \text{ and } 1$

Step 4: Write as $(x-4)(x+1) = 0$

Step 5: Set each factor = 0 and solve:

$$x - 4 = 0, \quad x + 1 = 0$$
$$+4 \; +4 \qquad -1 \; -1$$

Answers: $x = 4, \quad x = -1$

Practice

Solve.

1) $n^2 - 6n - 7 = 0$

2) $r^2 - 9r + 8 = 0$

3) $p^2 = 5p - 6$

4) $r^2 - 14r = -48$

5) $4r^2 - 10r - 4 = -7r + 3r^2$

6) $v^2 - 4v - 23 = -7v + 5$

Scan me for step-by-step solutions to ODD problems.

Now if this process doesn't work and you cannot factor the quadratic trinomial, then we must use what is known as the quadratic formula to solve for *x*.

Here is the Quadratic formula: $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Where a,b,c are constants in the form: $ax^2 + bx + c = 0$

NOTE: It is very important you rewrite the equation into the form $ax^2 + bx + c = 0$ before determining you're a,*b* and *c*.

Example 2

Solve $3x^2 - 1 = 10x$.

Rewrite $\begin{array}{r} 3x^2 - 1 = 10x \\ \underline{-10x \quad -10x} \end{array}$ as $3x^2 - 10x - 1 = 0$.

Now we can determine: $3x^2 - 10x - 1 = 0$

$\Uparrow \quad \Uparrow \quad \Uparrow$

$a=3 \quad b= -10 \quad c= -1$

Use these values to plug into the quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{10 \pm \sqrt{(-10)^2 - 4(3)(-1)}}{2(3)}$$

$$x = \frac{10 \pm \sqrt{112}}{6}$$

Leave in simplest radical form: $x = \frac{10 \pm 4\sqrt{7}}{6}$

(review Section 6 if you forgot)

Factor the numerator and reduce if possible: $x = \frac{\cancel{2}(5 \pm 2\sqrt{7})}{\cancel{6}} \quad = x = \frac{5 \pm 2\sqrt{7}}{3}$

Practice

Solve using the quadratic formula.

7) $x^2 - 4x - 96 = 0$

8) $6k^2 - k - 117 = 0$

9) $n^2 + 12n + 16 = -11$

10) $5n^2 - n - 51 = -3$

11) $4n^2 - 81 - 3n = -3n$

12) $8x^2 - 2x = 7x^2 + 10$

Scan me for step-by-step solutions to ODD problems.

Finally we are to review how to solve quadratic equations by using a process called

Completing the Square:

Procedure

To solve a quadratic equation of the form $ax^2 + bx - c = 0$ where $a = 1$ by completing the square:

1. Isolate the terms in x on one side of the equation.
2. Add the square of one-half the coefficient of x or $\left(\frac{1}{2}b\right)^2$ to both sides of the equation.
3. Write the square root of both sides of the resulting equation and solve for x.

Note: The procedure outlined above only works when $a = 1$. If the coefficient a is not equal to 1, divide each term by a, and then proceed with completing the square using the procedure outlined above.

Example 3

Solve $x^2 + 10x + 21 = 0$ by completing the square.

Step 1: Isolate all the x – terms on one side.

$$\begin{array}{r} x^2 + 10x + 21 = 0 \\ -21 \quad -21 \\ \hline x^2 + 10x = -21 \end{array}$$

Step 2: The b constant is 10. So we calculate $\left(\frac{b}{2}\right)^2 = \left(\frac{10}{2}\right)^2 = 25$. Now we add 25 to both sides.

$$x^2 + 10x + 25 = -21 + 25$$
$$x^2 + 10x + 25 = 4$$

Step 3: Factor and rewrite the binomial as a square

$$(x+5)(x+5)=4$$
$$(x+5)^2=4$$

Write the square root on both sides.

$$\sqrt{(x+5)^2}=\sqrt{4}$$
$$x+5=\pm 2$$

IMPORTANT: Don't forget to write the plus and minus symbol ($\pm$) after taking the square root of the constant.

Now solve for x.

$$x+5=\pm 2$$
$$-5 \quad -5$$
$$x=-2-5 \quad and \quad x=+2-5$$
$$x=-7 \quad and \quad x=-3$$

Practice

Solve by completing the square.

13) $x^2 - 20x + 19 = 0$

14) $p^2 - 18p + 45 = 0$

15) $x^2 + 16x - 57 = 0$

16) $k^2 + 4k + 3 = 0$

17) $-5x^2 - 92 = 8x - 6x^2 - 8$

18) $r^2 - 18r = 88$

Scan me for step-by-step solutions to ODD problems.

