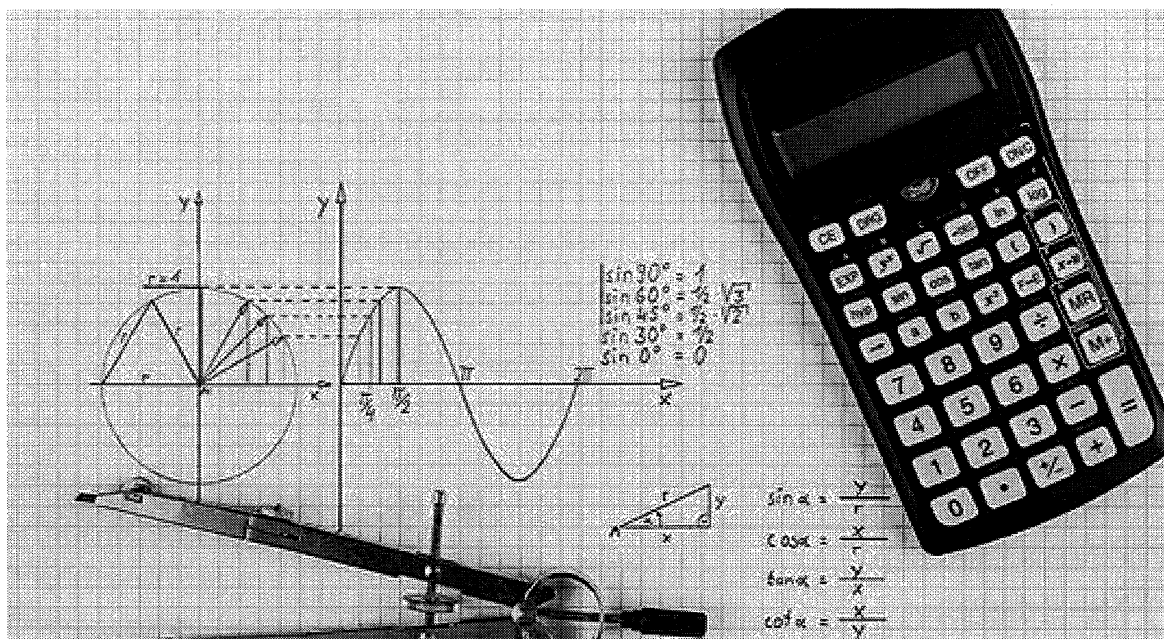


# SUMMER PACKET

## PREPARING FOR PRECALCULUS



## SUFFIELD PUBLIC SCHOOLS

Supply List	Video/ App Resources	Pre-Requisites
<ul style="list-style-type: none"> <li>Pencils</li> <li>Colored Pencils</li> <li>Graph Paper</li> <li>Graphing Calculator (TI 84 – Preferred))</li> <li>Ruler</li> </ul>	<ul style="list-style-type: none"> <li><a href="https://www.khanacademy.org/">https://www.khanacademy.org/</a></li> <li><a href="https://mathantics.com/">https://mathantics.com/</a></li> <li><a href="https://webmath.com/">https://webmath.com/</a></li> <li><a href="https://www.mathplanet.com/">https://www.mathplanet.com/</a></li> <li><a href="http://www.math.com/">http://www.math.com/</a></li> <li><a href="https://www.ixl.com/math/algebra-1">https://www.ixl.com/math/algebra-1</a></li> <li><a href="https://www.purplemath.com">https://www.purplemath.com</a></li> </ul>	<ul style="list-style-type: none"> <li>Adding and Subtracting Polynomials</li> <li>Simplifying Algebraic Expressions using Properties of exponents.</li> <li>Solving Linear Equations</li> <li>Graphing Linear Functions</li> <li>Writing Equations of Lines</li> <li>Solving and Graphing Systems of Linear Equations</li> <li>Solving and Graphing Absolute Value Equations</li> <li>Factoring</li> <li>Solving Quadratic Equations</li> <li>Graphing Quadratic Functions</li> <li>Solving Rational Equations</li> <li>Graphing Rational Functions</li> <li>Solving Radical Equations</li> <li>Graphing Radical Functions</li> </ul>

To Any student entering Precalculus in the fall –

To ensure your success in Precalculus, you need to be proficient in the foundational skills and concepts that you have learned over the course of Algebra 1, Geometry, and Algebra 2 as this will enable you to easily grasp the new concepts that will be covered, and apply them to solve mathematical and real-life problems. We highly recommend that you take time to review and solidify your knowledge of these topics over the course of the summer.

Attached is a summer review packet which has practice problems for several of the major topics that you have studied in Algebra 1, Geometry, and Algebra 2. There are some review notes and also links to on-line tutorials for these topics. Please take the time to work through the packet and use the resources that are provided for review when you do not remember how to solve a problem. You can certainly use other on-line resources that you can find for help, or ask relatives and/or friends for help. Please remember, that even though the completion of this packet is not required, this is an opportunity for you to review and make sure **you** are ready for Precalculus in the fall.

Learning Mathematics is like building a house; if your foundation is weak, you can't build high, and if there are gaps or floors that are not complete, the next floor is nearly impossible to build. **Our wish and hope for all our math students is not just to get by, but to excel in mathematics, and to ultimately enjoy and appreciate the beauty and power of mathematics** – a subject we love and hope we can get you excited to continue learning. We are providing you with resources to do as much preparation and solidification of the basic skills needed to engage with the course material fluently and confidently.

We wish you a happy summer, and hope you return in the fall with high expectations for yourself, the willingness to ask questions and reach out for help when you are struggling, and with the confidence that you can be successful.

## Simplifying Polynomial Expressions

Objectives: The Student will be able to –

- Apply the appropriate arithmetic operations and algebraic properties needed to simplify an algebraic expression.
- Simplify polynomial expressions using addition and subtract

### Polynomials:

To add/subtract polynomials, combine like terms.

EX:  $8x - 3y + 6 - (6y + 4x - 9)$  *Distribute the negative through the parentheses.*

$$= 8x - 3y + 6 - 6y - 4x + 9$$

*Combine like terms with similar variables.*

$$= 8x - 4x - 3y - 6y + 6 + 9$$
$$= 4x - 9y + 15$$

Simplifying Polynomials:

<https://www.youtube.com/watch?v=ZvL9aDGNHqA>

<https://www.khanacademy.org/math/algebra-home/alg-polynomials/alg-introduction-to-polynomials/v/simplify-a-polynomial>

<https://www.youtube.com/watch?v=ZvL9aDGNHqA>

Multiplying Polynomials:

<https://www.youtube.com/watch?v=fGThIRpWEE4>

<https://www.youtube.com/watch?v=ZvL9aDGNHqA>

Simplify:

1.  $8x - 9y + 16x + 12y$   $24x + 3y$

2.  $14y + 22 - 15y^2 + 23y$   $-15y^2 + 37y + 22$

3.  $5n - (3 - 4n)$   $9n - 3$

4.  $-2(11x - 3)$   $-22x + 6$

5.  $3(8z - 4w) + 2(10z - 6w)$   $44z - 24w$

6.  $5(3x - 4) - 2(4x - 6)$   $7x - 8$

Multiply the Following:

1.  $2x^2(4x^2 - 3x - 5)$   
 $8x^4 - 6x^3 - 10x^2$

2.  $(x + 5)(x - 8)$   
 $x^2 - 3x - 40$

3.  $(x + 4)(x - 4)$   
 $x^2 - 16$

4.  $(2x - 5)(3x + 2)$   
 $6x^2 - 11x - 10$

5.  $(3x + 4)^2$   
 $9x^2 + 24x + 16$

6.  $(2x - 5)(3x^2 + 4x - 2)$   
 $6x^3 - 7x^2 - 24x + 10$

## Solving Linear Equations

Objectives: The Student will be able to –

- Solve one and two step equations.
- Solve equations with variables on both sides.
- Solve Multi-step equations that need to be simplified first
- Solve Literal Equations for a specific variable.

Solving Linear Equations (One Step):

<https://www.khanacademy.org/math/algebra-home/alg-basic-eq-ineq/alg-old-school-equations/v/algebra-linear-equations-1>

Solving Linear Equations (Two Step):

<https://www.khanacademy.org/math/algebra-home/alg-basic-eq-ineq/alg-old-school-equations/v/algebra-linear-equations-2>

Solving Linear Equations (Multi - Step):

<https://www.khanacademy.org/math/algebra-home/alg-basic-eq-ineq/alg-old-school-equations/v/algebra-linear-equations-3>

<https://www.khanacademy.org/math/algebra-home/alg-basic-eq-ineq/alg-old-school-equations/v/algebra-linear-equations-4>

Solving Literal Equations:

<https://www.khanacademy.org/math/algebra-home/alg-basic-eq-ineq/alg-old-school-equations/v/solving-for-a-variable>

<https://www.youtube.com/watch?v=gqSfw2gmMsg>

Solve each equation. You must show all work.

