# ALGEBRA 2 COMMON CORE SUMMER SKILLS PACKET

***Our Purpose:*** *Completion of this packet over the summer before beginning Algebra 2 will be of great value to helping students successfully meet the academic challenges awaiting them in Algebra 2 and beyond.*

***To Parents/Guardians:*** *Teachers and administrators of White Plains High School actively encourage parents/guardians to engage in their child's learning. This Summer Review for students entering Algebra 2 has been developed to provide our students with a summer resource and practice to help refresh major topics from previous years.*
*By encouraging your child to work on this packet, you will help them enter Algebra 2 prepared with the required skills to reach their full potential. The packet has been designed to provide a review of Algebra 1 skills that are essential for student success in Algebra 2.*

*The White Plains High School Mathematics Department highly recommends that students work through this packet during the summer and bring it, with work shown and with any questions, to school on the first day of Algebra 2.*

***Helpful websites/links/solutions:***

- www.purplemath.com
- www.algebra.com
- https://www.mathplanet.com/
- Every problem with an * has a video solution. Just scan the QR code on that page.
- Solutions to all problems are on pages 51 – 57.

WPHS MATH DEPARTMENT

# ALGEBRA II – SUMMER PREP

## BASIC SKILLS AND TECHNIQUES NEEDED FOR A SUCCESSFUL YEAR

TOPIC A



## FUNCTIONS

- Identify the domain and range of relations and functions
- Determine whether relations are functions
- Notation and evaluation

TOPIC B



## ALGEBRAIC EXPRESSIONS

- Simplifying Expressions
- Combining Like Terms
- Distributive Property to combine like terms
- Multiplying Polynomials

TOPIC C



## SOLVING LINEAR SYSTEMS

- What is a Linear System?
- Solving Graphically
- Solving Algebraically

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# ALGEBRA II - SUMMER PREP

BASIC SKILLS AND TECHNIQUES NEEDED FOR A SUCCESSFUL YEAR

## TOPIC D — EXPONENTS

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- Adding/Subtracting
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## TOPIC E — RADICALS

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- Quadratic Equation

# TOPIC A

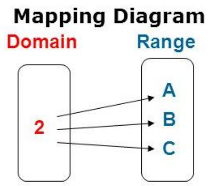
## FUNCTIONS

- Identify the domain and range of relations and functions
- Determine whether relations are functions
- Notation and evaluation

## Notes:

A **relation** is a pairing of input values with output values. It can be shown as a set of ordered pairs $(x,y)$, where $x$ is an input and $y$ is an output.

The set of input values for a relation is called the **domain**, and the set of output values is called the **range**.



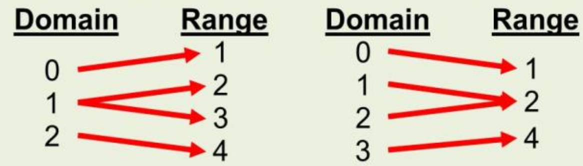

Set of Ordered Pairs: {(2, A), (2, B), (2, C)}

$(x, y)$ ⟶ (input, output) ⟶ (domain, range)

### Is this a function?

For a relation to be a function, **one input (x)** must have **exactly one output (y)**.

**For example, is this a function? Explain.**




This is **NOT** a function; the input of 1 has two different outputs.

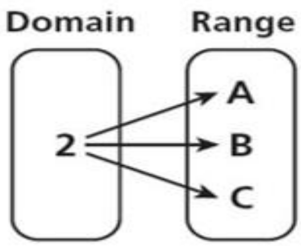
This is a function; all inputs have exactly one output.

| x | y |
|---|---|
| 0 | 2 |
| 3 | 4 |
| -3 | -2 |
| 2 | 4 |

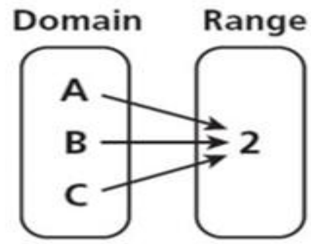
Find the Domain and Range

{(0,2),(3,4),(-3,-2),(2,4)}

Domain: {0,3,-3,2}  Range: {2,4,-2}



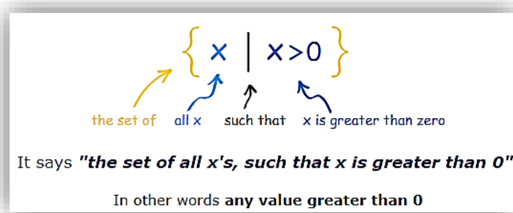

**Not a function:** The relationship from number to letter is *not* a function because the domain value 2 is mapped to the range values A, B, and C.



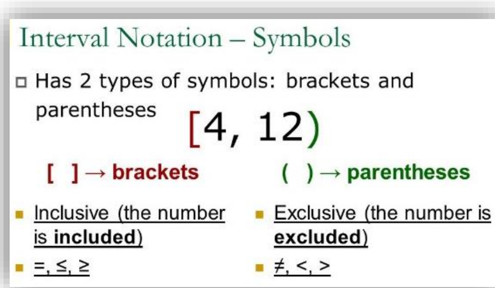

**Function:** The relationship from letter to number is a function because each letter in the domain is mapped to only one number in the range.

## Stating Domain and Range:

- We can use **set-builder notation**: $\{x|x\geq 4\}$, which translates to "all real numbers $x$ such that $x$ is greater than or equal to 4." Notice that **braces** are used to indicate a set.




- Another method, and probably our most commonly used, is **interval notation.** With **Interval Notation** solution sets are indicated with **parentheses or brackets**. The solutions to $x\geq 4$ are represented as $[4,\infty)$. This is perhaps the most useful method.



**<u>Practice:</u>**

1.) What is the range of the relation $y = 2x^2 + 5x$, if the domain is the set {-2, 0, 1, 2}?

2.) A function is defined by the equation $y = -3x - 4$. If the domain is $1 \leq x \leq 5$, what is the minimum value in the range of the function?

3.) The domain of $y = 3x + 2$ is $\{-1 \leq x \leq 4\}$. Which integer is **not** in the range?

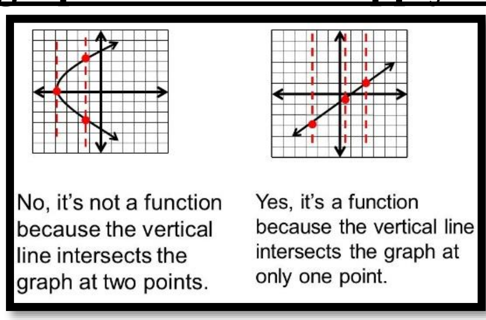
a) 0
b) -5
c) 11
d) -1

4.) Use interval notation to indicate all real numbers greater than or equal to $-2$.

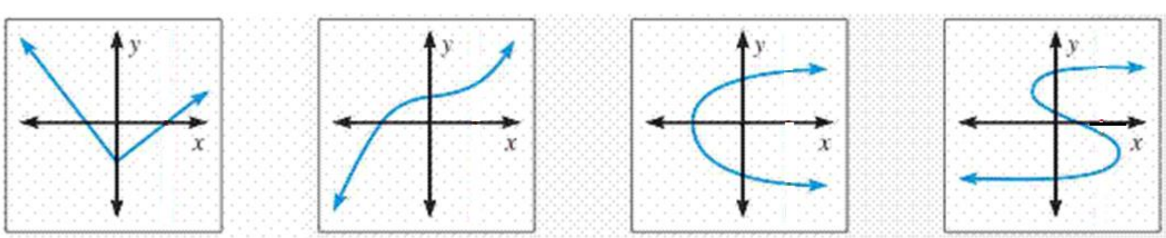
5.) Write the interval expressing all real numbers less than or equal to -1.

6.) Use interval notation to indicate all real numbers between $-3$ and $5$.

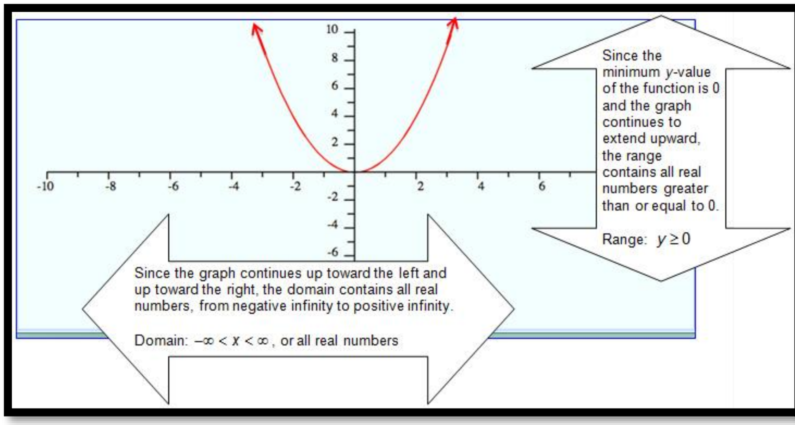
## <u>To determine if a graph is a function apply the vertical line test:</u>

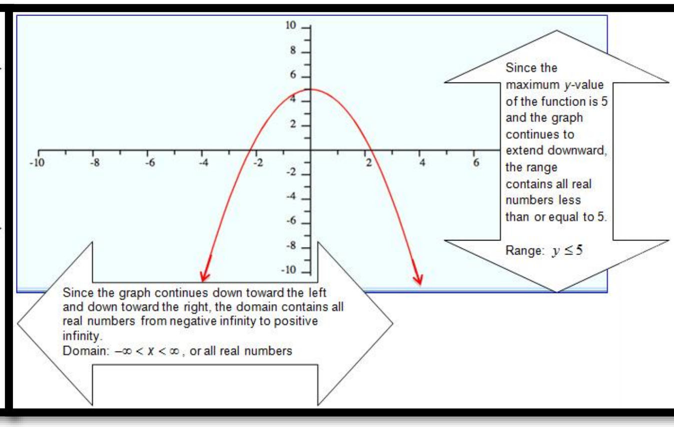



7.) Apply the vertical line test to determine which graph(s) below are function(s).

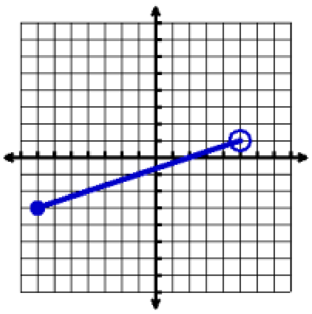


# Determine domain and Range of a function given the graph:



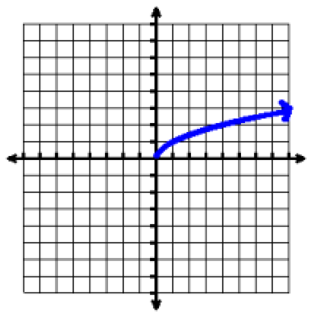




8.) State the domain and range of each graph below using Interval Notation:



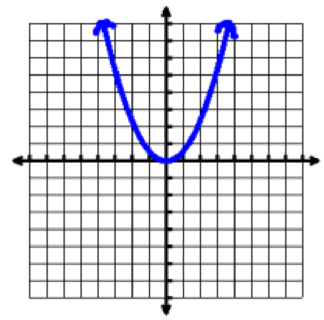
Domain: ____________

Range: _______________



Domain: ____________

Range: _______________

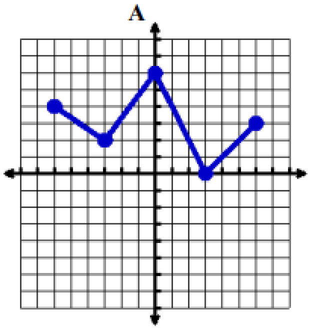


Domain: ____________

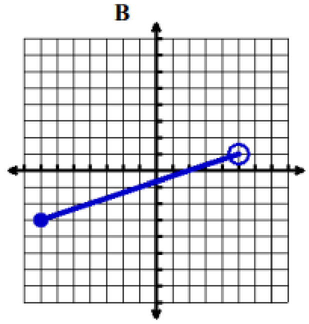
Range: _______________

9.) For each letter below state the domain and range of the graph:

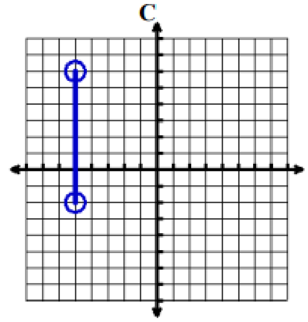
A



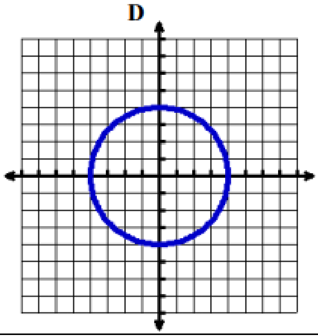
B



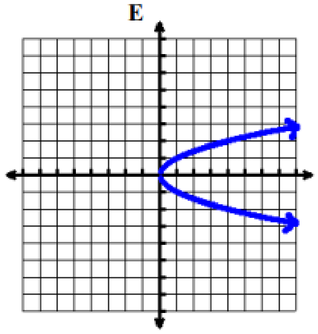
C



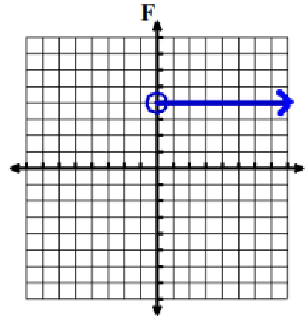
D



E

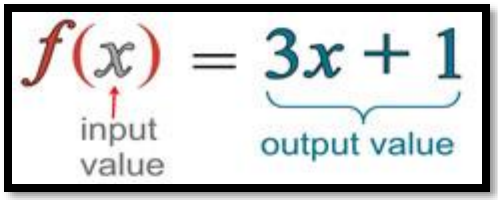


F



**Notes:**

Function notation is the way a function is written. Traditionally, functions are referred to by single letter names, such as *f*, *g*, *h* and so on.  Any letter(s), however, may be used to name a function.




**This is read as "f of x"**

## Function Notation:

- Identifies the independent variable in a problem.
    - $f(x) = x + 2b + c$, where the variable is "*x*".
- States which element of the function is to be examined.
- Find $f(2)$ when $f(x) = 3x$, is the same as saying, "Find $y$ when $x = 2$, for $y = 3x$."

## Evaluating a Function:

To evaluate a function, substitute the input (the given number or expression) for the function's variable (place holder, *x*). Replace the *x* with the number or expression.

**Numeric Example:** Given the function $f(x) = 2x - 5$, find $f(4)$

***Solution:*** Substitute 4 into the function in place of *x*.   $f(4) = 2(4) - 5 = 3$.

Therefore we can say the given function evaluated at x = 4 will yield a value of 3.

f(4) = 3

**Algebraic Example:** Find $g(2w)$ when $g(x) = x^2 - 2x + 1$.

***Solution:*** When substituting expressions, like 2*w*, into a function, using parentheses.

$$g(2w) = (2w)^2 - 2(2w) + 1$$

$$g(2w) = 4w^2 - 4w + 1$$

(The answer will appear in terms of *w*.)