Solutions

Section 1 Practice

1) 300 2) .00572 3) .00000097 4) .051 5) 4.8
6) .009038 7) 20,000 8) .00034 9) .346 10) 853

Section 2 Practice

1) P=28.4 cm, A=50.41cm^2
2) P=13.4m, A=11.1m^2
3) P=16.8 mi, A=13.3mi^2
4) P=11.7 km, A=4.2km^2
5) C=75.4 ft, A=452.4 ft^2
6) C=25.1 cm, A=50.3 cm^2
7) 2 cm 8) 3 m 9) 10.4 m 10) 7 ft 11) 5.2 mi

Section 3 Practice

1) 1331 mi^3
2) 108 km^3
3) 3801.33 in^3
4) 2714.34 ft^3
5) 6538.27 ft^3
6) 268.08 ft^3
7) 201.06 km^3
8) 150.8 ft^3

Section 4 Practice

1) $-\frac{7}{4}$
2) $-\frac{5}{6}$
3) $-\frac{13}{3}$
4) $\frac{28}{3}$
5) -1
6) $\frac{5}{4}$
7) Undefined
8) $-\frac{1}{3}$

Section 5 Practice

1) $y=\frac{7}{5}x+4$
2) $y=2x+1$
3) $y=-x+2$
4) $y=\frac{1}{2}x-4$
5) $y=-\frac{5}{4}x+2$
6) $y=\frac{1}{2}x+2$
7) $y=-\frac{1}{2}x-2$
8) $y=-2x-3$
9) $y=3x+2$
10) $y=-\frac{7}{4}x-4$
11) $y=\frac{4}{5}x-4$
12) $y=\frac{3}{5}x-3$
13) $y-1=0$
14) $y-3=\frac{8}{3}(x-3)$
15) $y+5=-4(x-2)$
16) $y-5=\frac{4}{3}(x-3)$
17) $y=-\frac{1}{5}(x-5)$
18) $y-5=-(x+3)$
19) $y+1=-2(x-2)$
20) $y+4=-\frac{6}{7}(x-2)$

Section 6 Practice

1) $4\sqrt{10}$
2) $\sqrt{70}$
3) $5\sqrt{2}$
4) $2\sqrt{6}$
5) $5\sqrt{6}$
6) 16
7) $4\sqrt{2}$
8) $\sqrt{210}$
9) $7\sqrt{10}$
10) 27
11) $-10\sqrt{3}$
12) $-12\sqrt{15}$
13) 72
14) $18\sqrt{5}$
15) -30
16) $12\sqrt{5}$
17) $3\sqrt{6}$
18) $\sqrt{5}+2\sqrt{3}$
19) $5\sqrt{5}+3\sqrt{6}$
20) $-3\sqrt{5}$
21) $3\sqrt{2}$
22) $2\sqrt{6}$
23) $10\sqrt{2}$
24) $-5\sqrt{6}$

Section 7 Practice

1) 12 cm
2) 3 m
3) 10 in
4) 15 yd
5) $3\sqrt{17}$ cm
6) $4\sqrt{11}$ yd
7) $2\sqrt{2}$ ft
8) $\sqrt{66}$ km

Section 8 Practice

1) $10x^3(4x^2+7x+7)$
2) $3(6v^2+5v+2)$
3) $3(10-7x+5x^7)$
4) $9(-10n^4+3n+6)$
5) $10x^4(7x^4+8x-5)$
6) $3(4x^3+5x-1)$
7) $(7p+3)(7p-3)$
8) $(2v+9)(2v-9)$
9) $(8b+1)(8b-1)$
10) $3(3v+2)(3v-2)$
11) $(p+9)(p+2)$
12) $(x-2)(x-7)$
13) $(n+3)(n-9)$
14) $(k+3)(k-7)$
15) $(v-2)(v-10)$
16) $(m+10)(m-8)$

Section 9 Practice

1) $\{7, -1\}$
2) $\{8, 1\}$
3) $\{3, 2\}$
4) $\{8, 6\}$
5) $\{-1, 4\}$
6) $\{4, -7\}$
7) $\{12, -8\}$
8) $\left\{\frac{9}{2}, -\frac{13}{3}\right\}$
9) $\{-3, -9\}$
10) $\left\{\frac{16}{5}, -3\right\}$
11) $\left\{\frac{9}{2}, -\frac{9}{2}\right\}$
12) $\{1+\sqrt{11}, 1-\sqrt{11}\}$
13) $\{19, 1\}$
14) $\{15, 3\}$
15) $\{3, -19\}$
16) $\{-1, -3\}$
17) $\{14, -6\}$
18) $\{22, -4\}$