1.  $5x - 2 = 33$   $x = 7$

2.  $140 = 4x + 36$   $x = 26$

3.  $8(3x - 4) = 196$   $x = \frac{19}{2}$

4.  $45x - 720 + 15x = 60$   $x = 13$

5.  $132 = 4(12x - 9)$   $x = \frac{7}{2}$

6.  $154 + 7x - 68 = 198$   $x = 16$

7.  $-5(3x - 8) + 6x = -131$   $x = 19$

8.  $-7x - 10 = 18 + 3x$   $x = -\frac{14}{5}$

9.  $12x + 8 - 15 = -2(3x - 82)$   $x = \frac{19}{2}$

10.  $-(12x - 6) = 2x - 14$   $x = \frac{10}{7}$

Solve the Literal Equations for the indicated variable.

11.  $Y + M = Z$ , for  $M$

$$M = Z - Y$$

12.  $2x - 3y = 9$ , for  $y$

$$y = \frac{9 - 2x}{-3} \text{ or } y = -3 + \frac{2}{3}x$$

13.  $2xy + w = 10$ , for  $x$

$$x = \frac{10 - w}{2y}$$

14.  $4x + y - 5h = 10y + k$ , for  $x$

$$x = \frac{9y + k + 5h}{4}$$

## Solving Absolute Value Equations

Objectives: The Student will be able to –

- Solve absolute value equations.

Solving Absolute Value Equations:

<https://www.youtube.com/watch?v=cHbhzQVd7Y>

<https://www.khanacademy.org/math/algebra-home/alg-absolute-value/alg-absolute-value-equations/v/absolute-value-equations>

Solve the following:

1.  $|2x| = 12$

2.  $|3x + 4| = 8$

3.  $2|3x - 2| + 5 = 15$

4.  $|5x + 2| = -7$

1.  $|2x| = 12$   
 $2x = 12$     $2x = -12$   
 $x = 6$     $x = -6$

2.  $|3x + 4| = 8$   
 $3x + 4 = 8$     $3x + 4 = -8$   
 $3x = 4$     $3x = -12$   
 $x = \frac{4}{3}$     $x = -4$

3.  $2|3x - 2| + 5 = 15$   
 $2|3x - 2| = 10$   
 $|3x - 2| = 5$   
 $3x - 2 = 5$     $3x - 2 = -5$   
 $3x = 7$     $3x = -3$   
 $x = \frac{7}{3}$     $x = -1$

4.  $|5x + 2| = -7$  //  
No Sol'n.

## Properties of Exponents

Objectives: The Student will be able to –

- Simplify expressions using the properties of exponents

### Exponents:

#### Recall the following rules of exponents:

1.  $a^1 = a$  Any number raised to the power of one equals itself.
2.  $1^a = 1$  One raised to any power is one.
3.  $a^0 = 1$  Any nonzero number raised to the power of zero is one.
4.  $a^m \cdot a^n = a^{m+n}$  When multiplying two powers that have the same base, add the exponents.
5.  $\frac{a^m}{a^n} = a^{m-n}$  When dividing two powers with the same base, subtract the exponents.
6.  $(a^m)^n = a^{mn}$  When a power is raised to another power, multiply the exponents.
7.  $a^{-n} = \frac{1}{a^n}$  and  $\frac{1}{a^{-n}} = a^n$  Any nonzero number raised to a negative power equals its reciprocal raised to the opposite positive power.

Properties of Exponents:

<https://www.youtube.com/watch?v=tePk4uUBrsU>

<https://www.youtube.com/watch?v=etMK3xViMAc>

<https://www.khanacademy.org/math/cc-eighth-grade-math/cc-8th-numbers-operations/cc-8th-exponent-properties/v/exponent-properties-involving-products>

Simplify each expression. Express answers with positive exponents.

$$1. x^5 \cdot x \cdot x^2 = x^8$$

$$2. \frac{m^{15}}{m^3} = m^{12}$$

$$3. (k^4)^5 = k^{20}$$

$$4. y^0 = 1$$

$$5. (p^4 n^2) \cdot (p^7 n^5) = p^{11} n^7$$

$$6. \frac{45y^3 z^{10}}{5y^3 z} = 9z^9$$

$$7. (4h^5 k^3) \cdot (15h^3 k^2) = 60h^8 k^5$$

$$8. \frac{12a^4 b^6}{36ab^2 c} = \frac{a^3 b^4}{3c}$$

$$9. (3m^2 y)^3 = 27m^6 y^3$$

$$10. (3x^4 y^2)^2 \cdot (2x^3 y^5 m^3)^2 = 36x^{14} y^{14} m^6$$

$$11. 4x^2 (3x^5 y^2)^0 = 4x^2$$

## Slopes of Lines & Equations of Lines

Objectives: The Student will be able to –

- Identify and calculate the slope of a line.
- Write the equation of a line in Point-Slope Form.
- Write the equation of a line in Slope-Intercept Form.
- Write the equation of a line in Standard Form.
- Write the equations of parallel and perpendicular lines.

Equations of Lines:

Slope-intercept form:  $y = mx + b$

Vertical line:  $x = c$  (slope is undefined)

Point-slope form:  $y - y_1 = m(x - x_1)$

Horizontal line:  $y = c$  (slope is zero)

Standard Form:  $Ax + By = C$

Slope:  $m = \frac{y_2 - y_1}{x_2 - x_1}$

Slopes:

<https://www.khanacademy.org/math/algebra/x2f8bb11595b61c86:linear-equations-graphs/x2f8bb11595b61c86:slope/v/slope-of-a-line-2>

[https://www.youtube.com/watch?v=ld\\_UqMLAXzY](https://www.youtube.com/watch?v=ld_UqMLAXzY)

Equations of Lines:

<https://www.youtube.com/watch?v=lzqTD0JWwhY>

<https://www.youtube.com/watch?v=xIzDjC11udE>

<https://www.youtube.com/watch?v=4u7MZvFqFJo>

Equations of Parallel and Perpendicular Lines:

<https://www.youtube.com/watch?v=LTb2-LE7StE>

<https://www.khanacademy.org/math/geometry/hs-geo-analytic-geometry/hs-geo-parallel-perpendicular-eq/v/perpendicular-lines-2>

Calculate the Slopes of the lines with following pairs of points.

1.  $(-1, 4)$  and  $(1, -2)$   $m = -3$
2.  $(3, 5)$  and  $(-3, 1)$   $m = \frac{2}{3}$
3.  $(2, 5)$  and  $(5, 10)$   $m = \frac{5}{3}$
4.  $(2, -5)$  and  $(-4, -5)$   $m = 0$
5.  $(3, -2)$  and  $(3, 7)$   $m = \frac{9}{0} = \text{undefined}$

Write the equation of the lines with given slope and a point on the line.

Write the equations of the lines in:

(a) Point - Slope Form.

(b) Slope - Intercept Form

(c) Standard Form

1. Slope =  $-3$  Point :  $(2, 3)$

2. Slope =  $5$  Point :  $(-2, 4)$

3. Slope =  $\frac{2}{3}$  Point :  $(3, -5)$

①  $y - 3 = -3(x - 2)$

(b)  $y = -3x + 9$

(c)  $3x + y = 9$

②  $y - 4 = 5(x + 2)$

(b)  $y = 5x + 14$

(c)  $-5x + y = 14$

③  $y + 5 = \frac{2}{3}(x - 3)$

(b)  $y = \frac{2}{3}x - 7$

(c)  $-2x + 3y = -21$

Write the equation of the lines through the given points on the line.

Write the equations of the lines in:

(a) Point - Slope Form.

(b) Slope - Intercept Form

(c) Standard Form

1. Points :  $(2, -3)$  and :  $(5, -12)$

2. Points :  $(-4, -5)$  and :  $(2, 19)$

3. Points :  $(5, -1)$  and :  $(10, 2)$

① slope =  $-3$

(a)  $y + 3 = -3(x - 2)$

or

$y + 12 = -3(x - 5)$

(b)  $y = -3x + 3$

(c)  $3x + y = 3$

② slope =  $4$

(a)  $y + 5 = 4(x + 4)$

or

$y - 19 = 4(x - 2)$

(b)  $y = 4x + 11$

(c)  $-4x + y = 11$

③ slope =  $\frac{3}{5}$

(a)  $y + 1 = \frac{3}{5}(x - 5)$

or

$y - 2 = \frac{3}{5}(x - 10)$

(b)  $y = \frac{3}{5}x - 4$

(c)  $-3x + 5y = -20$

4. Rewrite the equation into Slope-Intercept Form :  $8x - 2y = 24$ .  
Identify the slope and y-intercept.
5. Rewrite the equation into Slope-Intercept Form :  $3x + 5y = 30$ .  
Identify the slope and y-intercept.
6. Write the equation of the line through the point  $(3, 8)$   
and parallel to the line  $6x + 3y = 30$ .
7. Write the equation of the line through the point  $(-4, 5)$   
and perpendicular to the line  $3x - 9y = 18$ .