**PRACTICE:**

**1.** Let $g(x) = -5x + 2$. Evaluate each of the following:

**(a)** $g(-1)$ **(b)** $g(-2)$ **(c)** $g(0)$ **(d)** $g(5)$

**2.** Let $f(x) = 2x + 2$. Evaluate each of the following:

**(a)** $f(-3)$ **(b)** $f(6)$ **(c)** $f(-1)$ **(d)** $f(4)$

**3.** Let $g(x) = x^2 + 4x - 1$. Evaluate each of the following:

**(a)** $g(-4)$ **(b)** $g(8)$ **(c)** $g(-1)$ **(d)** $g(1)$

**4.** Let $f(x) = 3x^2 - 5x$. Evaluate each of the following:

**(a)** $f(2)$ **(b)** $f(-8)$ **(c)** $f(7)$ **(d)** $f(-1)$

**6.** Suppose $n(x) = 7x + 4$. Determine $x$ such that:

**(a)** $n(x) = 39$ **(b)** $n(x) = 0$ **(c)** $n(x) = 4$ **(d)** $n(x) = \frac{1}{3}$

**7.** Suppose $q(x) = -5x + 6$. Determine $x$ such that:

**(a)** $q(x) = 21$ **(b)** $q(x) = 0$ **(c)** $q(x) = -6$ **(d)** $q(x) = \frac{1}{4}$

**8.** Suppose $g(x) = -3x + 8$. Determine $x$ such that:

**(a)** $g(x) = 14$ **(b)** $g(x) = 0$ **(c)** $g(x) = -14$ **(d)** $g(x) = \frac{1}{5}$

**9.** Let $h(x) = x^2 + 2x - 5$. Evaluate each of the following:

**(a)** h(2m) **(b)** h(w) **(c)** h(4x) **(d)** $h(n^2)$

**TOPIC B**

**ALGEBRAIC EXPRESSIONS**

- Simplifying Expressions
- Combining Like Terms
- Distributive Property to combine like terms
- Multiplying Polynomials

# What Are Like Terms?

First, what are terms? **Terms** are connected by addition or subtraction signs. For example: The expression a + b contains two terms and the expression ab is one term. The expression ab is one term because it is connected with an understood multiplication sign. The expression 3a + b + 2c has three terms. The expression 2ab + 4c has two terms.

Second, what are like terms and why are they important? **Like terms** have the same variable(s) with the same exponent, such as 4x and 9x. More examples of like terms are:

- ***4ab and 7ab***
- ***$2x^2$ and $6x^2$***
- ***$4ab^2$ and $6ab^2$***
- ***5 and 11***

You can add and subtract like terms. When you add and subtract like terms, you are simplifying an algebraic expression. How do you add like terms? Simply add the numbers in front of the variables (coefficients) and keep the variables the same. (*In the expression 5x + 6y, 5 and 6 are the coefficients).*

**Example:** $2x + 3x + 7x$
Add the numbers in front of the variables. $2 + 3 + 7$
Don't change the variable. $12x$

**Example:** $4xy + 3xy$
Add the numbers in front of the variables. $4 + 3$
Don't change the variables. $7xy$

**Example:** $2x^2y - 5x^2y$
Subtract the numbers in front of the variables. $2 - 5$
Don't change the variables. $-3x^2y$

**Example:** $4x + 2y + 9 + 6x + 2$
(Hint: You can only add the like terms, $4x$ and $6x$, and the numbers 9 and 2.)
Add the like terms. $4x + 2y + 9 + 6x + 2$
$= 10x + 2y + 11$

## Practice

**Simplify the expressions by combining like terms.**

_______ **1.** $2x + 4x + 11x$

_______ **2.** $6x + 3y + 4y + 7x$

_______ **3.** $6x + 5 + 9x$

_______ **4.** $12x - 3x + 9x$

_______ **5.** $5 - 4x + 2x^2 + 7 - x$

_______ **6.** $9x - 2y - 5y + 2 - 4x$

_______ **7.** $12x + 12y + 5$

_______ **8.** $5x^2 + 7x + 5 + 3x^2 - 2x + 4$

_______ **9.** $-12x + 12x + 8$

_______ **10.** $3xy + 5x - 2y + 4yx + 11x - 8$

# Use Distributive Property to Combine Like Terms

How do you simplify an expression like 2(x+y) +3(x+2y)? You know you can't add x to y because they are not like terms. Here you will need to use the **distributive property**. The distributive property tells you to multiply the number and/or variable(s) outside the parentheses by every term inside the parentheses. **For example**:

Multiply 2 by $x$ and 2 by $y$. Then multiply 3 by $x$ and 3 by $2y$. If there is no number in front of the variable, it is understood to be 1, so 2 times $x$ means 2 times $1x$. To multiply, you multiply the numbers and the variable stays the same. When you multiply 3 by $2y$, you multiply 3 by 2 and the variable, $y$, stays the same, so you would get $6y$. After you have multiplied, you can then combine like terms.

**Example:**

Multiply $2(x + y)$ and $3(x + 2y)$. — $2(x + y) + 3(x + 2y)$

Combine like terms. — $2x + 2y + 3x + 6y$

$= 5x + 8y$

**Tip**

If there is no number (coefficient) in front of a variable, it is understood to be 1.

Here are two more examples using the distributive property.

**Example:** $2(x + y) + 3(x - y)$

Multiply 2 by $x$ and 2 by $y$. Then multiply 3 by $x$ and 3 by $(-y)$. When you multiply 3 by $(-y)$, this is the same as $3(-1y)$. The 1 is understood to be in front of the $y$ even though you don't see it. In this example, you can see how the parentheses are used to indicate multiplication.

Use the distributive property. — $2(x + y) + 3(x - y)$

Combine like terms. — $2x + 2y + 3x - 3y$

$= 5x - y$

**Example:** $2(2x + y) - 3(x + 2y)$

Use the distributive property to get rid of the parentheses. The subtraction sign in front of the 3 is the same as multiplying $(-3)(x)$ and $(-3)(2y)$.

Use the distributive property. — $2(2x + y) - 3(x + 2y)$

Combine like terms. — $4x + 2y - 3x - 6y$

$= x - 4y$

## Practice

Use the distributive property to simplify the expressions.

_______ **11.** $5y - 4(x - y)$

_______ **12.** $3(r + s) - 6(r + s)$

_______ **16.** $3(m + n) + 4(2m - 3) + 2(5 - n)$

_______ **17.** $3(x - y) + 5(2x + 3y)$

______ *13. $2s + 7(2s - 2r)$

______ 14. $-8(x + {}^{-}2y) + 4(x - 2y)$

______ *15. $5(a + b) + {}^{-}2(a + b) - (a - b)$

______ 18. $5 - 3(x + 7) + 8x$

______ *19. $-5(a + 3b) - 2(4a - 4b)$

______ 20. $12 + 3(x + 2y - 4) - 7(-2x - y)$

**Video Solutions:**

#13.)

#15.)

#19.)

# Multiplying Polynomials

## What Is a Polynomial?

A **polynomial** is a sum of numbers and positive integer powers of a single variable. For instance, 2, $x + 3$, and $5x^2 + 3$ are polynomials. In this lesson, you will multiply polynomials that have more than one term.

## Multiplying Expressions by a Monomial

A polynomial comprised of a single term is called a **monomial**. To multiply a polynomial with one term (monomial) by a polynomial consisting of more than one term, use the distributive property. You multiply the term outside the parentheses by every term inside the parentheses.

**Examples:** $2(a + b - 3) = 2a + 2b - 6$

$3x(x^2 + 2x) = 3x^3 + 6x^2$

### Practice

______ 1. $5(x - y + 2)$

______ 2. $7x(x - 3)$

______ 3. $8x^3(3x^2 + 2x - 5)$

______ 4. $-6(x - y - 7)$

______ 5. $3b(x^2 + 2xy + y)$

______ 6. $-7c^2(2a - 5ac)$

______ 7. $(x^2y - x)6y$

______ 8. $2a^2x(3x - 2ab + 10ax^3 + 8)$

______ 9. $-8xy^2(2x^3 - 3x^2y)$

______ 10. $4rs(-2rt + 7r^2s - 9s^2t^2)$

# Multiplying a Binomial by a Binomial

What is a binomial? A **binomial** is a polynomial consisting of two terms. To multiply a binomial by a binomial, you will use a method called "FOIL." This process is called FOIL because you work the problem in this order:

## **F**irst, **O**uter, **I**nner, **L**ast

**Example:** $(x+2)(x+3)$

| | |
|---|---|
| Multiply the **first** terms in each binomial. | $([x]+2)([x]+3)$<br>$=x^2$ |
| Multiply the two **outer** terms in each binomial. | $([x]+2)(x+[3])$<br>$=x^2+3x$ |
| Multiply the two **inner** terms in each binomial. | $(x+[2])([x]+3)$<br>$=x^2+3x+2x$ |
| Multiply the two **last** terms in each binomial. | $(x+[2])(x+[3])$<br>$=x^2+3x+2x+6$ |
| Simplify. | $=x^2+5x+6$ |

**Example:** $(x+3)(x-1)$

| | |
|---|---|
| Multiply the two **first** terms in each binomial. | $([x]+3)([x]-1)$<br>$=x^2$ |
| Multiply the two **outer** terms in each binomial. | $([x]+3)(x-[1])$<br>$=x^2-1x$ |
| Multiply the two **inner** terms in each binomial. | $(x+[3])([x]-1)$<br>$=x^2-1x+3x$ |
| Multiply the two **last** terms in each binomial. | $(x+[3])(x-[1])$<br>$=x^2-1x+3x-3$ |
| Simplify. | $=x^2+2x-3$ |

### Practice

Multiply these binomials.

_____*11. $(x+3)(x+6)$

_____12. $(x-4)(x-9)$

_____*13. $(2x+1)(3x-7)$

_____14. $(x+2)(x-3y)$

_____*15. $(5x+7)(5x-7)$

_____16. $(x+2)(x+4)$

_____17. $(x+6)(x-3)$

_____18. $(x-6)(x-2)$

_____19. $(x-1)(x+10)$

_____*25. $(a-b)(a+b)$

_____26. $(7x+2y)(2x-4y)$

_____*27. $(3x+2)(4x-3y)$

_____28. $(2x+y)(x-y)$

_____*29. $(5x-2)(6x-1)$

_____30. $(a+b)(c+d)$

_____31. $(7y-2)(7y+2)$

_____32. $(4x-1)(4x-1)$

_____33. $(7a+3)(2a-2)$

**Video Solutions:**
**#11,13,15**

**#25,27,29**

# Multiplying a Binomial by a Trinomial

**Example:** $(x+2)(x^2+2x+1)$

To work this problem, you need to multiply each term in the first polynomial with each term in the second polynomial.

Multiply $x$ by each term in the second polynomial. $x(x^2+2x+1)$
$= x^3+2x^2+x$

Multiply 2 by each term in the second polynomial. $2(x^2+2x+1)$
$= 2x^2+4x+2$

Simplify. $x^3+2x^2+x+2x^2+4x+2$
$= x^3+4x^2+5x+2$

**Example:** $(x+1)(x+1)(x+1)$

To work this problem, you need to multiply the first two factors. You will then multiply the result by the third factor.

Multiply the first two factors. $(x+1)(x+1)$
$= x^2+x+x+1$

Simplify. $= x^2+2x+1$

Then multiply the product by the third factor. $(x^2+2x+1)(x+1)$

Multiply. $x^2(x+1)$
$= x^3+x^2$

Multiply. $2x(x+1)$
$= 2x^2+2x$

Multiply. $1(x+1)$
$= x+1$

Simplify. $= x^3+x^2+2x^2+2x+x+1$
$= x^3+3x^2+3x+1$

## Practice

______**38.** $(x-2)(x^2-2x+1)$

______**39.** $(2x+3)(x^2+2x+5)$

______**40.** $(x+3)(x+2)(x-2)$

______**41.** $(3x+2)(x-1)(x+3)$

______**42.** $(x+y)(4x^2+3xy-1)$

______**43.** $(2a+b)(a+2b+8)$

______**44.** $(3x+2)(2x+3y-1)$

______***45.** $(5x+4)(2x^2+x+3)$

______**46.** $(2x-2)(3x^2+2x+3)$

______**47.** $(2x^2+y^2)(x^2-y^2)$

______**48.** $(x+2)(3x^2-5x+2)$

______***49.** $(2x-3)(x^3+3x^2-4x)$

______**50.** $(4a+b)(5a^2+2ab-b^2)$

______**51.** $(3y-7)(6y^2-3y+7)$

# Solving Linear Systems

## Graphically:

When you graphed linear equations previously, you found their graphs were always a straight line. If the graph of an equation is a straight line, the equation is a **linear equation**. There are other ways to determine if an equation is linear without graphing the equation. Equations of this type can be put into the form Ax + By = C, where A and B are not both equal to zero. This is called the standard form of a linear equation. In standard form, a linear equation can have no exponent greater than 1, cannot have any variables in the denominator if the equation contains both variables, and cannot have the product of variables in the equation.

The form we are comfortable with and mostly used for graphing is **y = mx + b**, where ***m is the slope*** of the line and ***b is the y-intercept*** (where the graph crosses the y-axis)

Here are examples of equations that *are* linear:

$2x + y = 11$

$\frac{2}{3}x + 3y = 12$

$5x - 2y = 16$

$x = 18$

Here are examples of equations that *are not* linear:

| | |
|---|---|
| $x^2 + y = 5$ | Equation contains a variable with a power greater than 1. |
| $\frac{2}{x} + y = 7$ | Equation contains the variable in the denominator. |
| $xy = 6$ | Equation contains the product of two variables. |

### Practice

Determine if the following equations are linear equations.