Equations of Parallel and Perpendicular Lines:

<https://www.youtube.com/watch?v=LTb2-LE7StE>

<https://www.khanacademy.org/math/geometry/hs-geo-analytic-geometry/hs-geo-parallel-perpendicular-eq/v/perpendicular-lines-2>

$$\begin{aligned} \textcircled{4} \quad 8x - 2y &= 24 & \text{slope} &= 4 \\ -2y &= -8x + 24 & y\text{-int} &= (0, -12) \\ y &= 4x - 12 \end{aligned}$$

$$\begin{aligned} \textcircled{5} \quad 3x + 5y &= 30 & \text{slope} &= -3/5 \\ 5y &= -3x + 30 & y\text{-int} &= (0, 6) \\ y &= -\frac{3}{5}x + 6 \end{aligned}$$

$$\begin{aligned} \textcircled{6} \quad 6x + 3y &= 30 & \text{slope} &= -2 \\ 3y &= -6x + 30 & \Rightarrow \text{slope} &= -2 \\ y &= -2x + 10 & \text{Point} &= (3, 8) \end{aligned}$$

Parallel lines have same slope

Point-slope Form

$$y - 8 = -2(x - 3)$$

$$\begin{aligned} \textcircled{7} \quad 3x - 9y &= 18 & \text{slope} &= \frac{1}{3} \Rightarrow \text{slope} &= -3 \\ -9y &= -3x + 18 & \text{Point} &= (-4, 5) & y - 5 &= -3(x + 4) \\ y &= \frac{1}{3}x - 2 & \text{Perpendicular lines have slopes} & & \\ & & \text{that are opposite reciprocals} & & \end{aligned}$$

## Graphing Linear Functions

Objectives: The Student will be able to –

- Graph lines given in Slope – Intercept form.
- Graph lines given in Point – Slope form.
- Graph lines given in Standard Form.
- Identify the equation of a line from its graph.

Graphing Linear Functions

<https://www.khanacademy.org/math/algebra/x2f8bb11595b61c86:forms-of-linear-equations/x2f8bb11595b61c86:standard-form/v/plotting-x-y-relationships>

<https://www.youtube.com/watch?v=BtcKotD6Ni8>

[https://www.youtube.com/watch?v=Ft2\\_QtXAnh8](https://www.youtube.com/watch?v=Ft2_QtXAnh8)

Graphing Linear Functions:

<https://www.youtube.com/watch?v=zihUOJgDkv0>

<https://www.khanacademy.org/math/in-in-grade-9-ncert/xfd53e0255cd302f8:linear-equations-in-two-variables/xfd53e0255cd302f8:graph-of-a-linear-equation-in-two-variables/v/graphs-of-linear-equations>

[https://www.youtube.com/watch?v=UgtMbCI4G\\_I](https://www.youtube.com/watch?v=UgtMbCI4G_I)

## Graphing Linear Functions

Objectives: The Student will be able to –

- Graph lines given in Slope – Intercept form.
- Graph lines given in Point – Slope form.
- Graph lines given in Standard Form.
- Identify the equation of a line from its graph.

Graphing Linear Functions:

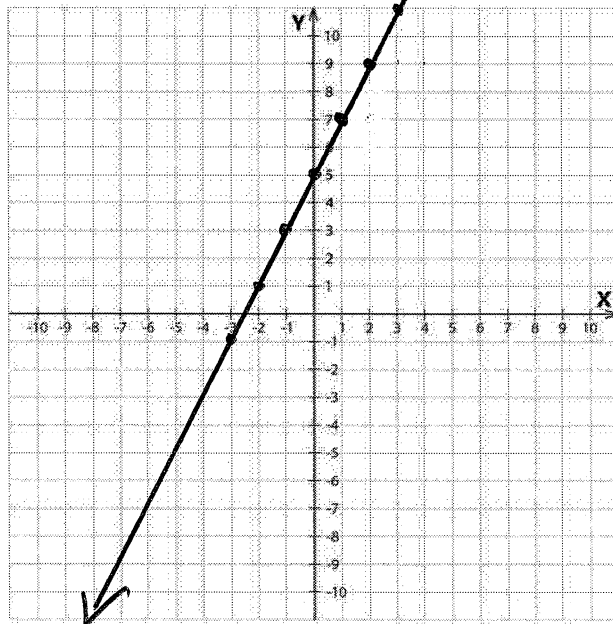
<https://www.youtube.com/watch?v=zihUOJgDkv0>

<https://www.khanacademy.org/math/in-in-grade-9-ncert/xfd53e0255cd302f8:linear-equations-in-two-variables/xfd53e0255cd302f8:graph-of-a-linear-equation-in-two-variables/v/graphs-of-linear-equations>

[https://www.youtube.com/watch?v=UgtMbCl4G\\_I](https://www.youtube.com/watch?v=UgtMbCl4G_I)

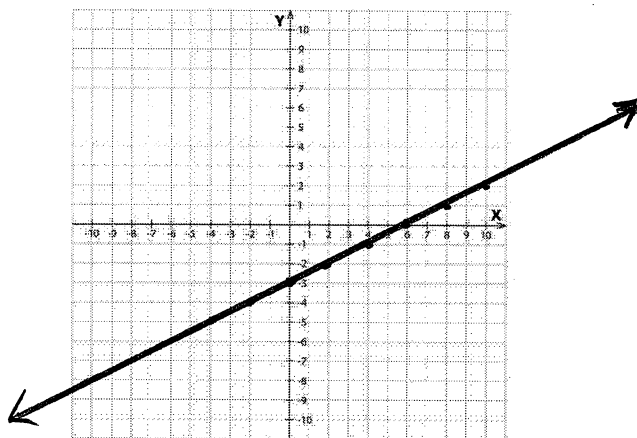
1. Graph the line :  $y = 2x + 5$

Slope = 2 Y-intercept = (0, 5)



2. Graph the line :  $y = \frac{1}{2}x - 3$

Slope =  $\frac{1}{2}$  Y-intercept =  $(0, -3)$



y-int  $(0, -3)$

x-int (y=0)

$$0 = \frac{1}{2}x - 3$$

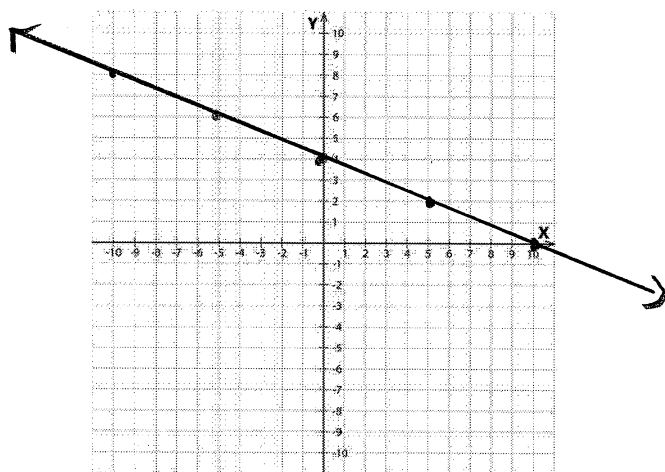
$$3 = \frac{1}{2}x$$

$$6 = x$$

$(6, 0)$

3. Graph the line :  $y = -\frac{2}{5}x + 4$

Slope =  $-\frac{2}{5}$  Y-intercept =  $(0, 4)$



y-int:  $(0, 4)$

x-int (y=0)

$$0 = -\frac{2}{5}x + 4$$

$$-4 = -\frac{2}{5}x$$

$$-20 = -2x$$

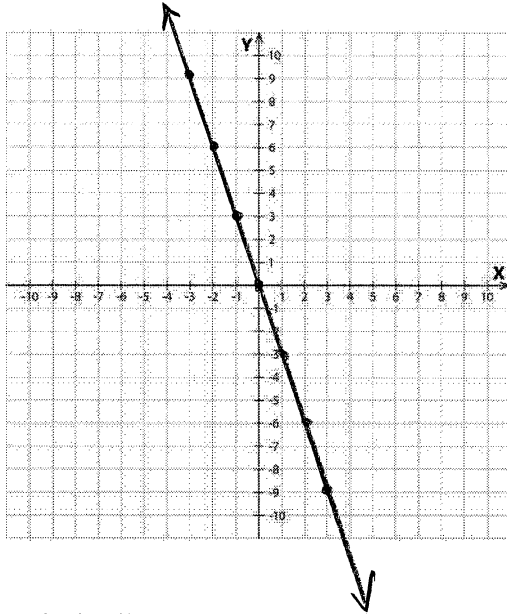
$$10 = x$$

$(10, 0)$

4. Graph the line :  $y = -3x$

Slope = -3 Y-intercept = (0,0)

y-int (0,0)  
x-int (0,0)

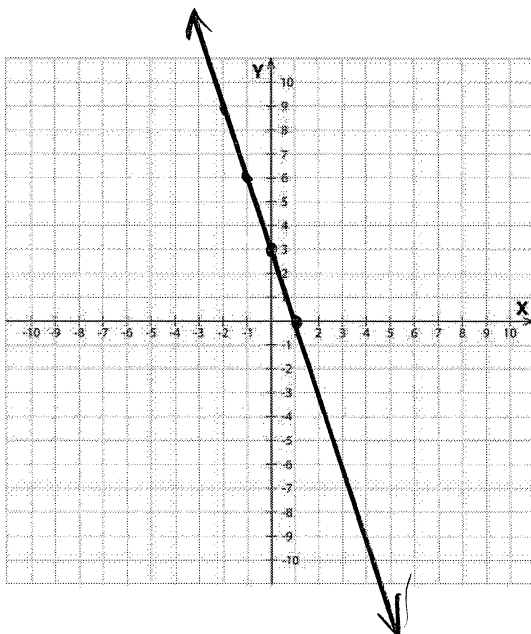


5. Graph the line :  $3x + y = 3$

X-intercept = (1,0) Y-intercept = (0,3)

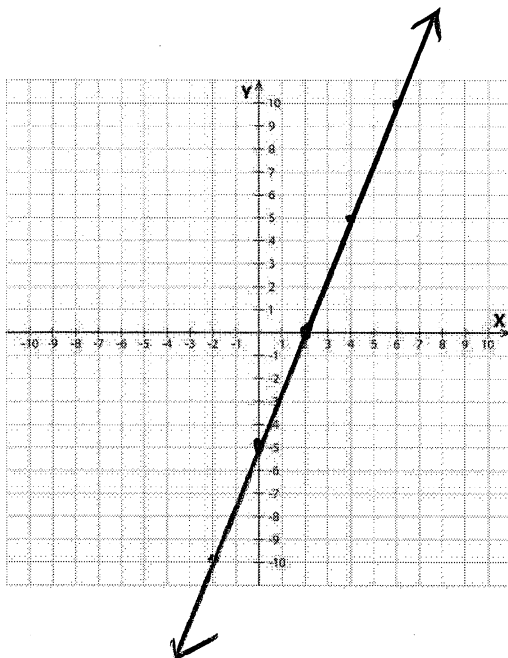
y-int (x=0)  
 $3x + y = 3$   
 $3(0) + y = 3$   
 $y = 3$   
(0,3)

x-int (y=0)  
 $3x + y = 3$   
 $3x + 0 = 3$   
 $3x = 3$   
 $x = 1$   
(1,0)



6. Graph the line :  $5x - 2y = 10$

X-intercept =  $(2, 0)$  Y-intercept =  $(0, -5)$

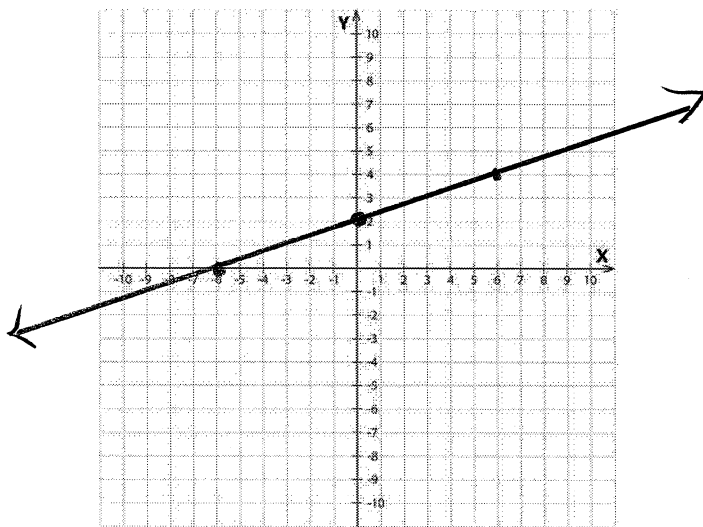


$$\begin{aligned} \text{x-int (y=0)} \\ 5x - 2y &= 10 \\ 5x - 2(0) &= 10 \\ 5x &= 10 \\ x &= 2 \\ (2, 0) \end{aligned}$$

$$\begin{aligned} \text{y-int (x=0)} \\ 5x - 2y &= 10 \\ 5(0) - 2y &= 10 \\ -2y &= 10 \\ y &= -5 \\ (0, -5) \end{aligned}$$

7. Graph the line :  $-2x + 6y = 12$

X-intercept = \_\_\_\_\_ Y-intercept = \_\_\_\_\_



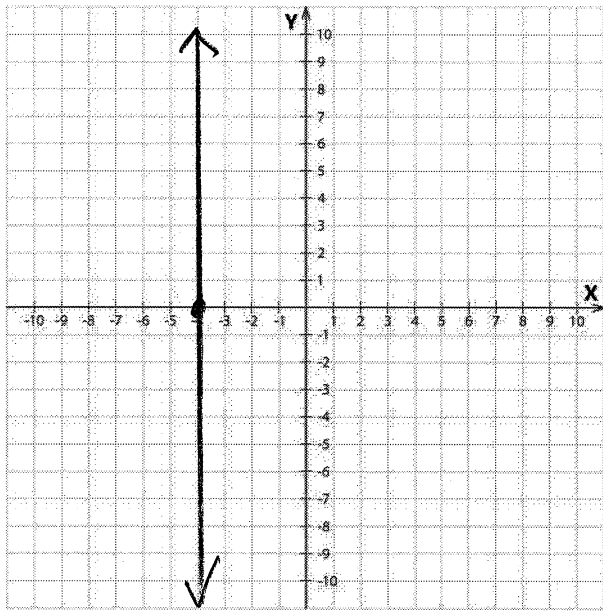
$$\begin{aligned} \text{x-int (y=0)} \\ -2x + 6y &= 12 \\ -2x + 6(0) &= 12 \\ -2x &= 12 \\ x &= -6 \\ (-6, 0) \end{aligned}$$

$$\begin{aligned} \text{y-int (x=0)} \\ -2x + 6y &= 12 \\ -2(0) + 6y &= 12 \\ 6y &= 12 \\ y &= 2 \\ (0, 2) \end{aligned}$$

8. Graph the line :  $x = -4$

X-intercept =  $(-4, 0)$  Y-intercept =  $\emptyset$

$x = \text{constant} \rightarrow \text{Vertical Line}$



## G. Graph Inequalities in two variables

Objectives: The Student will be able to –

- Graph inequalities in two variables

Graphing Inequalities in two variables:

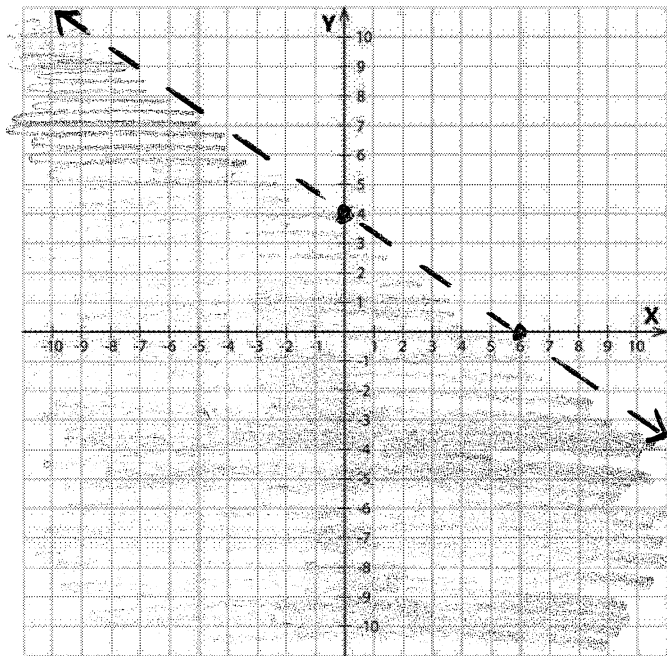
<https://www.khanacademy.org/math/algebra/x2f8bb11595b61c86:inequalities-systems->