_______ **1.** $5x + y = 13$

_______ **2.** $6x - 3y = 8$

_______ **3.** $y = 12$

_______ **4.** $x + y^2 = 11$

_______ **5.** $3x + 2xy - 4y = 5$

_______ **6.** $\frac{3}{4}x + 2y = 5$

_______ **7.** $\frac{3}{x} + y = 17$

_______ **8.** $y^2 + 5x + 6 = 0$

_______ **9.** $x + y^3 = 11$

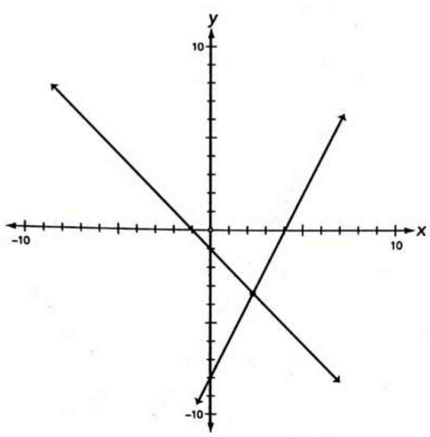
_______ **10.** $3x + 2y + 5 = x - y + 7$

# What Is a System of Linear Equations?

A **system** consists of two or more equations with the same variables. If you have two different variables, you need at least two equations. There are several methods of solving systems of linear equations. In this lesson, you will solve systems of equations graphically. When you graphed linear equations in Lesson 8, the graph, which was a straight line, was a picture of the answers; you had an infinite number of solutions. However, a system of linear equations has more than one equation, so its graph will consist of more than one line. You'll know that you've solved a system of linear equations when you determine the point(s) of intersection of the lines. Since two distinct lines can intersect in only one point, that means such a system of linear equations has one solution. What if the lines don't intersect? When the lines do not intersect, the system has no solutions.

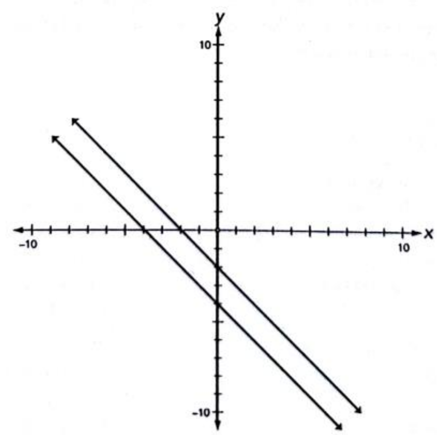
Two distinct lines can intersect in only one point, or they do not intersect at all. However there is a third possibility: The lines could coincide, which means they are the same line. If the lines coincide, there are an infinite number of solutions, since every point on the line is a point of intersection. **<u>The three cases mentioned are shown below:</u>**

**<u>Case 1:</u>** The lines intersect in one point.
Here we have **<u>ONE SOLUTION</u>**.



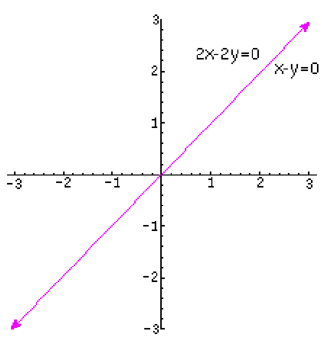

**Slopes different, intersect at 1 point**

**<u>Case 2:</u>** The lines do not intersect.
Here we have **<u>NO SOLUTION</u>**.




**Slopes same, different y-intercepts**

**<u>Case 3:</u>** The lines coincide.
Here we have an infinite number of solutions.




**Slopes same, same y-intercepts**

You can determine the nature of the solutions of a system without actually graphing them. Once you have placed the equation into slope intercept form (y=mx+b), you can compare the slopes and y-intercepts of each equation. Use the graphs on the previous page to determine which case you have.

**Case 1:** The slopes and $y$-intercepts are the same, so the lines coincide. There will be an infinite number of solutions.

$y = 2x + 3$
$y = 2x + 3$

**Case 2:** The slopes are the same, but the $y$-intercepts are different. The lines will be parallel, so there is no solution.

$y = 2x + 4$
$y = 2x - 5$

**Case 3:** The slopes are different. The lines will intersect, so there will be one solution.

$y = 3x + 2$
$y = 2x + 3$

## Practice

Without graphing the system, determine the number of solutions the system will have.

______**21.** $y = 3x - 5$
$y = 3x + 2$

______**22.** $x + 3y = 10$
$2x + 6y = 20$

______**23.** $3y - 2x = 6$
$2y - 3x = 4$

______**24.** $2x + 3y = 6$
$3x - y = 2$

______**25.** $y + 3 = 3x + 5$
$3y = 9x$

______**26.** $3x + y = 6$
$3x - y = 6$

______**27.** $4x - 3y = 12$
$-4x + 3y = -12$

______**28.** $3x + 3y = 15$
$-2x - 2y = 8$

# Solving Linear Systems

**Algebraically**: We will use two methods to solve systems algebraically, ELIMINATION and SUBSTITUTION.

### How to Use the Elimination Method

Graphs serve many useful purposes, but using algebra to solve a system of equations can be faster and more accurate than graphing a system. A system of equations consists of equations often involving more than one variable. When you use the elimination method of solving systems of equations, the strategy is to eliminate all the variables except one. When you have only one variable left in the equation, then you can solve it.

**Example**

Solve the system: $x + y = 10$

$x - y = 4$

Add the equations. $(x + y) + (x - y) = 10 + 4$

$2x - 0y = 14$

Drop the $0y$. $2x = 14$

Divide both sides of the equation by 2. $\frac{2x}{2} = \frac{14}{2}$

Simplify both sides of the equation. $x = 7$

You have solved for the variable $x$. To solve for the variable $y$, substitute the value of the $x$ variable into one of the original equations. It does not matter which equation you use. $x + y = 10$

Substitute 7 in place of the $x$ variable. $7 + y = 10$

Subtract 7 from both sides of the equation. $7 - 7 + y = 10 - 7$

Simplify both sides of the equation. $y = 3$

You solve a system of equations by finding the value of all the variables. In the previous example, you found that $x = 7$ and $y = 3$. Write your answer as the ordered pair (7,3). To check your solution, substitute the values for $x$ and $y$ into *both* equations.

**Example**

Solve the system: $x + y = 6$

$-x + y = -4$

Add the two equations. $0x + 2y = 2$

Drop the $0x$. $2y = 2$

Divide both sides of the equation by 2. $\frac{2y}{2} = \frac{2}{2}$

Simplify both sides of the equation. $y = 1$

Use one of the original equations to solve for $x$. $x + y = 6$

Substitute 1 in place of $y$. $x + 1 = 6$

Subtract 1 from both sides of the equation. $x + 1 - 1 = 6 - 1$

Simplify both sides of the equation. $x = 5$

Write the solution of the system as an ordered pair: (5,1).

**Check:** $x + y = 6$

Substitute the values of $x$ and $y$ into the first equation. $5 + 1 = 6$

Simplify. $6 = 6$

$-x + y = -4$

Substitute the values of $x$ and $y$ into the second equation. $-(5) + 1 = -4$

Simplify. $-4 = -4$

Did you get the right answer? Yes! You got true statements when you substituted the value of the variables into both equations, so you can be confident that you solved the system of equations correctly.

## Practice

Solve the systems of equations using the elimination method.

_____ **1.** $3x + 4y = 8$
$-3x + y = 12$

_____ **2.** $4x - 2y = -5$
$-x + 2y = 2$

_____ **3.** $-5x + 2y = 10$
$5x - 3y = -10$

_____ **4.** $4x + 2y = 12$
$-4x + y = 3$

_____ **5.** $2x - 3y = 9$
$x + 3y = 0$

_____ **6.** $2x - y = 2$
$-2x + 4y = -5$

_____ **7.** $x + y = 4$
$2x - y = -1$

_____ **8.** $3x + 4y = 17$
$-x + 2y = 1$

_____ **9.** $7x + 3y = 11$
$2x + y = 3$

_____ **10.** $0.5x + 5y = 28$
$3x - y = 13$

_____ **11.** $3(x + y) = 18$
$5x + y = -2$

_____ **12.** $\frac{1}{2}x + 2y = 11$
$2x - y = 17$

_____ **13.** $5x + 8y = 25$
$3x - 15 = y$

_____ **14.** $6y + 3x = 30$
$2y + 6x = 0$

_____ **15.** $3x = 5 - 7y$
$2y = x - 6$

_____ **16.** $3x + y = 20$
$\frac{x}{3} + 10 = y$

_____ **17.** $2x + 7y = 45$
$3x + 4y = 22$

_____ **18.** $3x - 5y = -21$
$2(2y - x) = 16$

## When You Can't Easily Eliminate a Variable

Sometimes, you can't easily eliminate a variable by simply adding the equations. Take a look at the following example. What should you do?

**Example**

Solve the system: $x + y = 24$

$2x + y = 3$

If you were to add this system of equations the way it is, you would be unable to eliminate a variable. However, if one of the $y$ variables were negative, you would be able to eliminate the $y$ variable. You can change the equation to eliminate the $y$ variable by multiplying one of the equations by $-1$. You can use either equation. Save time by choosing the equation that looks easier to manipulate.

| | |
|---|---|
| Multiply both sides of the first equation by $-1$. | $-1(x + y) = -1 \cdot 24$ |
| Simplify both sides of the equation. | $-x - y = -24$ |
| Add the second equation to the modified first equation. | $2x + y = 3$ |
| | $x = -21$ |
| | $x + y = 24$ |
| Substitute the value of $x$ into one of the original equations. | $-21 + y = 24$ |
| Add 21 to both sides of the equation. | $-21 + 21 + y = 24 + 21$ |
| Simplify both sides of the equation. | $y = 45$ |

The solution to the system of equations is $(-21,45)$.

**Example**

Solve the system: $2x + y = 4$

$3x + 2y = 6$

How will you eliminate a variable in this system? If you multiply the first equation by $-2$, you can eliminate the $y$ variable.

| | |
|---|---|
| | $-2(2x + y) = -2 \cdot 4$ |
| | $-4x - 2y = -8$ |
| Add the second equation to the modified first equation. | $3x + 2y = 6$ |
| | $-x = -2$ |
| Multiply both sides of the equation by $-1$. | $-1 \cdot -x = -1 \cdot -2$ |
| | $x = 2$ |
| Substitute 2 in place of the $x$ variable in one of the original equations. | |
| | $2x + y = 4$ |
| | $2 \cdot 2 + y = 4$ |
| Simplify. | $4 + y = 4$ |
| Subtract 4 from both sides of the equation. | $4 - 4 + y = 4 - 4$ |
| Simplify. | $y = 0$ |

The solution of the system is $(2,0)$.

## Practice

Solve the systems of equations using the elimination method.

_______ **19.** $2x + 4y = 10$
$-3x + 3y = -6$

_______ **20.** $-4x - 5y = -12$
$8x - 3y = -28$

_______ **21.** $7x - 7y = -7$
$21x - 21y = -21$

_______ **22.** $5x - 5y = 5$
$25x - 25y = 25$

_______ **23.** $8x - 4y = 16$
$4x + 5y = 22$

_______ **24.** $5x - 3y = 31$
$2x + 5y = 0$

_______ **25.** $3x - 4y = 6$
$-5x + 6y = -8$

_______ **26.** $6x - 2y = -6$
$-5x + 5y = 5$

_______ **27.** $-5x + 2y = 10$
$3x + 6y = 66$

_______ **28.** $2x + 2 = 3y - x - 4$
$5x + 2y + 1 = 5$

_______ **29.** $x = 3y + 6$
$3x = 6y - 15$

_______ **30.** $3x + 2y = x - 6$
$3(x + 2y) = 3$

### How to Use the Substitution Method

Some systems of equations are easier to solve using the substitution method instead of the elimination method. Study the following examples.

**Example**

Solve the system: $y = 2x$
$2x + y = 12$

The first equation says that $y = 2x$. If you substitute $2x$ in place of $y$ in the second equation, you can eliminate a variable. $2x + 2x = 12$

Combine similar terms. $4x = 12$

Divide both sides of the equation by 4. $\frac{4x}{4} = \frac{12}{4}$

Simplify both sides of the equation. $x = 3$

Substitute 3 in place of $x$ in one of the original equations. $y = 2x$

$y = 2 \cdot 3$

$y = 6$

The solution of the system is (3,6).

## Practice

Solve the systems of equations using the substitution method.

______**31.** $x = 2y$
$4x + y = 18$

______**32.** $y = 2x$
$3x + 6y = 30$

______**33.** $y = 4x - 1$
$6x - 2y = 28$

______**34.** $y = 2x + 1$
$3x + 2y = 9$

______**35.** $x = 2y + 1$
$3x - y = 13$

______**38.** $x + y = 3$
$3x + 101 = 7y$

______**39.** $5x + y = 3.6$
$y + 21x = 8.4$

______**40.** $8x - y = 0$
$10x + y = 27$

______**41.** $y + 3x = 0$
$y - 3x = 24$

______**42.** $2x + y = 2 - 5y$
$x - y = 5$

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## Mixed Practice

Solve the systems of equations using any method.

______*__45.__ $x + 2y = 10$
$2x - 2y = -4$

______*__46.__ $y = -6x$
$2x - y = 16$

______*__47.__ $4x + 2y = 18$
$3x - y = 1$

______**48.** $x = 5y - 2$
$2x - y = 14$

______**49.** $3x + 7y = 4$
$6x + 2y = -4$

______**50.** $2x - 5y = 10$
$3x + 2y = -4$

**Video Solutions:**

**#45,46**

**#47**

## Applications

Use a system of equations to solve the following word problems. First, work through the example problem given.

**Example:** Your book club is having a pizza party. The pepperoni pizza is \$8, and the combination pizza is \$12. You need 9 pizzas, and you have \$80 to spend. How many of each kind can you get?

Let $x$ = number of pepperoni pizzas.

Let $y$ = number of combination pizzas.

| | |
|---|---|
| | $x + y = 9$ |
| | $8x + 12y = 80$ |
| Multiply the first equation by –8. | $-8(x + y) = -8 \cdot 9$ |
| Simplify. | $-8x - 8y = -72$ |
| Add the second equation to the altered first equation. | $\underline{8x + 12y = 80}$ |
| | $4y = 8$ |
| Divide both sides of the equation by 4. | $\frac{4y}{4} = \frac{8}{4}$ |
| Simplify both sides of the equation. | $y = 2$ |
| Substitute 2 in place of $y$ in one of the original equations. | $x + y = 9$ |
| | $x + 2 = 9$ |
| Subtract 2 from both sides of the equation. | $x + 2 - 2 = 9 - 2$ |
| Simplify. | $x = 7$ |

You can get 7 pepperoni pizzas and 2 combination pizzas.

**EXAMPLES:**

**51.** The admission prices for the last baseball game of the season were \$3 for students and \$4 for adults. One hundred tickets were sold, and the gate receipts were \$340. How many of each kind of ticket were sold?

**52.** Jolynn drove 370 miles in 6 hours. She drove 70 miles per hour on the highway and 45 miles per hour on the back roads. How many hours did she spend on the backroads?

**TOPIC D** **EXPONENTS**

- Adding/Subtracting
- Multiplying
- Dividing
- Power to Power

# Working with Exponents

*You will add, subtract, multiply and divide expressions with exponents. Along with raising expressions to a power.*

# What is an Exponent?

An exponent tells you how many times a factor is multiplied. An exponent appears as a raised number of the number or variable to its immediate left. For example, in the expression $4^3$, the three is the exponent. The expression $4^3$ shows that four is a factor three times. That means four times four times four. Here are some examples:

$$5^2 = 5 \cdot 5$$
$$2^3 = 2 \cdot 2 \cdot 2$$
$$a^2 = a \cdot a$$
$$2x^3y^2 = 2 \cdot x \cdot x \cdot x \cdot y \cdot y$$

## Remember:

When we combine like terms, we added the numbers in front of the variables (coefficients) and left the variables the same. As follows:

$$3x + 4x = 7x$$
$$2x^2 + 7x^2 = 9x^2$$
$$3xy + 6xy = 9xy$$
$$5x^3 - 3x^3 = 2x^3$$

What do you do with exponents when you are adding? NOTHING!! You only add the coefficients. The variables and their exponents stay the same.

# Multiplying with Exponents

The rules for multiplying expressions with exponents may appear to be confusing. You treat exponents differently from ordinary numbers. You would think that when you are multiplying, you would multiply the exponents. However, that's not true. When you are multiplying expressions, you add the exponents. Here's an example of how to simplify an expression.

**Example:** $x^2 \cdot x^3 =$

$(x \cdot x)(x \cdot x \cdot x) =$

$x^5$

You can see that you have 5 $x$'s, which is written as $x^5$. To get $x^5$ for an answer, you *add* the exponents instead of multiplying them.

**Example:** $a^3 \cdot a^4 =$

$(a \cdot a \cdot a)(a \cdot a \cdot a \cdot a) =$

$a^7$

The factors of $a^3$ are $a \cdot a \cdot a$. The factors of $a^4$ are $a \cdot a \cdot a \cdot a$. The factored form of $a^3 \cdot a^4$ is $a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a$. When you write the problem out in factor form, you can see that you have 7 $a$'s. The easy way to get 7 is to add the exponents.

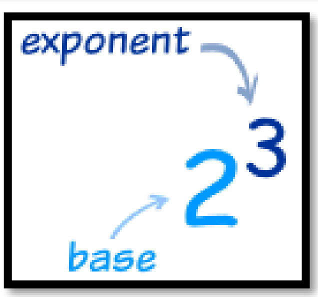
What would you do if you see an expression like $x^{20}$, and you want to multiply it by $x^{25}$? You can see that writing out the factors of $x^{20} \cdot x^{25}$ would take a long time. Think about how easy it would be to make a mistake if you wrote out all the factors. It is much more efficient and fast to use the rule for multiplying exponents:

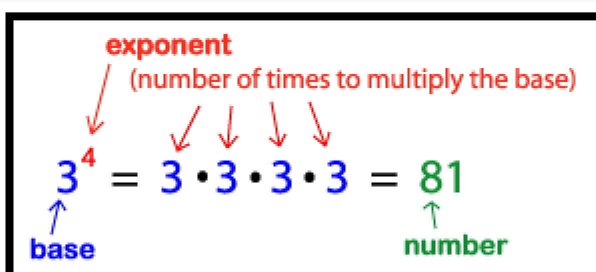
***When you are multiplying such terms (same base), you add the exponents.***

**What Is a Base?**

In the expression $x^4$, the $x$ is the **base**, and the 4 is the exponent. You can multiply variables only when the base is the same. You can multiply $a^2 \cdot a^4$ to get $a^6$. However, you cannot add the exponents in $a^3 \cdot b^4$ because the bases are different.

To multiply $2x^2 \cdot 3x^4$, multiply the numbers, keep the variable the same, and add the exponents. Your answer would be $6x^6$. When you multiply $5a^3 \cdot 2a^4$, you get $10a^7$.



**Remember:**

*If there is no exponent, the exponent is understood to be 1.*

***For example:*** **$x \cdot x^2 = x^1 \cdot x^2 = x^3$**

**Practice**

**Simplify the expressions using the rules for adding and multiplying with exponents.**

_____ **1.** $11x + 17x$

_____ **2.** $3x + 4x^2$

_____ **3.** $a^7 \cdot a^3$

_____ **4.** $x^2y + 5x^2y$

_____ **5.** $3x \cdot 4x^2$

_____ **6.** $ab^2 \cdot a^2b^3$

_____ **7.** $2m^5n^2 \cdot 6mn^9$

_____ **8.** $16a^4b^2c + 5abc - 4a^4b^2c$

_____ **9.** $7xy + 7xy^2 - 7x^2y$

_____ **10.** $3x^2y + 4x \cdot 5xy$

# Dividing with Exponents

Now you know that when you multiply expressions with exponents, you add the exponents. ***However when you divide, you SUBTRACT the exponents.*** Take a look at some examples:

**Examples:**

$$\frac{x^5}{x^2} = \frac{x \cdot x \cdot x \cdot x \cdot x}{x \cdot x} = x^{5-2} = x^3$$

$$\frac{x^2}{x^5} = \frac{x \cdot x}{x \cdot x \cdot x \cdot x \cdot x} = \frac{1}{x^{5-2}} = \frac{1}{x^3}$$

$$\frac{a^3b^2}{ab^5} = \frac{a \cdot a \cdot a \cdot b \cdot b}{a \cdot b \cdot b \cdot b \cdot b \cdot b} = \frac{a^{3-1}}{b^{5-2}} = \frac{a^2}{b^3}$$

**Tip:** When subtracting exponents, always subtract the larger exponent from the smaller exponent.

- If the larger exponent is in the numerator, the variable and exponent will be in the numerator. (First example above)
- If the larger exponent is in the denominator, the variable and the exponent will be in the denominator. (Second example above)

Writing out the factors when you work problems takes too long. So, you can use the rule for exponents when you divide. When you divide expressions that contain exponents like $\frac{4x^5}{2x^3}$, divide the coefficients (4 and 2) and subtract the exponents (5 and 3). The answer for the expression is $\frac{4x^5}{2x^3} = 2x^2$.

**<u>Remember:</u>**

*A quantity raised to the zero power is 1.*

***For example:*** $= \frac{x^3}{x^3} = x^0$

*However, a number divided by itself is always 1. Therefore* $x^0 = 1$.

## Practice

Simplify the expressions.

_______ **11.** $\frac{y^8}{y^3}$

_______ **12.** $\frac{a^3}{a^6}$

_______ **13.** $\frac{-b^7}{b^3}$

_______ **14.** $\frac{a^4b}{a^2}$

_______ **15.** $\frac{10xy^2}{2xy}$

_______ **16.** $\frac{2ab^3}{18a^3b^7}$

_______ **17.** $\frac{16xy^5z^4}{4y^2z^6}$

_______ **18.** $\frac{25r^5s^3}{5r^6s}$

_______ **19.** $\frac{8x^3y^7}{10xy^{10}}$

_______ **20.** $\frac{3m^7n^4}{9m^4n^4}$

# <u>What to Do with Exponents</u>
# <u>When You Raise a Quantity to a Power</u>

How do you simplify the expression $(x^3)^2$? Remember that an exponent tell you how many times a quantity is a factor.

**Examples:**

$$(x^3)^2 = (x \cdot x \cdot x)(x \cdot x \cdot x) = x^6$$
$$(a^2b^3)^2 = (a \cdot a \cdot b \cdot b \cdot b)(a \cdot a \cdot b \cdot b \cdot b) = a^4b^6$$
$$(3a^3)^2 = (3 \cdot a \cdot a \cdot a)(3 \cdot a \cdot a \cdot a) = 3^2a^6 = 9a^6$$

*From the previous examples, you can see that if you multiply the exponents, you will get the correct answer.*

***If you raise a quantity to a power, you multiply the exponents.***

## Practice

Simplify the expressions.

______**21.** $(x^5)^2$

______**22.** $(c^4)^5$

______**23.** $(a^2b^3)^2$

______**24.** $(xy^5)^3$

______**25.** $(m^6n^2)^3$

______**26.** $(2x^3)^3$

______**27.** $[(r^5s^2)^2]^2$

______**28.** $[(xy^3)^3]^2$

______**29.** $[(2abc^4)^3]^3$

______**30.** $(3x^2y)^3 + (5xy^2)^3$

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## Mixed Practice

Simplify the expressions.

______ * **31.** $xyz(x^6yz^2)$

______ * **32.** $3ab(6a + 5b)$

______ * **33.** $\frac{18x^4y^7}{9xy}$

______**34.** $\frac{64rst}{16rst}$

______ * **35.** $(x^2y^3)^3 \cdot 2x^3$

______ * **36.** $2x^2 \cdot 3y^5 + 4xy^3 \cdot 2xy^2$

______**37.** $(3a^4b^5)^3$

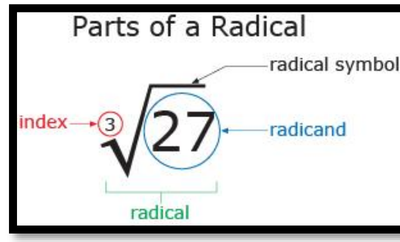
______ * **38.** $(2xy^5)^3 - 8xy^8(x^2y^7)$

______**39.** $\frac{12x^3y^2}{2xy} + 5xy^2$

______**40.** $\frac{3a^2b^3 \cdot 2a^4b^7}{8a^5b^{12}}$

**TOPIC E** — **RADICALS**

- Simplifying Radicals
- Solving Radical Equations
- Solving Quadratic Equations that have a Radical Answer




# RADICALS

*Here we will review the definition of a radical and show you how to simplify them. We will also review all operations with radicals (add, subtract, multiply, divide)*

## What Is a Radical?

You have seen how the addition in $x + 5 = 11$ can be undone by subtracting 5 from both sides of the equation. You have also seen how the multiplication in $3x = 21$ can be undone by dividing both sides by 3. Taking the **square root** (also called a **radical**) is the way to undo the exponent from an equation like $x^2 = 25$.

The exponent in $7^2$ tells you to square 7. You multiply $7 \cdot 7$ and get $7^2 = 49$.

The **radical sign** $\sqrt{\ }$ in $\sqrt{36}$ tells you to find the positive number whose square is 36. In other words, $\sqrt{36}$ asks: What number times itself is 36? The answer is $\sqrt{36} = 6$ because $6 \cdot 6 = 36$.

The number inside the radical sign is called the **radicand**. For example, in $\sqrt{9}$, the radicand is 9.

# Square Roots of Perfect Squares

The easiest radicands to deal with are perfect squares. Because they appear so often, it is useful to learn to recognize the first few perfect squares:

**Squares**

| | | |
|---|---|---|
| $1^2 = 1 \times 1 = 1$ | $5^2 = 5 \times 5 = 25$ | $9^2 = 9 \times 9 = 81$ |
| $2^2 = 2 \times 2 = 4$ | $6^2 = 6 \times 6 = 36$ | $10^2 = 10 \times 10 = 100$ |
| $3^2 = 3 \times 3 = 9$ | $7^2 = 7 \times 7 = 49$ | $11^2 = 11 \times 11 = 121$ |
| $4^2 = 4 \times 4 = 16$ | $8^2 = 8 \times 8 = 64$ | $12^2 = 12 \times 12 = 144$ |

**Square Roots**

| | | |
|---|---|---|
| $\sqrt{1} = \pm 1$ | $\sqrt{25} = \pm 5$ | $\sqrt{81} = \pm 9$ |
| $\sqrt{4} = \pm 2$ | $\sqrt{36} = \pm 6$ | $\sqrt{100} = \pm 10$ |
| $\sqrt{9} = \pm 3$ | $\sqrt{49} = \pm 7$ | $\sqrt{121} = \pm 11$ |
| $\sqrt{16} = \pm 4$ | $\sqrt{64} = \pm 8$ | $\sqrt{144} = \pm 12$ |

***It is even easier to recognize when a variable is a perfect square, because the exponent is even.***

**Rule:** $\sqrt{a^2} = a$

$$\sqrt{x^2} = x$$

Any even power is a perfect square.

$$\sqrt{x^4} = x^2$$

$$\sqrt{x^{10}} = x^5$$

$$\sqrt{x^{90}} = x^{45}$$

The square root exponent is half of the original exponent.

**Example:** $\sqrt{64x^2y^{10}}$

| | |
|---|---|
| Write as a square. | $\sqrt{8xy^5 \cdot 8xy^5}$ |
| Evaluate. | $8xy^5$ |

You could also have split the radical into parts and evaluated them separately:

**Example:** $\sqrt{64x^2y^{10}}$

| | |
|---|---|
| Split into perfect squares. | $\sqrt{64 \cdot x^2 \cdot y^{10}}$ |
| Write as squares. | $\sqrt{8 \cdot 8} \cdot \sqrt{x \cdot x} \cdot \sqrt{y^5 \cdot y^5}$ |
| Evaluate. | $8 \cdot x \cdot y^5$ |
| Multiply together. | $8xy^5$ |

If your radical has a coefficient like $3\sqrt{25}$, evaluate the square root before multiplying: $3\sqrt{25} = 3 \cdot 5 = 15$.

## Practice

Simplify the radicals.

______ *1. $\sqrt{12}$

______ 2. $\sqrt{49}$

______ *3. $\sqrt{81}$

______ 4. $\sqrt{500}$

______ 5. $\sqrt{144}$

______ 6. $-\sqrt{64}$

______ 7. $4\sqrt{4}$

______ *8. $-2\sqrt{9}$

______ 9. $\sqrt{a^2}$

______ 10. $5\sqrt{36}$

______ 11. $\sqrt{1,600}$

______ 12. $8\sqrt{0}$

______ 13. $\sqrt{3n^2}$

______ *14. $\sqrt{24x^5}$

______ 15. $\sqrt{0.04}$

______ 16. $\sqrt{100x^4}$

______ 17. $-2\sqrt{4a^8}$

______ 18. $3\sqrt{25x^2y^{18}}$

______ *19. $-4\sqrt{400a^6b^2}$

______ 20. $5\sqrt{144x^4y^2}$

# Simplifying Radicals

Not all radicands are perfect squares. There is no whole number that, multiplied by itself, equals 5. With a calculator, you can get a decimal that squares very close to 5, but it won't come out exactly. The only precise way to represent the square root of 5 is to write $\sqrt{5}$. It cannot be simplified any further.

***There are three rules for knowing when a radical cannot be simplified any further:***

1. ***The radicand contains no factor, other than 1, that is a perfect square.***
2. ***The radicand is not a fraction.***
3. ***The radical is not in the denominator of a fraction.***

## When the Radicand Contains a Factor That Is a Perfect Square

To determine if a radicand contains any factors that are perfect squares, factor the radicand completely. All the factors must be prime. A number is prime if its only factors are one and the number itself. A prime number cannot be factored any further.

For example, here's how you simplify $\sqrt{12}$. The number 12 can be factored into $2 \cdot 6$. This is not completely factored because 6 is not prime. The number 6 can be further factored $2 \cdot 3$. The number 12 completely factored is $2 \cdot 2 \cdot 3$.