[graphs/x2f8bb11595b61c86:graphing-two-variable-inequalities/v/graphing-linear-inequalities-in-two-variables-example-2](https://www.khanacademy.org/math/algebra/x2f8bb11595b61c86:inequalities-systems-graphs/x2f8bb11595b61c86:graphing-two-variable-inequalities/v/graphing-linear-inequalities-in-two-variables-example-2)

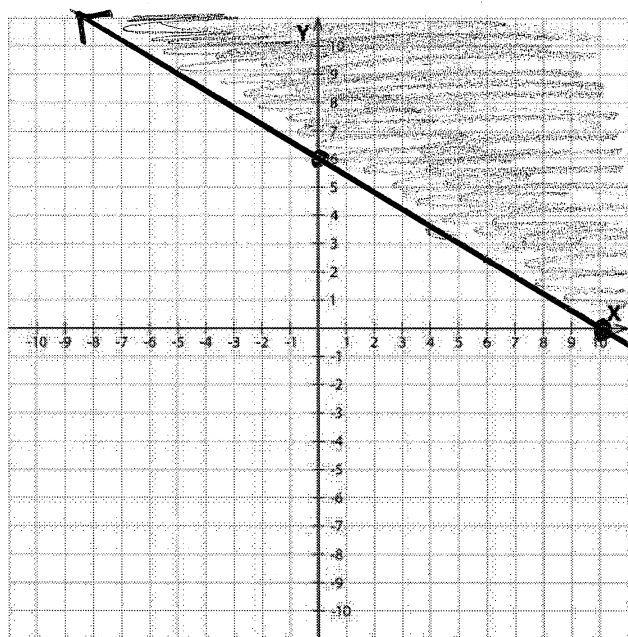
1. Graph the inequality:  $2x + 3y < 12$

$x$ -intercept =  $(6, 0)$

$y$ -intercept =  $(0, 4)$

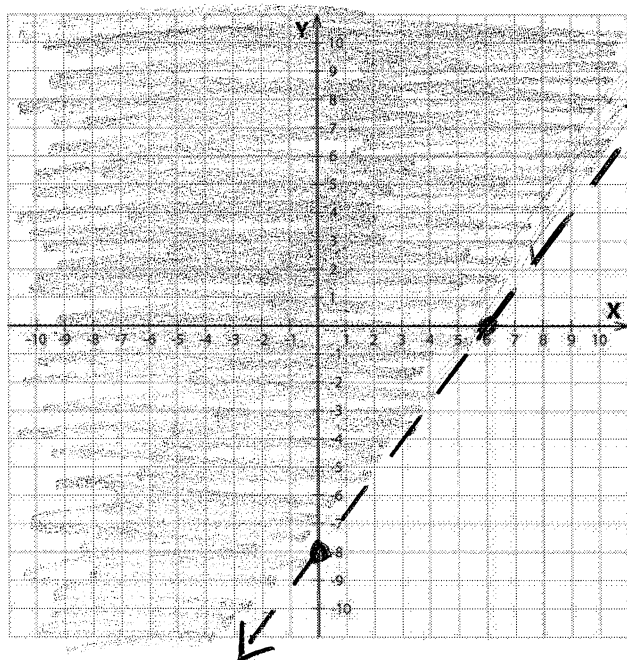


2. Graph the inequality :  $3x + 5y \geq 30$



x-intercept (10, 0)  
y-intercept (0, 6)

3. Graph the inequality :  $4x - 3y < 24$



x-intercept (6, 0)  
y-intercept (0, -8)

# Solving Systems of Equations

Objectives: The Student will be able to –

- Solve systems of linear equations by graphing
- Solve systems of linear equations by elimination
- Solve systems of linear equations by substitution

## Systems of Equations:

$$\begin{cases} 3x + y = 6 \\ 2x - 2y = 4 \end{cases}$$

### **Substitution:**

Solve 1 equation for 1 variable

Rearrange.

Plug into 2<sup>nd</sup> equation.

Solve for the other variable.

Then plug answer back into an original equation to solve for the 2<sup>nd</sup> variable.

$$y = 6 - 3x$$

Solve 1<sup>st</sup> equation for y

$$2x - 2(6 - 3x) = 4$$

Plug into 2<sup>nd</sup> equation

$$2x - 12 + 6x = 4$$

Distribute

$$8x = 16 \text{ and } x = 2$$

Simplify

### **Elimination:**

Find opposite coefficients for 1 variable

Multiply equation(s) by constant(s).

Add equations together (lose 1 variable)

Solve for variable.

$$6x + 2y = 12$$

Multiply 1<sup>st</sup> equation by 2

$$2x - 2y = 4$$

coefficients of y are opposite

$$8x = 16$$

Add

$$x = 2$$

Simplify.

Plug x=2 back into the original equation

$$\begin{aligned} 6 + y &= 6 \\ y &= 0 \end{aligned}$$

Solving Systems by graphing:

<https://www.khanacademy.org/math/cc-eighth-grade-math/cc-8th-systems-topic/cc-8th-systems-graphically/v/solving-linear-systems-by-graphing>

<https://www.youtube.com/watch?v=Pd4hwS8qHms>

Solving Systems by Elimination:

<https://www.khanacademy.org/math/algebra-home/alg-system-of-equations/alg-equivalent-systems-of-equations/v/solving-systems-of-equations-by-elimination>

<https://www.youtube.com/watch?v=HL2fDIOMLJO>

Solving Systems by Substitution:

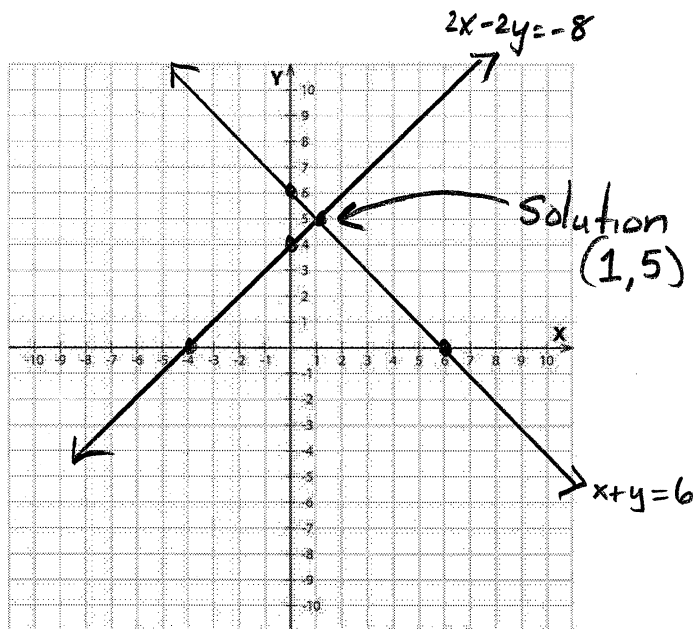
<https://www.khanacademy.org/math/algebra-home/alg-system-of-equations/alg-solving-systems-of-equations-with-substitution/v/solving-linear-systems-by-substitution>

<https://www.youtube.com/watch?v=cblHUeq3bkE>

1. Solve the system of equations by graphing:

$$x + y = 6$$

$$2x - 2y = -8$$



$$x + y = 6$$

$$x\text{-intercept } (6, 0)$$

$$y\text{-intercept } (0, 6)$$

$$2x - 2y = -8$$

$$x\text{-intercept } (-4, 0)$$

$$y\text{-intercept } (0, 4)$$

2. Solve the system of linear equations by elimination:

$$3x + 2y = -2$$

$$4x - 3y = 20$$

$$(2, -4)$$

$$\begin{array}{r} 3[3x + 2y = -2] \quad 9x + 6y = -6 \\ 2[4x - 3y = 20] \quad 8x - 6y = 40 \\ \hline 17x = 34 \end{array}$$

$$17x = 34$$

$$x = 2$$

3. Solve the system of linear equations by substitution:

$$5x - 3y = 2$$

$$y = 3x + 6$$

$$y = 3x + 6$$

$$5x - 3y = 2$$

$$5x - 3(3x + 6) = 2$$

$$5x - 9x - 18 = 2$$

$$-4x - 18 = 2$$

$$-4x = 20$$

$$x = -5$$

$$y = 3x + 6$$

$$y = 3(-5) + 6$$

$$y = -15 + 6$$

$$y = -9$$

$$(-5, -9)$$

$$3x + 2y = -2$$

$$3(2) + 2y = -2$$

$$6 + 2y = -2$$

$$2y = -8$$

$$y = -4$$

# Factoring

## Factoring:

Follow these steps in order to factor polynomials.