The radical $\sqrt{12}$ can be written as $\sqrt{2 \cdot 2 \cdot 3}$. This can be split up into $\sqrt{2 \cdot 2} \cdot \sqrt{3}$. Since $\sqrt{2 \cdot 2} = 2$, the simplified form of $\sqrt{12}$ is $2\sqrt{3}$.

## Additional Examples:

**Example:** $\sqrt{18}$

| | |
|---|---|
| Factor completely. | $\sqrt{2 \cdot 3 \cdot 3}$ |
| Separate out the perfect square $3 \cdot 3$. | $\sqrt{3 \cdot 3} \cdot \sqrt{2}$ |
| Simplify. | $3\sqrt{2}$ |

**Example:** $\sqrt{60}$

| | |
|---|---|
| Factor completely. | $\sqrt{6 \cdot 10}$ |
| Neither 6 nor 10 is prime. Both can be factored further. | $\sqrt{2 \cdot 3 \cdot 2 \cdot 5}$ |
| Separate out the perfect square $2 \cdot 2$. | $\sqrt{2 \cdot 2} \cdot \sqrt{3 \cdot 5}$ |
| Because $\sqrt{3 \cdot 5}$ contains no perfect squares, it cannot be simplified further. | $2\sqrt{15}$ |

**Example:** $\sqrt{32}$

Factor completely. $\sqrt{2 \cdot 16}$

The number 16 is not prime. It can be factored. $\sqrt{2 \cdot 2 \cdot 8}$

The number 8 is not prime. It can be factored. $\sqrt{2 \cdot 2 \cdot 2 \cdot 4}$

The number 4 is not prime. It can be factored. $\sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}$

You have two sets of perfect squares, $2 \cdot 2$ and $2 \cdot 2$. The square root of each is 2, so you have two square roots of 2. The square roots go outside the radical. You then multiply the numbers that are outside the radical.

$2 \cdot 2\sqrt{2}$

Simplify. The product of 2 times 2 gives you 4. $4\sqrt{2}$

**Shortcut:** You may have noticed in the first step, $\sqrt{2 \cdot 16}$, that 16 is a perfect square, and the square root of 16 is 4. This would have given you the answer $4\sqrt{2}$. Use the shorter method whenever you see one.

## Examples involving variables:

**Example:** $\sqrt{50x^3}$

Factor completely. $\sqrt{2 \cdot 5 \cdot 5 \cdot x \cdot x \cdot x}$

Separate the perfect square $5 \cdot 5$ and $x \cdot x$. $\sqrt{5 \cdot 5} \cdot \sqrt{x \cdot x} \cdot \sqrt{2 \cdot x}$

Simplify. $5x\sqrt{2x}$

**Example:** $\sqrt{9x^2y^3}$

Rewrite the radicand as the product of perfect squares. $\sqrt{9 \cdot x^2 \cdot y^2 \cdot y}$

Take out the square roots. $3xy\sqrt{y}$

$$\sqrt{75x^3}$$

$$\sqrt{25 \cdot 3 \cdot x^2 \cdot x}$$

$$\sqrt{25} \cdot \sqrt{3} \cdot \sqrt{x^2} \cdot \sqrt{x}$$

$$5 \cdot \sqrt{3} \cdot x \cdot \sqrt{x} = 5x\sqrt{3x}$$

## Practice

Simplify the radicals.

______21. $\sqrt{8}$

______22. $\sqrt{20}$

______23. $\sqrt{54}$

______24. $\sqrt{40}$

______25. $\sqrt{72}$

______26. $\sqrt{27}$

______27. $\sqrt{28}$

______28. $\sqrt{160}$

______29. $\sqrt{200}$

______30. $\sqrt{44}$

______31. $\sqrt{225}$

______32. $\sqrt{500}$

______33. $\sqrt{1{,}200}$

______34. $\sqrt{11}$

______35. $\sqrt{3x^2y^2}$

______36. $\sqrt{4b^6}$

______37. $\sqrt{8c^4d}$

______38. $\sqrt{80a^2b^3c^4}$

______39. $\sqrt{20a^5b^6c}$

______40. $\sqrt{500d^{13}}$

---

### When the Radicand Contains a Fraction

The radicand cannot be a fraction. If you get rid of the denominator in the radicand, then you no longer have a fraction. This process is called **rationalizing the denominator.** Your strategy will be to make the denominator a perfect square. To do that, you multiply the denominator by itself. However, if you multiply the denominator of a fraction by a number, you must multiply the numerator of the fraction by the same number. Take a look at the following examples.

**Example:** $\sqrt{\frac{1}{2}}$

Make the denominator a perfect square. $\sqrt{\frac{1}{2} \cdot \frac{2}{2}}$

Take out the square roots. One is a perfect square and so is $2 \cdot 2$. $\frac{1}{2}\,\frac{\sqrt{1.2}}{\sqrt{2.2}} = \frac{\sqrt{2}}{2}$

**Example:** $\sqrt{\frac{2}{3}}$

Make the denominator a perfect square. $\sqrt{\frac{2}{3} \cdot \frac{3}{3}}$

The number 1 is considered a factor of all numbers. If the numerator does not contain a perfect square, then 1 will be the perfect square and will be in the numerator. Take the square root of 1 in the numerator and $3 \cdot 3$ in the denominator. The product of $2 \cdot 3$ will give you 6 for the radicand.

$$\frac{\sqrt{2.3}}{\sqrt{3.3}} = \frac{1}{3}\sqrt{6}$$

**Example:** $\sqrt{\frac{3x}{2}}$

Make the denominator a perfect square. $\sqrt{\frac{3x}{2} \cdot \frac{2}{2}}$

Take the square roots. $\frac{\sqrt{3x \cdot 2}}{\sqrt{2 \cdot 2}} = \frac{1}{2}\sqrt{6x}$

## Practice

Simplify the radicals.

_____*41. $\sqrt{\frac{2}{5}}$

_____42. $\sqrt{\frac{2x^2}{3}}$

_____*43. $\sqrt{\frac{a^2b^2}{2}}$

_____44. $\sqrt{\frac{2}{7x}}$

_____*45. $\sqrt{\frac{5x^3}{3}}$

_____46. $\sqrt{\frac{20}{11}}$

_____47. $\sqrt{\frac{3x^2}{4}}$

_____48. $\frac{3}{\sqrt{5}}$

_____*49. $\sqrt{\frac{56}{4}}$

_____50. $\frac{\sqrt{160}}{\sqrt{2}}$

_____*51. $\frac{\sqrt{150}}{\sqrt{3}}$

# When a Radical is in the Denominator

When you have a radical in the denominator, the expression is not in simplest form. The expression $\frac{2}{\sqrt{3}}$ contains a radical in the denominator. To get rid of the radical in the denominator, rationalize the denominator. In other words, make the denominator a perfect square. To do that, you need to multiply the denominator by itself.

| | |
|---|---|
| **Example:** $\frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$ | $\frac{2\sqrt{3}}{\sqrt{9}}$ |
| Simplify. | |
| The number 9 is a perfect square. | $\frac{2\sqrt{3}}{3}$ |

| | |
|---|---|
| **Example:** $\frac{5}{\sqrt{2}}$ | |
| Rationalize the denominator. | $\frac{5}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$ |
| Simplify. | $\frac{5\sqrt{2}}{\sqrt{4}}$ |
| Take the square root of 4. | $\frac{5\sqrt{2}}{2}$ |

| | |
|---|---|
| **Example:** $\frac{\sqrt{6}}{\sqrt{2}}$ | |
| Rationalize the denominator. | $\frac{\sqrt{6}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$ |
| Simplify. | $\frac{\sqrt{12}}{\sqrt{4}}$ |
| You aren't finished yet because both radicands contain perfect squares. | $\frac{\sqrt{3 \cdot 4}}{\sqrt{4}}$ |
| Take the square root of 4. | $\frac{2\sqrt{3}}{2}$ |
| Finished? Not quite. You can divide 2 into 2, or cancel the 2's. | $\sqrt{3}$ |

**Practice**

**Simplify the radicals.**

______*52. $\frac{3}{\sqrt{7}}$

______53. $\frac{6}{\sqrt{5}}$

______*54. $\frac{\sqrt{2}}{\sqrt{3}}$

______55. $\frac{2\sqrt{3}}{\sqrt{6}}$

______56. $\frac{5}{\sqrt{x}}$

______*57. $\frac{3}{\sqrt{2y}}$

______*58. $\frac{7\sqrt{2}}{\sqrt{7}}$

______59. $\frac{\sqrt{8ab}}{\sqrt{b}}$

# Adding and Subtracting Radicals

You can add and subtract radicals if the radicands are the same. For example, you can add $3\sqrt{2}$ and $5\sqrt{2}$ because the radicands are the same. To add or subtract radicals, you add the number in front of the radicals and leave the radicand the same. When you add $3\sqrt{2} + 5\sqrt{2}$, you add the 3 and the 5, but the radicand $\sqrt{2}$ stays the same. The answer is $8\sqrt{2}$.

**Example:** $2\sqrt{5} + 7\sqrt{5}$

Add the numbers in front of the radicals. $9\sqrt{5}$

**Example:** $11\sqrt{5} - 4\sqrt{5}$

Subtract the numbers in front of the radicals. $7\sqrt{5}$

**Example:** $4\sqrt{3} + 2\sqrt{5} + 6\sqrt{3}$

You can add only the radicals that are the same. $10\sqrt{3} + 2\sqrt{5}$

**Example:** $5\sqrt{8} + 6\sqrt{8}$

Add the radicals. $11\sqrt{8}$

But $\sqrt{8}$ contains a factor that is a perfect square, so you aren't finished because your answer is not in simplest form. $11\sqrt{2 \cdot 4}$

Take out the square root of 4. $2 \cdot 11\sqrt{2}$

Simplify. $22\sqrt{2}$

## Practice

Add and subtract the radicals. Simplify the radical when necessary.

______ *60. $3\sqrt{7} + 8\sqrt{7}$

______ 61. $11\sqrt{3} - 8\sqrt{3}$

______ 62. $5\sqrt{2} + 6\sqrt{2} - 3\sqrt{2}$

______ *63. $3\sqrt{6} + 2\sqrt{2} - 5\sqrt{6}$

______ 64. $4\sqrt{a} + 13\sqrt{a}$

______ *65. $\sqrt{75} - \sqrt{20} + 3\sqrt{5}$

______ 66. $9\sqrt{x} - 4\sqrt{y} + 2\sqrt{x}$

______ 67. $\sqrt{12} + 3\sqrt{3}$

______ *68. $2\sqrt{18} + 6\sqrt{2}$

______ 69. $3\sqrt{5} + \sqrt{20} - \sqrt{7}$

# Multiplying and Dividing Radicals

To multiply radicals like $4\sqrt{3}$ by $2\sqrt{2}$, you multiply the numbers in front of the radicals: 4 times 2. Then multiply the radicands: 3 times 2. The answer is $8\sqrt{6}$.

**Example:** $5\sqrt{3} \cdot 2\sqrt{2}$
Multiply the numbers in front of the radicals. Then multiply the radicands.
$10\sqrt{6}$

**Example:** $2\sqrt{6} \cdot 3\sqrt{3}$
Multiply the numbers in front of the radicals. Then multiply the radicands.
$6\sqrt{18}$

However, you are not finished yet because 18 contains the factor 9, which is a perfect square. $6\sqrt{2 \cdot 9}$
Take out the square root of 9. $3 \cdot 6\sqrt{2}$
Simplify. $18\sqrt{2}$

***Remember:***

*When you multiply or divide radicals, the radicands do not have to be the same.*

To divide the radical $4\sqrt{6}$ by $2\sqrt{3}$, divide the numbers in front of the radicals. Then divide the radicands. The answer is $2\sqrt{2}$.

**Example:** $\frac{10\sqrt{6}}{5\sqrt{2}}$
Divide the numbers in front of the radical. Then divide the radicands.
$2\sqrt{3}$

**Example:** $\frac{8\sqrt{20}}{4\sqrt{5}}$
Divide the numbers in front of the radicals. Then divide the radicands.
$2\sqrt{4}$

However, you aren't finished yet because 4 is a perfect square.
Take the square root of 4. $2 \cdot 2$
Simplify. $4$

***Remember:***

*If there is no number in front of the radical sign it is assumed to be 1.*

## Practice

Multiply and divide the radicals. Simplify the radicals when necessary.

______*70. $7\sqrt{3} \cdot 5\sqrt{2}$

______*71. $\frac{14\sqrt{6}}{7\sqrt{2}}$

______72. $-3\sqrt{5} \cdot 4\sqrt{2}$

______73. $\frac{24\sqrt{10}}{12\sqrt{2}}$

______*74. $3\sqrt{a} \cdot 4\sqrt{b}$

______75. $\frac{8\sqrt{x^2}}{4\sqrt{x}}$

______76. $\frac{6\sqrt{20}}{2\sqrt{5}}$

______77. $3\sqrt{10} \cdot 5\sqrt{10}$

______*78. $4\sqrt{6} \cdot 5\sqrt{3}$

______*79. $\frac{30\sqrt{15}}{4\sqrt{10}}$

## Mixed Practice

Here's a chance to review what you learned in this lesson.

______80. $\sqrt{32}$

______81. $4\sqrt{18}$

______82. $3\sqrt{25x^6y}$

______83. $13\sqrt{5} + 5\sqrt{13}$

______84. $(6\sqrt{2})(3\sqrt{10})$

______85. $2\sqrt{24} + {}^{-}2\sqrt{27}$

______86. $\sqrt{32x^4y^7z^3}$

______87. $\frac{\sqrt{2a}}{\sqrt{4}}$

______88. $\frac{7}{\sqrt{x}}$

______89. $\sqrt{400x^4}$

______90. $\frac{\sqrt{3x}}{\sqrt{3}}$

______91. $5\sqrt{2} + {}^{-}7\sqrt{2}(4\sqrt{3})$

______92. $6\sqrt{3} \cdot 6\sqrt{3} + 10$

______93. $(-2\sqrt{7})^2$

______94. $2[-4\sqrt{2}(5\sqrt{5})]$

# What is a Radical Equation?

What is a radical equation? An equation is not considered a radical equation unless the radicand contains a variable, like $\sqrt{x} = 3$. You know that squaring something is the opposite of taking the square root. To solve such a radical equation, you square both sides.

| | |
|---|---|
| **Example:** $\sqrt{x} = 3$ | |
| Square both sides. | $(\sqrt{x})^2 = 3^2$ |
| Simplify. | $\sqrt{x} \cdot \sqrt{x} = 3 \cdot 3$ |
| Multiply. | $\sqrt{x \cdot x} = 9$ |
| Take the square root of $x^2$. | $x = 9$ |

Solving an equation like $x^2 = 25$ requires a little extra thought. Plug x = 5 and you see that $5^2 = 25$. This means that x=5 is a solution. However, if you plug in x = -5, you see that $(-5)^2 = 25$. This means that x = -5 is also a solution, so our initial equation has two solutions x=5 and x = -5. This happens so often that there is a special symbol ± that means plus or minus. You say that x = ± 5 is the solution to $x^2 = 25$.

Remember that every quadratic equation has two solutions.

| | |
|---|---|
| **Example:** $x^2 = 24$ | |
| Take the square root of both sides. | $\sqrt{x^2} = \sqrt{24}$ |
| The answer could be + or –. | $x = \pm\sqrt{24}$ |
| Factor out perfect squares. | $x = \pm\sqrt{4 \cdot 6}$ |
| Simplify. | $x = \pm 2\sqrt{6}$ |

## Practice

Solve the radical equations. Show your steps.

______ *1. $x^2 = 81$

______ 2. $x^2 = 50$

______ *3. $\sqrt{x} = 8$

______ 4. $\sqrt{n} = 3 \cdot 2$

______ 5. $x^2 = 49$

______ 6. $x^2 = 135$

______ 7. $\sqrt{n} = 11$

# Solving Complex Radical Equations

***Remember:***

*Before squaring both sides of an equation, get the radical on a side by itself.*

**Example:** $\sqrt{x} + 1 = 5$

| | |
|---|---|
| Subtract 1 from both sides of the equation. | $\sqrt{x} + 1 - 1 = 5 - 1$ |
| Simplify. | $\sqrt{x} = 4$ |
| Square both sides of the equation. | $(\sqrt{x})^2 = 4^2$ |
| Simplify. | $x = 16$ |

**Example:** $3\sqrt{x} = 15$

| | |
|---|---|
| Divide both sides of the equation by 3. | $\frac{3\sqrt{x}}{3} = \frac{15}{3}$ |
| Simplify. | $\sqrt{x} = 5$ |
| Square both sides of the equation. | $(\sqrt{x})^2 = 5^2$ |
| Simplify. | $x = 25$ |

**Example:** $2\sqrt{x} + 2 = 18$

| | |
|---|---|
| Subtract 2 from both sides of the equation. | $2\sqrt{x} + 2 - 2 = 18 - 2$ |
| Simplify. | $2\sqrt{x} = 16$ |
| Divide both sides of the equation by 2. | $\frac{2\sqrt{x}}{2} = \frac{16}{2}$ |
| Simplify. | $\sqrt{x} = 8$ |
| Square both sides of the equation. | $(\sqrt{x})^2 = 8^2$ |
| Simplify. | $x = 64$ |

## Practice

Solve the radical equations. Simplify the radical when necessary. Show all your steps.

______*8. $\sqrt{x}+5=7$

______ 9. $\sqrt{a}-6=5$

______*10. $4\sqrt{x}=20$

______11. $5\sqrt{w}=10$

______12. $3\sqrt{x}+2=8$

______13. $4\sqrt{x}-3=17$

______14. $\sqrt{a+3}=7$

______*15. $\sqrt{x-5}+2=11$

______16. $3\sqrt{x+2}=21$

______17. $2\sqrt{a+3}=6$

______18. $9-4\sqrt{y}=-11$

______19. $3+6\sqrt{c}=27$

______20. $\sqrt{3x+2}=1$

______*21. $\sqrt{x}=\frac{1}{9}$

______22. $\sqrt{a}=2\sqrt{3}$

______23. $5\sqrt{2x+1}+2=17$

______24. $6\sqrt{2x+1}=18$

______*25. $-3\sqrt{x+2}+2=-4$

______26. $15+\sqrt{5x}=20$

______27. $\sqrt{4x+1}+7=12$

______28. $5\sqrt{3x+6}=30$

**TOPIC F**

**FACTORING POLYNOMIALS**

- Greatest Common Factor
- Difference of Two Squares
- Trinomials where a = 1
- Trinomials where a > 1
- Quadratic Equation

# Factoring Polynomials

*We will start by factoring expressions using the greatest common factor, difference of two squares, and the trinomial method.*

# What Is Factoring?

Factoring is the opposite of multiplying. It undoes what multiplication does. When you factor an algebraic expression, you end up with quantities called **factors** which, when multiplied, will give you the original expression.

# Finding the Greatest Common Factor

The first type of factoring we will review is the GCF method. With this method, you look for the greatest factor that is common amongst two or more expressions or numbers.

**Example:** What is the greatest common factor of 18 and 27?

Factors of 18 are: 1, 2, 3, 6, 9, 18

Factors of 27 are: 1, 3, 9, 27

The greatest factor both numbers have is 9, so the greatest common factor of 18 and 27 is 9.

Another approach would be to factor each number until all its factors are prime factors.

**Example:** What is the greatest common factor of 12 and 24?

The prime factors of 12 are: $2 \cdot 2 \cdot 3$

The prime factors of 24 are: $2 \cdot 2 \cdot 2 \cdot 3$

The factors that both 12 and 24 have in common are $2 \cdot 2 \cdot 3$, which equals 12. So you know that the greatest common factor of 12 and 24 is 12.

**Example:** What is the greatest common factor of $8x^3y$ and $24x^2y^2$?

The prime factors of $8x^3y$ are: $2 \cdot 2 \cdot 2 \cdot x \cdot x \cdot x \cdot y$

The prime factors of $24x^2y^2$ are: $2 \cdot 2 \cdot 2 \cdot 3 \cdot x \cdot x \cdot y \cdot y$

The factors that both numbers have in common are $2 \cdot 2 \cdot 2 \cdot x \cdot x \cdot y$, which equals $8x^2y$.

**Example:** What is the greatest common factor of $30a^2b$ and $40a^3b^3$?

The prime factors of $30a^2b$ are: $2 \cdot 3 \cdot 5 \cdot a \cdot a \cdot b$

The prime factors of $40a^3b^3$ are: $2 \cdot 2 \cdot 2 \cdot 5 \cdot a \cdot a \cdot a \cdot b \cdot b \cdot b$

Both numbers have $2 \cdot 5 \cdot a \cdot a \cdot b$ in common, so the greatest common factor is $10a^2b$.

# Factoring Using the GCF Method

To factor an expression like 2x+6, find the greatest common factor of both terms. In this expression, the factors of 2x are 2 and x, and the factors of 6 are 2 and 3, so the GCF is 3. To factor the expression, put the GCF outside the parentheses and put what is left inside the parentheses.

**Example:** $2x + 6 = 2(x + 3)$

## Practice

Factor using the greatest common factor method.

______ *1. $5x + 25$

______ 2. $100a + 300$

______ *3. $15a^2b^2 + 15ab^2$

______ 4. $22xy + 11x$

______ *5. $5x + 9$

______ 6. $16x^2 + 20x$

______ 7. $x^2y + 3x$

______ 8. $8x^3 - 2x^2 + 4x$

______ *9. $-6x^3 + 18x^2y$

______ 10. $10x^4y^2 - 50x^3y + 70$

______ 11. $6a^2 - 39ab$

______ 12. $12a^3b^2c + 4a^5bc^2$

______ 13. $22x^4y + 55x^2y^2$

______ 14. $8x^2 + 12x + 20$

______ *15. $5f^3 - 15f + 25$

______ 16. $30a^3b^2 + 20a^2b + 35a^4b^2$

# Factoring Using the Difference of Two Squares Method

Another type of factoring is the difference of two squares. You will find this method easy if you have reviewed the concept of squares.

A number or expression multiplied by itself equals a **perfect square**.

**Examples:**

4 because $2 \cdot 2 = 4$
9 because $3 \cdot 3 = 9$
25 because $5 \cdot 5 = 25$
$a^2$ because $a \cdot a = a^2$
$16b^2$ because $4b \cdot 4b = 16b^2$
$d^{10}$ because $d^5 \cdot d^5 = d^{10}$

## *Remember:*

***Any even numbered exponent is a perfect square.*** $n^{12} = n^6 \cdot n^6$

The method of factoring using the DOTS (Difference of Two Squares) is an easy pattern to remember. The pattern is (x + y)(x – y), where x is the square root of the first term, and y is the square root of the second term.

**Example:** $x^2 - 4$

Both $x^2$ and 4 are perfect squares, and they are connected by a subtraction sign, which is why the expression is called the *difference* of two squares. To factor the expression, take the square root of the first term and the square root of the second term. Write it like this: $(x + 2)(x - 2)$.

***Remember:***

***The sum of two squares ($n^2 + 4$) CANNOT be factored.***

Here are two more examples:

**Example:** $y^2 - 9$

$= (y + 3)(y - 3)$

**Example:** $16a^2 - 25b^2$

$= (4a + 5b)(4a - 5b)$

## ***Remember:***

***The expressions (n+6)(n-6) and (n-6)(n+6) are equivalent due to the commutative property.***

## Practice

Factor these expressions using the difference of two squares method.

______* **17.** $16r^2 - 121$

______ **18.** $a^2 - 9$

______* **19.** $x^2y^2 - 49$

______ **20.** $b^2 - 100$

______* **21.** $r^2 - s^2$

______ **22.** $36b^2 - 100$

______* **23.** $a^6 - b^6$

______ **24.** $y^2 - 64$

______ **25.** $4x^2 - 1$

______* **26.** $25x^2 - 4y^2$

______ **27.** $x^4 + 1$

______ **28.** $x^4 - 16$

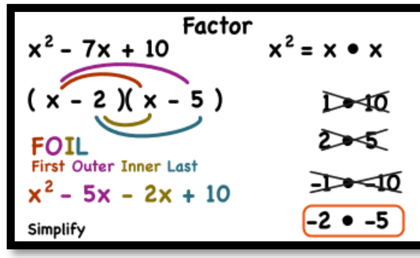
______ **29.** $b^{10} - 36$

______ **30.** $16a^2 - 25b^2$

# Factoring Using the Trinomial Method
# When leading coefficient is 1 (a = 1)

Before factoring you should place your trinomial in descending order:
$ax^2 + bx + c$ and follow the steps below:

- Identify a, b, and c in the **trinomial** $ax^2 + bx + c$.
- Write down all **factor** pairs of c.
- Identify which **factor** pair's sums up to b.
- Substitute **factor** pairs into two binomials.




## *Hint:*

*When the constant, c is positive, both factor pairs will have the same sign!*

Positive so both factor pairs will have the same sign

$n^2 + 6n + 5 = (n + 5)(n + 1)$

Positive so both factor pairs will be positive

**OR**

Positive so both factor pairs will have the same sign

$n^2 - 5n + 6 = (n - 3)(n - 2)$

Negative so both factor pairs will be negative

## Practice

Factor using the trinomial method.

______*31. $x^2 + 9x + 18$

______32. $x^2 + 6x + 8$

______*33. $x^2 - 4x + 3$

______34. $x^2 + 7x + 12$

______*35. $x^2 - 10x + 16$

______36. $x^2 - 15x + 14$

______*37. $x^2 + 9x + 20$

______38. $x^2 - 12x + 20$

______39. $x^2 - 9x + 20$

______40. $x^2 - 11x + 30$

## ***Hint:***

*When the constant, c is negative, the factor pairs will have different signs!*

Negative so factor pairs will have different signs

$n^2 - 4n - 12 = (n - 6)(n + 2)$

Negative so the number with the larger absolute value will be negative

**OR**

Negative so factor pairs will have different signs

$n^2 + 4n - 12 = (n + 6)(n - 2)$

Positive so the number with the larger absolute value will be positive

## Practice

Factor the trinomials.

______*41. $a^2 - a - 20$

______42. $r^2 - 7r - 18$

______*43. $x^2 - 6x - 7$

______44. $x^2 + 3x - 10$

______45. $x^2 - 3x - 10$

______46. $x^2 - 7x - 8$

______*47. $x^2 + 6x - 16$

______48. $x^2 - 4x - 21$

______*49. $x^2 + x - 30$

______50. $x^2 - 3x - 18$

# Factoring Using the Trinomial Method
# When leading coefficient is  not 1 (a > 1)

Here we will use something called the ac method, or splitting the middle. As before we want our trinomial in the form $\mathbf{ax^2 + bx + c}$ so that we can identify our a, b and c values. You will start by multiplying the **a** and **c** values. Find the factor pairs for this product so that you can rewrite the middle term as the sum of two factors. Factor by grouping.

$$6x^2 + 17x + 12$$

72

| | |
|---|---|
| 1 | 72 |
| 2 | 36 |
| 3 | 24 |
| 4 | 18 |
| 6 | 12 |
| 8 | 9 |

$$6x^2 + 8x + 9x + 12$$

$$(6x^2 + 8x) + (9x + 12)$$

$$2x(3x + 4) + 3(3x + 4)$$

$$(3x + 4)(2x + 3)$$

Steps:

1) Find factors of 72 that add up to 17
2) Rewrite the polynomial so that the middle term is a sum of the 2 factors you found
3) Factor by grouping

$$2n^2 + 9n + 10$$

- What is "ac"? a=2, c=10 …. a•c = 20
- Find a factor pair of 20 that sums to 'b'. b=9
  - 5 & 4     5•4=20 and 5+4=9
- Split your 'b' term into like terms using 5&4

$$2n^2 + 4n + 5n + 10$$

- Factor by Grouping

$$2n(n + 2) + 5(n + 2)$$

- Factor out what's common (The parentheses)

$$(n + 2)(2n + 5)$$

$2x^2 + x - 6$

$2x^2 + 4x - 3x - 6$

$2x(x+2) - 3(x+2)$

$(x+2)(2x-3)$

12
-1 and 12
-2 and 6
-3 and 4
1 and -12
2 and -6
3 and -4

## Practice

Factor using the trinomial method.

______ * **1.** $2x^2 + 7x + 3$

______ **2.** $5x^2 + 13x - 6$

______ * **3.** $7x^2 + 3x - 4$

______ **4.** $4x^2 - 21x + 5$

______ **5.** $6x^2 + x - 7$

______ **6.** $8x^2 - 3x - 5$

______ * **7.** $8x^2 - 6x - 5$

______ **8.** $9x^2 + 3x - 2$

______ **9.** $6x^2 - 17x + 5$

______ **10.** $5x^2 - 7x - 6$

### Factoring Using Any Method

When you factor algebraic expressions, you need to analyze the expression to determine which type of factoring to use. Remember the three methods of factoring you have learned:

1. greatest common factor
2. difference of two squares
3. trinomials

## Practice

Factor using the appropriate method.