**STEP 1:** Look for a GCF in ALL of the terms.

**STEP 2:** How many terms does the polynomial have?

**2 Terms** a) is it the difference of two squares?  $a^2 - b^2 = (a + b)(a - b)$

**EX:**  $x^2 - 25 = (x + 5)(x - 5)$

b) Is it the sum or difference of two cubes?  $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$   
 $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

**EX:**  $m^3 + 64 = (m + 4)(m^2 - 4m + 16)$   
 $p^3 - 125 = (p - 5)(p^2 + 5p + 25)$

**3 Terms**

**EX:**

$$x^2 + bx + c = (x + \_)(x + \_)$$

$$x^2 + 7x + 12 = (x + 3)(x + 4)$$

$$x^2 - bx - c = (x - \_)(x - \_)$$

$$x^2 - 5x + 4 = (x - 1)(x - 4)$$

$$x^2 + bx - c = (x - \_)(x + \_)$$

$$x^2 + 6x - 16 = (x - 2)(x + 8)$$

$$x^2 - bx - c = (x - \_)(x + \_)$$

$$x^2 - 2x - 24 = (x - 6)(x + 4)$$

**4 Terms**—Factor by Grouping

- Pair up first two terms and last two terms.
- Factor out GCF of each pair of numbers.
- Factor out front parentheses that the terms have in common.
- Put leftover terms in parentheses.

$$\begin{aligned} \text{Ex: } x^3 + 3x^2 + 9x + 27 &= (x^3 + 3x^2) + (9x + 27) \\ &= x^2(x + 3) + 9(x + 3) \\ &= (x + 3)(x^2 + 9) \end{aligned}$$

Factoring Polynomials

<https://www.youtube.com/watch?v=U6FndtdgpcA>

<https://www.youtube.com/watch?v=mXvt9OumKH8>

<https://www.khanacademy.org/math/algebra/x2f8bb11595b61c86:quadratics-multiplying-factoring/x2f8bb11595b61c86:factor-quadratics-intro/v/factor-completely-common-factor>

Factor Completely:

1.  $x^2 + 4x - 12$

$$(x+6)(x-2)$$

2.  $x^2 + 8x + 15$

$$(x+3)(x+5)$$

3.  $x^2 - 3x - 40$

$$(x-8)(x+5)$$

4.  $4x^2 - 25$

$$(2x+5)(2x-5)$$

5.  $x^2 - 11x + 30$

$$(x-6)(x-5)$$

6.  $2x^3 + 32x^2 + 128x$

$$2x(x^2 + 16x + 64)$$

$$2x(x+8)(x+8)$$

7.  $x^3 + 3x^2 - 4x - 12$

$$x^2(x+3) - 4(x+3)$$

$$(x^2 - 4)(x+3)$$

$$(x+2)(x-2)(x+3)$$

8.  $9x^3 + 12x^2 - 45x$

$$3x(3x^2 + 4x - 15)$$

$$3x(3x-5)(x+3)$$

9.  $6x^3 - 18x^2$

$$6x^2(x-3)$$

10.  $a^3 - 8b^3$

$$(a-2b)(a^2 + 2ab + 4b^2)$$

11.  $n^2 - 2n - np + 2p$

$$n(n-2) - p(n-2)$$

$$(n-p)(n-2)$$

12.  $12x^4 - 2x^3 - 24x^2$

$$2x^2(6x^2 - x - 12)$$

$$2x^2(2x-3)(3x+4)$$

## Solving Quadratic Equations

Objectives: The Student will be able to –

- Solve quadratic equations by using square roots.
- Solve quadratic equations by factoring.
- Solve quadratic equations by using the Quadratic Formula

Solving Quadratic Equations by Square Roots:

<https://www.khanacademy.org/math/algebra/x2f8bb11595b61c86:quadratic-functions-equations/x2f8bb11595b61c86:untitled-1082/v/simple-quadratic-equation>

Solving Quadratic Equations by Factoring:

<https://www.khanacademy.org/math/algebra/x2f8bb11595b61c86:quadratic-functions-equations/x2f8bb11595b61c86:quadratics-solve-factoring/v/example-1-solving-a-quadratic-equation-by-factoring>

<https://www.youtube.com/watch?v=qeByhTF8WEw>

<https://www.youtube.com/watch?v=zc2CpyRtivY>

Solving Quadratic Equations using the Quadratic Formula:

<https://www.khanacademy.org/math/algebra/x2f8bb11595b61c86:quadratic-functions-equations/x2f8bb11595b61c86:quadratic-formula-a1/v/using-the-quadratic-formula>

<https://www.youtube.com/watch?v=IINAJI36-10>

## Solve Quadratic Equations

Objectives: The Student will be able to –

- Solve quadratic equations by using square roots.
- Solve quadratic equations by factoring.
- Solve quadratic equations by using the Quadratic Formula

### Solve by using Square Roots:

1.  $3x^2 = 9$

$$\textcircled{1} \quad 3x^2 = 9 \\ x^2 = 3$$

2.  $2x^2 - 9 = 15$

$$x = \pm\sqrt{3}$$

3.  $(2x-5)^2 - 5 = 31$

$$\textcircled{2} \quad 2x^2 - 9 = 15$$

$$2x^2 = 24$$

$$x^2 = 12$$

$$x = \pm\sqrt{12}$$

$$x = \pm 2\sqrt{3}$$

$$\textcircled{3} \quad (2x-5)^2 - 5 = 31$$

$$(2x-5)^2 = 36$$

$$2x-5 = \pm\sqrt{36}$$

$$2x-5 = \pm 6$$

$$2x-5 = 6$$

$$2x = 11$$

$$x = \frac{11}{2}$$

$$2x-5 = -6$$

$$2x = -1$$

$$x = -\frac{1}{2}$$

Solving Quadratic Equations by Square Roots:

<https://www.khanacademy.org/math/algebra/x2f8bb11595b61c86:quadratic-functions-equations/x2f8bb11595b61c86:untitled-1082/v/simple-quadratic-equation>

### Solve by Factoring:

1.  $x^2 - 25 = 0$   $x = -5$   
 $(x+5)(x-5) = 0$   $x = 5$

2.  $x^2 - 8x + 15 = 0$   $x = 5$   
 $(x-5)(x-3) = 0$   $x = 3$

3.  $x^2 + 11x + 30 = 0$   $x = -5$   
 $(x+5)(x+6) = 0$   $x = -6$

4.  $x^2 - 3x - 40 = 0$   $x = 8$   
 $(x-8)(x+5) = 0$   $x = -5$

5.  $x^2 + 5x - 24 = 0$   $x = -8$   
 $(x+8)(x-3) = 0$   $x = 3$

6.  $3x^2 + 12x - 36 = 0 \rightarrow 3(x+6)(x-2)$   $x = -6$   
 $3(x^2 + 4x - 12) = 0$   $x = 2$

7.  $6x^2 + x - 12 = 0$   $x = 4/3$   
 $(3x-4)(2x+3) = 0$   $x = -3/2$

8.  $x^2 + 9x - 12 = 3x + 4$   $x = -8$   
 $x^2 + 6x - 16 = 0$   $x = 2$   
 $(x+8)(x-2) = 0$

Solving Quadratic Equations by Factoring:

<https://www.khanacademy.org/math/algebra/x2f8bb11595b61c86:quadratic-functions-equations/x2f8bb11595b61c86:quadratics-solve-factoring/v/example-1-solving-a-quadratic-equation-by-factoring>

<https://www.youtube.com/watch?v=qeByhTF8WEw>

<https://www.youtube.com/watch?v=zc2CpyRtjvY>

Solve by using the Quadratic Formula:

1.  $2x^2 + 6x + 3 = 0$

2.  $3x^2 - 2x + 2 = 7$

3.  $5x^2 - 4x + 2 = 4x^2 - 6x + 8$

Solving Quadratic Equations using the Quadratic Formula:

<https://www.khanacademy.org/math/algebra/x2f8bb11595b61c86:quadratic-functions-equations/x2f8bb11595b61c86:quadratic-formula-a1/v/using-the-quadratic-formula>