______ **11.** $2x^2 - 3x - 2$

______ **12.** $3x^2 + 9x - 12$

______ **13.** $5x^2 - 9x - 2$

______ **14.** $4x^2 + 11x - 3$

______ **15.** $4x^2 + 8x + 4$

______ **16.** $5x^2 + 12x + 4$

______ **17.** $36x^2 + 72x + 36$

______ **18.** $9x^2 + 18x + 9$

______ **26.** $b^2 + 3b - 18$

______ **27.** $n^2 - 2n - 35$

______ **28.** $24x + 6$

______ **29.** $9x^2 - 100$

______ **30.** $5x^2 + 7x - 6$

______ **31.** $6x^2 - 7x - 3$

______ **32.** $r^2 - 5r - 24$

______ **33.** $f^2 + 5f - 36$

______19. $7x^2 - 48x - 7$

______20. $4x^2 - 7x - 2$

______21. $4x^2 + 12x - 16$

______22. $4x^2 + 20x + 16$

______23. $49x^2 - 4$

______24. $c^2 - 11c + 30$

______25. $a^2 - b^2$

______34. $3x^3y + 6x^2y^2 - 9xy^3$

______35. $15x^2 - 7x - 2$

______36. $25a^2 - 64$

______37. $48x^3y^3 - 18x^4y$

______38. $6x^2 + 25x + 11$

______39. $10mn + 5m^2n^3 - 20m^3n^2$

______40. $25x^2 + 1$

### Factoring Using More Than One Method

Sometimes it may be necessary to use more than one method of factoring on the same expression. Always check for the greatest common factor first.

**Example:** $4x^2 - 4$

Look for the greatest common factor first. $4(x^2 - 1)$

You aren't finished because $x^2 - 1$ is the difference of two squares. $= 4(x - 1)(x + 1)$

**Example:** $2x^2 + 8x + 8$

Look for the greatest common factor. $2(x^2 + 4x + 4)$

Factor the trinomial. $2(x + 2)(x + 2)$

## Practice

Factor completely.

______*41. $3x^2 - 27$

______42. $4x^2 - 64$

______*43. $2x^2 + 12x + 18$

______44. $2x^2 + 4x - 6$

______45. $3x^2 + 21x + 30$

______46. $4x^6 - 100$

______*47. $27x^2 - 75y^2$

______*48. $12x^2 - 36x - 21$

______49. $3x^2 - 24x + 36$

______*50. $x^4 - 81$

# Quadratic Equations

## What Is a Quadratic Equation?

A **quadratic equation** is an equation of the form $ax^2 + bx + c = 0$, where $a$, $b$, and $c$ are numbers. The graph of a quadratic equation will be a smooth U-shaped curve, unlike a linear equation, whose graph is always a straight line.

Here are some examples of quadratic equations:

$x^2 = 4$

$x^2 + 3 = 0$

$2x^2 + 5 = 10$

$x^2 + 4x + 4 = 0$

$5x^2 - 1 = 0$

When solving Quadratic equations, one of the methods you will use is factoring (if possible).

**Example:** $x^2 - 3x - 4 = 0$

| | |
|---|---|
| Factor the trinomial $x^2 - 3x - 4$. | $(x-4)(x+1) = 0$ |
| Set each factor equal to zero. | $x - 4 = 0$ and $x + 1 = 0$ |
| Solve the equation. | $x - 4 = 0$ |
| Add 4 to both sides of the equation. | $x - 4 + 4 = 0 + 4$ |
| Simplify. | $x = 4$ |
| Solve the equation. | $x + 1 = 0$ |
| Subtract 1 from both sides of the equation. | $x + 1 - 1 = 0 - 1$ |
| Simplify. | $x = -1$ |

The two solutions for the quadratic equation are 4 and –1.

**Example:** $2x^2 - 33 = -1$

| | |
|---|---|
| Add 1 to both sides of the equation. | $2x^2 - 33 + 1 = -1 + 1$ |
| Simplify both sides of the equation. | $2x^2 - 32 = 0$ |
| Take out the common factor. | $2(x^2 - 16) = 0$ |
| Factor the difference of two squares. | $2(x-4)(x+4) = 0$ |
| Disregard the 2 and set the other factors equal to zero. | $x - 4 = 0$ and $x + 4 = 0$ |

The solutions are 4 and –4.

## Practice

Solve the quadratic equations using factoring.

______ *1. $a^2 + 2a - 24 = 0$

______ 2. $b^2 - b - 6 = 0$

______ 3. $x^2 + 15x + 50 = 0$

______ 4. $y^2 - 8y + 7 = 0$

______ 5. $r^2 + 12r + 27 = 0$

______ 6. $x^2 + 8x - 48 = 0$

______ 7. $x^2 + 7x - 18 = 0$

______ 8. $x^2 + 4x = 45$

______ *9. $x^2 - 6 = 30 - 3x^2$

______ 10. $x^2 + 11x = -24$

______ *11. $2x^2 - 16x - 18 = 0$

______ 12. $6x^2 - 12x + 6 = 0$

______ 13. $x^2 - 4x = 45$

______ 14. $2x^2 - 5x - 7 = 0$

______ *15. $3x^2 - 20x = 7$

______ 16. $2x^2 + 2x - 4 = 0$

______ 17. $x^2 + 2x = 15$

______ 18. $2x^2 + 6x - 20 = 0$

______ 19. $16x^2 + 2x = 2x + 9$

______ 20. $15x^2 + 84x = 36$

# QUADRATIC FORMULA

*Not all quadratic equations can be solved using factoring. There is another method called the Quadratic Formula.*

To use the quadratic formula, you need to know the $a$, $b$, and $c$ of the equation. However, before you can determine what $a$, $b$, and $c$ are, the equation must be in $ax^2 + bx + c = 0$ form. The equation $5x^2 + 2x = 9$ must be transformed to $ax^2 + bx + c = 0$ form.

**Example:** $5x^2 + 2x = 9$

Subtract 9 from both sides of the equation. $5x^2 + 2x - 9 = 9 - 9$

Simplify. $5x^2 + 2x - 9 = 0$

In this equation, $a$ is 5, $b$ is 2, and $c$ is $-9$.

## *Practice*

Find $a$, $b$, and $c$ (in that order) in the following quadratic equations.

______ **1.** $4x^2 + 8x + 1 = 0$

______ **2.** $x^2 - 4x + 10 = 0$

______ **3.** $2x^2 + 3x = 0$

______ **4.** $6x^2 - 8 = 0$

______ **5.** $4x^2 = 7$

______ **6.** $3x^2 = 0$

______ **7.** $2x^2 = -3x + 4$

______ **8.** $9x^2 + 2 = 7x$

The **quadratic formula** is a formula that allows you to solve any quadratic equation—no matter how simple or difficult. If the equation is written $ax^2 + bx + c = 0$, then the two solutions for $x$ will be $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. It is the $\pm$ in the formula that gives us the two answers: one with + in that spot, and one with −. The formula contains a radical, which is one of the reasons you studied radicals in the previous lesson. To use the formula, you substitute the values of $a$, $b$, and $c$ into the formula and then carry out the calculations.

| | |
|---|---|
| **Example:** $3x^2 - x - 2 = 0$ | |
| Determine $a$, $b$, and $c$. | $a = 3, b = -1$, and $c = -2$ |
| Take the quadratic formula. | $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ |
| Substitute in the values of $a$, $b$, and $c$. | $\frac{-(^-1) \pm \sqrt{(-1)^2 - 4 \cdot 3 \cdot -2}}{2 \cdot 3}$ |
| Simplify. | $\frac{1 \pm \sqrt{1 - ^-24}}{6}$ |
| Simplify more. | $\frac{1 \pm \sqrt{25}}{6}$ |
| Take the square root of 25. | $\frac{1 \pm 5}{6}$ |
| The solutions are 1 and $\frac{-2}{3}$. | $\frac{1+5}{6} = \frac{6}{6} = 1$ and $\frac{1-5}{6} = \frac{-4}{6} = -\frac{2}{3}$ |

## Practice

Solve the equations using the quadratic formula.

_______ *9. $4x^2 + 8x = 0$

_______ 10. $3x^2 = 12x$

_______ 11. $x^2 - 25 = 0$

_______ 12. $x^2 + 4x = 5$

_______ 13. $x^2 + 4x - 21 = 0$

_______ 14. $x^2 + 11x + 30 = 0$

_______ *15. $2x^2 + 5x + 3 = 0$

_______ 16. $6x^2 + 1 = 5x$

### Solving Quadratic Equations That Have a Radical in the Answer

Some equations will have radicals in their answers. The strategy for solving these equations is the same as the equations you just solved. Take a look at the following example.

| | |
|---|---|
| **Example:** | $3m^2 - 3m = 1$ |
| Subtract 1 from both sides of the equation. | $3m^2 - 3m - 1 = 1 - 1$ |
| Simplify. | $3m^2 - 3m - 1 = 0$ |
| Use the quadratic formula with $a = 3$, $b = -3$, and $c = -1$. | $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ |
| Substitute the values for $a$, $b$, and $c$. | $\frac{-(^-3) \pm \sqrt{(-3)^2 - 4 \cdot 3 \cdot -1}}{2 \cdot 3}$ |
| Simplify. | $\frac{3 \pm \sqrt{9 - {}^-12}}{6}$ |
| Simplify. | $\frac{3 \pm \sqrt{21}}{6}$ |

The solution to the equation is $m = \frac{3 \pm \sqrt{21}}{6}$ because one answer is $m = \frac{3 + \sqrt{21}}{6}$ and the other answer is $m = \frac{3 - \sqrt{21}}{6}$.

## Practice

Solve the equations using the quadratic formula. Leave your answers in radical form.

_______ *17. $x^2 - 3x + 1 = 0$

_______ 18. $2x^2 - 2x = 5$

_______ 19. $y^2 - 5y + 2 = 0$

_______ *20. $r^2 - 7r - 3 = 0$

_______ 21. $x^2 - 3x - 5 = 0$

_______ 22. $4x^2 - 5x - 1 = 0$

_______ *23. $m^2 + 11m - 1 = 0$

_______ 24. $x^2 + 5x = 3$

_______ 25. $2x^2 + 3x - 3 = 0$

_______ 26. $x^2 - 4x + 2 = 0$

_______ 27. $3y^2 + 3y - 3 = 0$

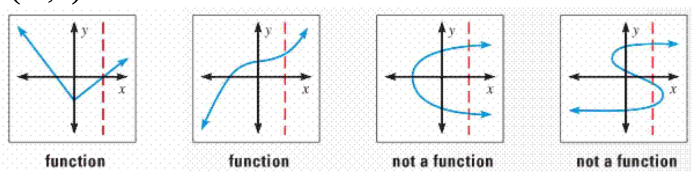
_______ 28. $x^2 + 2x - 6 = 0$

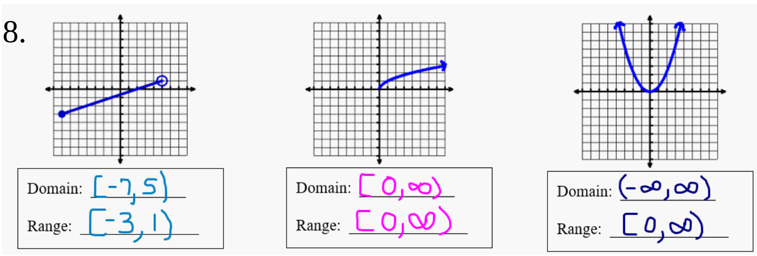
_______ 29. $2x^2 + x - 5 = 0$

_______ 30. $a^2 - 2a - 7 = 0$

# ANSWERS: TOPIC A

(Pages 2 – 3)

1. {-2,0,7,18}
2. -19
3. B (-5)
4. $[-2,\infty)$
5. $(-\infty,-1]$
6. (-3,5)
7. 

8. 


9.

| A | B | C |
|---|---|---|
| Domain: [-6, 6] | Domain: [-7, 5) | Domain: x = -5 |
| Range: [0, 6] | Range: [-3, 1) | Range: (-2, 6) |

| D | E | F |
|---|---|---|
| Domain: [-4, 4] | Domain: [0, ∞) | Domain: (0, ∞) |
| Range: [-4, 4] | Range: (-∞, ∞) | Range: y = 4 |

# ANSWERS: TOPIC A

(Pages 5 – 6)

| | | | |
|---|---|---|---|
| 1a. 7 | b. 12 | c. 2 | d. -23 |
| 2a. -4 | b. 14 | c. 0 | d. 10 |
| 3a. -1 | b. 95 | c. -4 | d. 4 |
| 4a. 2 | b. 232 | c. 112 | d. 8 |
| 6a. x = 5 | b. x = -4/7 | c. x = 0 | d. x = -11/21 |
| 7a. x = -3 | b. x = 6/5 | c. x = 12/5 | d. x = 23/20 |
| 8a. x = -2 | b. x = 8/3 | c. x = 22/3 | d. x = 13/5 |

9a. $4m^2+4m-5$ b. $w^2+2w-5$ c. $16x^2+8x-5$ d. $n^4+2n^2-5$

## ANSWERS: TOPIC B

**(Pages 7 – 9)**

1. $17x$
2. $13x + 7y$
3. $15x + 5$
4. $18x$
5. $2x^2 - 5x + 12$
6. $5x - 7y + 2$
7. $12x + 12y + 5$
8. $8x^2 + 5x + 9$
9. $8$
10. $7xy + 16x - 2y - 8$ (Note: $xy = yx$)
11. $-4x + 9y$
12. $-3r - 3s$
13. $16s - 14r$
14. $-4x + 8y$
15. $2a + 4b$
16. $11m + n - 2$
17. $13x + 12y$
18. $5x - 16$
19. $-13a - 7b$
20. $17x + 13y$

**(Pages 9 – 11)**

1. $5x - 5y + 10$
2. $7x^2 - 21x$
3. $24x^5 + 16x^4 - 40x^3$
4. $-6x + 6y + 42$
5. $3bx^2 + 6bxy + 3by$
6. $-14ac^2 + 35ac^3$
7. $6x^2y^2 - 6xy$
8. $6a^2x^2 - 4a^3bx + 20a^3x^4 + 16a^2x$
9. $-16x^4y^2 + 24x^3y^3$
10. $-8r^2st + 28r^3s^2 - 36rs^3t^2$
11. $x^2 + 9x + 18$
12. $x^2 - 13x + 36$
13. $6x^2 - 11x - 7$
14. $x^2 - 3xy + 2x - 6y$
15. $25x^2 - 49$
16. $x^2 + 6x + 8$
17. $x^2 + 3x - 18$
18. $x^2 - 8x + 12$
19. $x^2 + 9x - 10$
20. $x^2 + 2x + 1$
21. $2x^2 + 11x + 5$
22. $9x^2 - 1$
23. $20x^2 + 6x - 2$
24. $a^2 + 2ab + b^2$
25. $a^2 - b^2$
26. $14x^2 - 24xy - 8y^2$
27. $12x^2 + 8x - 9xy - 6y$
28. $2x^2 - xy - y^2$
29. $30x^2 - 17x + 2$
30. $ac + ad + bc + bd$
31. $49y^2 - 4$
32. $16x^2 - 8x + 1$
33. $14a^2 - 8a - 6$