<https://www.youtube.com/watch?v=IINAJl36-10>

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

①  $2x^2 + 6x + 3 = 0$

$a = 2$

$b = 6$

$c = 3$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(2)(3)}}{2(2)}$$

$$x = \frac{-6 \pm \sqrt{36 - 24}}{4}$$

$$x = \frac{-6 \pm \sqrt{12}}{4}$$

$$x = \frac{-6 \pm 2\sqrt{3}}{4}$$

$$x = \frac{-3 \pm \sqrt{3}}{2}$$

②  $3x^2 - 2x + 2 = 7$

$$3x^2 - 2x - 5 = 0$$

$a = 3$

$b = -2$

$c = -5$

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(3)(-5)}}{2(3)}$$

$$x = \frac{2 \pm \sqrt{4 - (-60)}}{6}$$

$$x = \frac{2 \pm \sqrt{64}}{6}$$

$$x = \frac{2 \pm 8}{6}$$

$$x = \frac{10}{6} \text{ or } \frac{-6}{6}$$

$$x = \frac{5}{3} \text{ or } -1$$

$$5x^2 - 4x + 2 = 4x^2 - 6x + 8$$

$$x^2 + 2x - 6 = 0$$

$a = 1$

$b = 2$

$c = -6$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(-6)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{4 - (-24)}}{2}$$

$$x = \frac{-2 \pm \sqrt{28}}{2}$$

$$x = \frac{-2 \pm 2\sqrt{7}}{2}$$

$$x = -1 \pm \sqrt{7}$$

# Radicals

## **Radicals:**

To simplify means that 1) no radicand has a perfect square factor and

2) there is no radical in the denominator (rationalize).

Recall the **Product Property**  $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$  and the **Quotient Property**  $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

**Examples:** Simplify  $\sqrt{24} = \sqrt{4} \cdot \sqrt{6}$  find the perfect square factor

$$= 2\sqrt{6} \quad \text{simplify}$$

Simplify  $\sqrt{\frac{7}{2}} = \frac{\sqrt{7}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$  multiply numerator & denominator by  $\sqrt{2}$

$$= \frac{\sqrt{14}}{\sqrt{4}} = \frac{\sqrt{14}}{2} \quad \text{multiply straight across and simplify}$$

If the denominator contains 2 terms, multiply the numerator and denominator by

conjugate of the denominator (the conjugate of  $3 + \sqrt{2}$  is  $3 - \sqrt{2}$ )

## Simplifying Radicals

<https://www.youtube.com/watch?v=6QJtWfliyZo>

<https://www.youtube.com/watch?v=U7AXF75rC00>

<https://www.khanacademy.org/math/algebra/x2f8bb11595b61c86:rational-exponents-radicals/x2f8bb11595b61c86:simplifying-square-roots/v/simplifying-square-roots-1>

## Rationalizing the denominator

<https://www.youtube.com/watch?v=KIZPKMDwg6M>

<https://www.youtube.com/watch?v=gY5TvIHg4Vk>

Simplify each of the following:

$$1. \sqrt{32} = 4\sqrt{2}$$

$$2. \sqrt{(2x)^8} = (2x)^4 = 16x^4$$

$$3. \sqrt[3]{-64} = -4$$

$$4. \sqrt{49m^2n^8} = 7mn^4$$

$$5. \sqrt{\frac{11}{9}} = \frac{\sqrt{11}}{\sqrt{9}} = \frac{\sqrt{11}}{3}$$

$$6. \sqrt{60} \cdot \sqrt{105}$$

Factorization tree for 60: 2, 30; 2, 15; 2, 3, 5

Factorization tree for 105: 3, 35; 3, 5, 7

Prime factors: 2, 2, 3, 5, 3, 5, 7

$$2 \cdot 3 \cdot 5 \sqrt{7} = 30\sqrt{7}$$

$$7. (\sqrt{5} - \sqrt{6})(\sqrt{5} + \sqrt{2})$$

Rationalize the Denominator:

$$8. \frac{1}{\sqrt{2}}$$

$$9. \frac{2}{\sqrt{3}}$$

$$10. \frac{3}{2 - \sqrt{5}}$$

$$⑦ (\sqrt{5} - \sqrt{6})(\sqrt{5} + \sqrt{2})$$

$$\sqrt{25} + \sqrt{10} - \sqrt{30} - \sqrt{12}$$

$$5 + \sqrt{10} - \sqrt{30} - 2\sqrt{3}$$

$$⑧ \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$⑨ \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$⑩ \frac{3}{2 - \sqrt{5}} \cdot \frac{2 + \sqrt{5}}{2 + \sqrt{5}}$$

$$= \frac{3(2 + \sqrt{5})}{4 + 2\sqrt{5} - 2\sqrt{5} - \sqrt{25}}$$

$$= \frac{3(2 + \sqrt{5})}{4 - 5} = -3(2 + \sqrt{5})$$

# Complex Numbers:

## Complex Numbers:

Form of complex number:  $a + bi$

Where  $a$  is the real part and the  $b$  is the imaginary part

Always make these substitutions  $\sqrt{-1} = i$  and  $i^2 = -1$

To simplify: pull out the  $\sqrt{-1}$  before performing any operation

Example:  $\sqrt{-5} = \sqrt{-1} \cdot \sqrt{5}$  Pull out  $\sqrt{-1}$  Example:  $(i\sqrt{5})^2 = i\sqrt{5} \cdot i\sqrt{5}$   
 $= i\sqrt{5}$  Make substitution  $= i^2 \sqrt{25} = (-1)(5) = -5$

Treat  $i$  like any other variable when  $+$ ,  $-$ ,  $\times$ , or  $\div$  (but always simplify  $i^2 = -1$ )

Example:  $2i(3 + i) = 2(3i) + 2i(i)$  Distribute  
 $= 6i + 2i^2$  Simplify  
 $= 6i + 2(-1)$  Substitute  
 $= -2 + 6i$  Simplify and rewrite in complex form

Since  $i = \sqrt{-1}$ , no answer can have an 'i' in the denominator. RATIONALIZE!

## Intro to Complex Numbers:

<https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:complex/x2ec2f6f830c9fb89:complex-num/v/complex-number-intro>

## Adding/Subtracting Complex Numbers

<https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:complex/x2ec2f6f830c9fb89:complex-add-sub/v/adding-complex-numbers>

## Multiplying Complex Numbers:

<https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:complex/x2ec2f6f830c9fb89:complex-mul/v/multiplying-complex-numbers>

## Dividing Complex Numbers

<https://www.youtube.com/watch?v=EfRRpVB62Ko>

## Operations with Complex Numbers

<https://www.youtube.com/watch?v=OQz1ydBcQSA>

Perform the indicated operations and simplify:

1.  $\sqrt{-49}$   
 $+7i$

2.  $6\sqrt{-12}$   
 $6i \cdot \sqrt{12}$   
 $6i \cdot 2\sqrt{3}$   
 $12\sqrt{3}i$

3.  $(2-3i)+(5+7i)$   
 $7+4i$

4.  $(3+6i)-(8-4i)$   
 $-5+10i$

5.  $-6(2-8i)+3(5+7i)$   
 $-12+48i+15+21i$   
 $3+69i$

6.  $(3-4i)^2$   $-7-24i$   
 $(3-4i)(3-4i)$   
 $9-12i-12i+16i^2$   
 $9-24i-16$

7.  $(6-2i)(3+5i)$   
 $18+30i-6i-10i^2$   
 $18+24i+10$   
 $28+24i$

8.  $(3-4i)(3+4i)$   
 $9+12i-12i-16i^2$   
 $9-16i^2$   
 $9+16$   
 $25$

9.  $\frac{1+6i}{5i} \cdot \frac{-5i}{-5i}$   
 $= \frac{-5i-30i^2}{-25i^2}$   
 $= \frac{-5i+30}{25}$   
 $= \frac{-i+6}{5}$

10.  $\frac{2+3i}{4-5i}$

$\frac{2+3i}{4-5i} \cdot \frac{4+5i}{4+5i}$

numerator  
 $(2+3i)(4+5i)$   
 $8+10i+12i+15i^2$   
 $8+22i+15i^2$   
 $8+22i-15$   
 $-7+22i$

denominator  
 $(4-5i)(4+5i)$   
 $16+20i-20i-25i^2$   
 $= 16-25i^2$   
 $= 16+25$   
 $= 41$

$\frac{-7+22i}{41}$

## Graphing Quadratic Functions

Objectives: The Student will be able to –

- Graph quadratic functions given in vertex form.
- Graph quadratic functions given in standard form.
- Graph quadratic functions given in intercept form.
- Identify the intercepts of a quadratic function.
- Identify the vertex of a quadratic function
- Identify the axis of symmetry of a quadratic function
- Determine the concavity of a quadratic function.