# ANSWERS: TOPIC B

**(Pages 9 – 11)**

38. $x^3 - 4x^2 + 5x - 2$
39. $2x^3 + 7x^2 + 16x + 15$
40. $x^3 + 3x^2 - 4x - 12$
41. $3x^3 + 8x^2 - 5x - 6$
42. $4x^3 + 7x^2y + 3xy^2 - x - y$
43. $2a^2 + 2b^2 + 5ab + 16a + 8b$
44. $6x^2 + 9xy + x + 6y - 2$
45. $10x^3 + 13x^2 + 19x + 12$
46. $6x^3 - 2x^2 + 2x - 6$
47. $2x^4 - x^2y^2 - y^4$
48. $3x^3 + x^2 - 8x + 4$
49. $2x^4 + 3x^3 - 17x^2 + 12x$
50. $20a^3 + 13a^2b^2 - 2ab^2 - b^3$
51. $18y^3 - 51y^2 + 42y - 49$

# ANSWERS: TOPIC C

**(Page 12)**

1. yes
2. yes
3. yes
4. no
5. no
6. yes
7. no
8. no
9. no
10. yes

**(Page 14)**

21. $\varnothing$
22. infinite
23. one

**(Page 14)**

24. one
25. $\varnothing$
26. one
27. infinite
28. $\varnothing$

**(Page 16)**

1. $(-\frac{8}{3}, 4)$
2. $(-1, \frac{1}{2})$
3. $(-2, 0)$
4. $(\frac{1}{2}, 5)$
5. $(3, -1)$
6. $(\frac{1}{2}, -1)$
7. $(1, 3)$
8. $(3, 2)$
9. $(2, {}^{-}1)$
10. $(6, 5)$
11. $({}^{-}2, 8)$
12. $(10, 3)$
13. $(5, 0)$
14. $({}^{-}2, 6)$
15. $(4, {}^{-}1)$
16. $(3, 11)$
17. $({}^{-}2, 7)$
18. $({}^{-}2, 3)$

**(Page 18)**

19. $(3, 1)$
20. $(-2, 4)$
21. $(-1, 0)$
22. $(0, -1)$
23. $(3, 2)$
24. $(5, -2)$
25. $(-2, -3)$
26. $(-1, 0)$
27. $(2, 10)$
28. $(0, 2)$
29. $(-27, -11)$
30. $(-7, 4)$

# ANSWERS: TOPIC C

**(Page 19 and 20)**

**31.** (4,2)
**32.** (2,4)
**33.** (–13,–53)
**34.** (1,3)
**35.** (5,2)
**36.** (–2,–4)
**37.** (3, 15)
**38.** (⁻8, 11)
**39.** (0.3,2.1)

**40.** $(\frac{3}{2},12)$
**41.** (⁻4, 12)
**42.** (4,⁻1)

**45.** (2,4)
**46.** (2,–12)
**47.** (2,5)
**48.** (8,2)
**49.** (–1,1)
**50.** (0,–2)
**51.** $x + y = 100$
$3x + 4y = 340$
Solution: $x = 60, y = 40$
So, 60 student tickets and
**52.** $x + y = 6$
$70x + 45y = 370$
Solution: $x = 4, y = 2$
So, 2 hours on back roads

# ANSWERS: TOPIC D

**(Page 23 - 25)**

**1.** $28x$
**2.** This is already simplified.
**3.** $a^{10}$
**4.** $6x^2y$
**5.** $12x^3$
**6.** $a^3b^5$
**7.** $12m^6n^{11}$
**8.** $12a^4b^2c + 5abc$
**9.** This is already simplified.
**10.** $23x^2y$
**11.** $y^5$
**12.** $\frac{1}{a^3}$
**13.** $-b^4$
**14.** $a^2b$
**15.** $5y$
**16.** $\frac{1}{9a^2b^4}$
**17.** $\frac{4xy^3}{z^2}$
**18.** $\frac{5s^2}{r}$
**19.** $\frac{4x^2}{5y^3}$
**20.** $\frac{m^3}{3}$
**21.** $x^{10}$
**22.** $c^{20}$
**23.** $a^4b^6$
**24.** $x^3y^{15}$
**25.** $m^{18}n^6$
**26.** $8x^9$
**27.** $r^{20}s^8$
**28.** $x^6y^{18}$
**29.** $512a^9b^9c^{36}$
**30.** $27x^6y^3 + 125x^3y^6$
**31.** $x^7y^2z^3$
**32.** $18a^2b + 15ab^2$
**33.** $2x^3y^6$
**34.** 4
**35.** $2x^9y^9$
**36.** $14x^2y^5$
**37.** $27a^{12}b^{15}$
**38.** 0
**39.** $6x^2y + 5xy^2$
**40.** $\frac{3a}{4b^2}$

# ANSWERS:  TOPIC E

**(Pages 27 – 35)**

1. $2\sqrt{3}$
2. 7
3. 9
4. $10\sqrt{5}$
5. 12
6. –8
7. 8
8. –6
9. $a$
10. 30
11. 40
12. 0
13. $n\sqrt{3}$
14. $2x^2\sqrt{6x}$
15. 0.2
16. $10x^2$
17. $-4a^4$
18. $15xy^9$
19. $-80a^3b$
20. $60x^2y$
21. $2\sqrt{2}$
22. $2\sqrt{5}$
23. $3\sqrt{6}$
24. $2\sqrt{10}$
25. $6\sqrt{2}$
26. $3\sqrt{3}$
27. $2\sqrt{7}$
28. $4\sqrt{10}$
29. $10\sqrt{2}$
30. $2\sqrt{11}$
31. 15
32. $10\sqrt{5}$
33. $20\sqrt{3}$
34. prime
35. $xy\sqrt{3}$
36. $2b^3$
37. $2c^2\sqrt{2d}$
38. $4abc^2\sqrt{5b}$
39. $2a^2b^3\sqrt{5ac}$
40. $10d^6\sqrt{5d}$
41. $\frac{1}{5}\sqrt{10}$
42. $\frac{x}{3}\sqrt{6}$
43. $\frac{ab}{2}\sqrt{2}$
44. $\frac{1}{7x}\sqrt{14x}$
45. $\frac{x}{3}\sqrt{15x}$
46. $\frac{2}{11}\sqrt{55}$
47. $\frac{x\sqrt{3}}{2}$
48. $\frac{3\sqrt{5}}{5}$
49. $\sqrt{14}$
50. $4\sqrt{5}$
51. $5\sqrt{2}$
52. $\frac{3}{7}\sqrt{7}$
53. $\frac{6}{5}\sqrt{5}$
54. $\frac{1}{3}\sqrt{6}$
55. $\sqrt{2}$
56. $\frac{5}{x}\sqrt{x}$
57. $\frac{3}{2y}\sqrt{2y}$
58. $\sqrt{14}$
59. $2\sqrt{2a}$
60. $11\sqrt{7}$
61. $3\sqrt{3}$
62. $8\sqrt{2}$
63. $2\sqrt{2} - 2\sqrt{6}$
64. $17\sqrt{a}$
65. $5\sqrt{3} + \sqrt{5}$
66. $11\sqrt{x} - 4\sqrt{y}$
67. $5\sqrt{3}$
68. $12\sqrt{2}$
69. $5\sqrt{5} - \sqrt{7}$
70. $35\sqrt{6}$
71. $2\sqrt{3}$
72. $-12\sqrt{10}$
73. $2\sqrt{5}$
74. $12\sqrt{ab}$
75. $2\sqrt{x}$
76. 6
77. 150
78. $60\sqrt{2}$
79. $\frac{15}{4}\sqrt{6}$
80. $4\sqrt{2}$
81. $12\sqrt{2}$
82. $15x^3\sqrt{y}$
83. Cannot simplify—no like terms
84. $36\sqrt{5}$
85. $4\sqrt{6} + {}^{-}6\sqrt{3}$
86. $4x^2y^3z\sqrt{2yz}$
87. $\frac{\sqrt{2a}}{2}$
88. $\frac{7\sqrt{x}}{x}$
89. $20x^2$

## ANSWERS: TOPIC E

(pg 35)

90. $\sqrt{x}$
91. $5\sqrt{2} - 28\sqrt{6}$
92. 118
93. 28
94. $-40\sqrt{10}$

## ANSWERS: TOPIC E

(pg 36-38)

1. $\pm 9$
2. $\pm 5\sqrt{2}$
3. 64
4. 36
5. $x = \pm 7$
6. $x = \pm 3\sqrt{15}$
7. 121
8. 4
9. 121
10. 25
11. 4
12. 4
13. 25
14. 46
15. 86
16. 47
17. 6
18. 25
19. 16
20. $-\frac{1}{3}$
21. $\frac{1}{81}$
22. 12
23. 4
24. 4
25. 2
26. 5
27. 6
28. 10

## ANSWERS: TOPIC F

(pg 40-43)

1. $5(x+5)$
2. $100(a+3)$
3. $15ab(ab+b)$
4. $11x(2y+1)$
5. prime—can't be factored
6. $4x(4x+5)$
7. $x(xy+3)$
8. $2x(4x^2-x+2)$
9. $-6x^2(x-3y)$, or $6x^2(-x+3y)$
10. $10(x^4y^2-5x^3y+7)$
11. $3a(2a-13b)$
12. $4a^3bc(3b+a^2c)$
13. $11x^2y(2x^2+5y)$
14. $4(2x^2+3x+5)$
15. $5(f^3-3f+5)$
16. $5a^2b(6ab+4+7a^2b)$
17. $(4r+11)(4r-11)$
18. $(a+3)(a-3)$
19. $(xy+7)(xy-7)$
20. $(b+10)(b-10)$
21. $(r+s)(r-s)$
22. $(6b+10)(6b-10)$
23. $(a^3+b^3)(a^3-b^3)$
24. $(y+8)(y-8)$
25. $(2x+1)(2x-1)$
26. $(5x+2y)(5x-2y)$
27. prime—can't be factored (because it is the sum of squares)
28. $(x^2+4)(x^2-4)$
29. $(b^5-6)(b^5+6)$
30. $(4a-5b)(4a+5b)$
31. $(x+3)(x+6)$
32. $(x+4)(x+2)$
33. $(x-1)(x-3)$
34. $(x+4)(x+3)$
35. $(x-8)(x-2)$
36. $(x-14)(x-1)$
37. $(x+5)(x+4)$
38. $(x-10)(x-2)$
39. $(x-4)(x-5)$
40. $(x-6)(x-5)$
41. $(a+4)(a-5)$
42. $(r-9)(r+2)$
43. $(x+1)(x-7)$
44. $(x+5)(x-2)$
45. $(x-5)(x+2)$
46. $(x-8)(x+1)$
47. $(x+8)(x-2)$
48. $(x-7)(x+3)$
49. $(x+6)(x-5)$
50. $(x-6)(x+3)$

## ANSWERS: TOPIC F

(pg 45-46)

1. $(2x+1)(x+3)$
2. $(5x-2)(x+3)$
3. $(7x-4)(x+1)$
4. $(4x-1)(x-5)$
5. $(6x+7)(x-1)$
6. $(8x+5)(x-1)$
7. $(4x-5)(2x+1)$
8. $(3x-1)(3x+2)$
9. $(3x-1)(2x-5)$
10. $(5x+3)(x-2)$
11. $(2x+1)(x-2)$
12. $(3x-3)(x+4)$
13. $(5x+1)(x-2)$
14. $(4x-1)(x+3)$
15. $(2x+2)(2x+2)$
16. $(5x+2)(x+2)$
17. $(6x+6)(6x+6)$
18. $(3x+3)(3x+3)$
19. $(7x+1)(x-7)$
20. $(4x+1)(x-2)$
21. $(4x-4)(x+4)$
22. $(4x+4)(x+4)$
23. $(7x-2)(7x+2)$
24. $(c-6)(c-5)$
25. $(a+b)(a-b)$
26. $(b+6)(b-3)$
27. $(n-7)(n+5)$
28. $6(4x+1)$
29. $(3x-10)(3x+10)$
30. $(5x-3)(x+2)$
31. $(3x+1)(2x-3)$
32. $(r-8)(r+3)$
33. $(f+9)(f-4)$
34. $3xy(x^2+2xy-3y^2)$
35. $(5x+1)(3x-2)$
36. $(5a+8)(5a-8)$
37. $6x^3y(8y^2-3x)$
38. $(2x+1)(3x+11)$
39. $5mn(2+mn^2-4m^2n)$
40. prime—can't be factored
41. $3(x+3)(x-3)$
42. $4(x+4)(x-4)$
43. $2(x+3)(x+3)$
44. $2(x-1)(x+3)$
45. $3(x+5)(x+2)$
46. $4(x^3+5)(x^3-5)$
47. $3(3x+5y)(3x-5y)$
48. $3(2x-7)(2x+1)$
49. $3(x-2)(x-6)$
50. $(x^2+9)(x+3)(x-3)$

## ANSWERS: TOPIC F

(pg 48)

1. –6,4
2. 3,–2
3. –10,–5
4. 7,1
5. –9,–3
6. –12,4
7. –9,2
8. –9,5
9. –3,3
10. –8,–3
11. 9,–1
12. 1 (double root)
13. 9,–5
14. $\frac{7}{2} = 3\frac{1}{2}, -1$
15. $-\frac{1}{3}, 7$
16. –2,1
17. –5,3
18. 2,–5
19. $-\frac{3}{4}, \frac{3}{4}$
20. $\frac{2}{5}, -6$

**(pg 49-50)**

1. 4,8,1
2. 1,–4,10
3. 2,3,0
4. 6,0,–8
5. 4,0,–7
6. 3,0,0
7. 2,3,–4
8. 9,–7,2
9. 0,–2
10. 0,4
11. 5,–5
12. –5,1
13. –7,3
14. –5,–6
15. $-\frac{3}{2}, -1$
16. $\frac{1}{2}, \frac{1}{3}$
17. $\frac{3 \pm \sqrt{5}}{2}$
18. $\frac{1 \pm \sqrt{11}}{2}$
19. $\frac{5 \pm \sqrt{17}}{2}$
20. $\frac{7 \pm \sqrt{61}}{2}$
21. $\frac{3 \pm \sqrt{29}}{2}$
22. $\frac{5 \pm \sqrt{41}}{8}$
23. $\frac{-11 \pm 5\sqrt{5}}{2}$
24. $\frac{-5 \pm \sqrt{37}}{2}$
25. $\frac{-3 \pm \sqrt{33}}{4}$
26. $4 \pm \sqrt{2}$
27. $\frac{1 \pm \sqrt{5}}{2}$
28. $-2 \pm \sqrt{7}$
29. $\frac{-1 \pm \sqrt{41}}{10}$
30. $1 \pm 2\sqrt{2}$