Graphing Quadratic Functions:

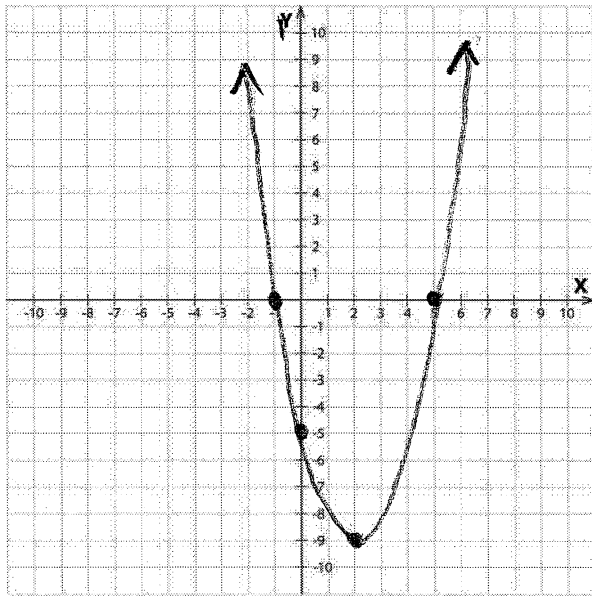
[https://www.youtube.com/watch?v=Hq2Up\\_1lh5E](https://www.youtube.com/watch?v=Hq2Up_1lh5E)

[https://www.youtube.com/watch?v=OHH7fX\\_M8Ns](https://www.youtube.com/watch?v=OHH7fX_M8Ns)

<https://www.youtube.com/watch?v=y99lNRqLiBA>

1. Graph :  $f(x) = (x-2)^2 - 9$

Identify all key points : Intercepts, Vertex, Axis of Symmetry.



Vertex:  $(2, -9)$

Concave up

y-int (x=0)

$y = (x-2)^2 - 9$

$y = (-2)^2 - 9$

$y = 4 - 9$

$y = -5$

$(0, -5)$

x-ints (y=0)

$0 = (x-2)^2 - 9$

$9 = (x-2)^2$

$\pm 3 = x-2$

$x-2 = 3$        $x-2 = -3$

$x = 5$        $x = -1$

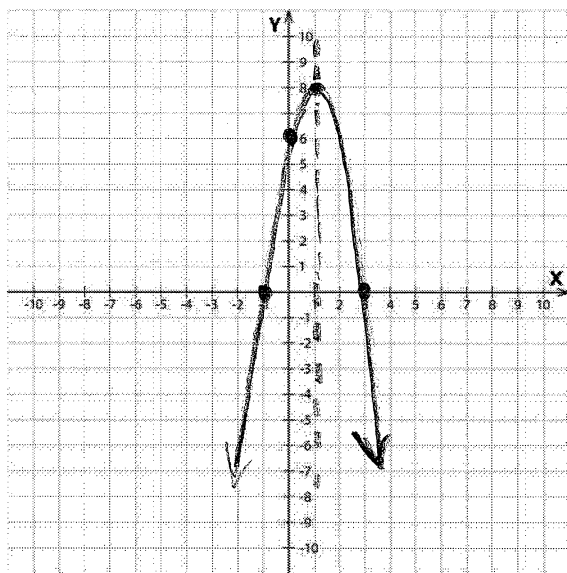
$(5, 0)$        $(-1, 0)$

Axis of symmetry

$x = 2$

2. Graph:  $f(x) = -2(x-1)^2 + 8$

Identify all key points : Intercepts, Vertex, Axis of Symmetry.



Vertex:  $(1, 8)$

Concave down

y-int ( $x=0$ )

$$y = -2(0-1)^2 + 8$$

$$y = -2(-1)^2 + 8$$

$$y = -2(1) + 8$$

$$y = -2 + 8$$

$$y = 6$$

$$(0, 6)$$

x-int ( $y=0$ )

$$0 = -2(x-1)^2 + 8$$

$$-8 = -2(x-1)^2$$

$$4 = (x-1)^2$$

$$\pm 2 = x-1$$

$$x-1=2$$

$$x=3$$

$$(3, 0)$$

$$x-1=-2$$

$$x=-1$$

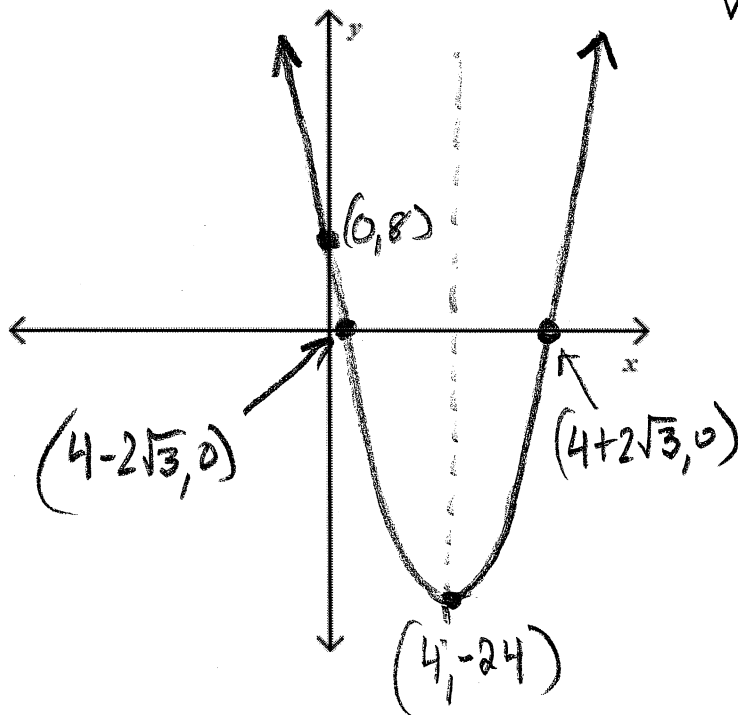
$$(-1, 0)$$

Axis of symmetry

$$x = 1$$

3. Graph:  $f(x) = 2(x-4)^2 - 24$

Identify all key points : Intercepts, Vertex, Axis of Symmetry.



Vertex:  $(4, -24)$

Concave up

y-intercept ( $x=0$ )

$$y = 2(x-4)^2 - 24$$

$$y = 2(-4)^2 - 24$$

$$y = 2(16) - 24$$

$$y = 32 - 24$$

$$y = 8$$

$$(0, 8)$$

x-intercepts ( $y=0$ )

$$0 = 2(x-4)^2 - 24$$

$$24 = 2(x-4)^2$$

$$12 = (x-4)^2$$

$$\pm \sqrt{12} = x-4$$

$$\pm 2\sqrt{3} = x-4$$

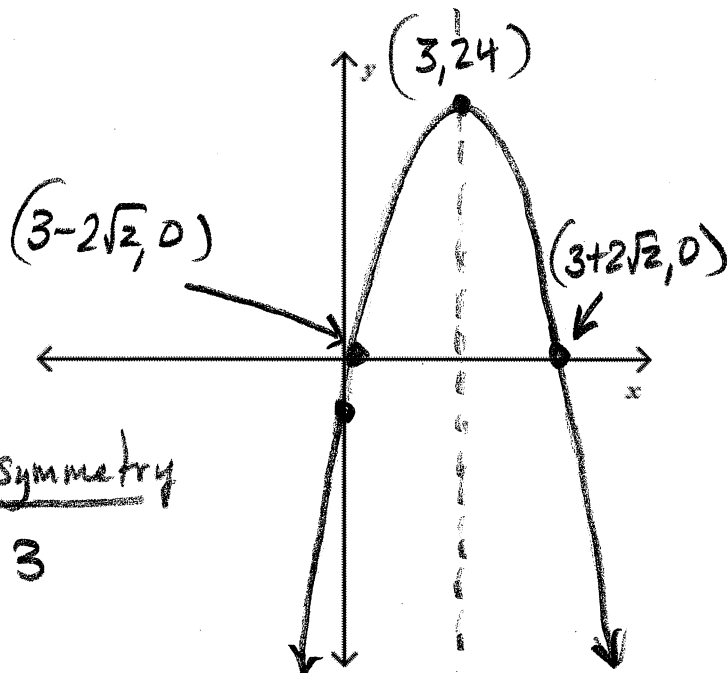
$$x = 4 + 2\sqrt{3}$$

$$x = 4 - 2\sqrt{3}$$

Axis of symmetry:  
 $x = 4$

4. Graph:  $f(x) = -3(x-3)^2 + 24$

Identify all key points : Intercepts, Vertex, Axis of Symmetry.



Vertex:  $(3, 24)$

Concave down

y-int (x=0)

$$y = -3(x-3)^2 + 24$$

$$y = -3(-3)^2 + 24$$

$$y = -3(9) + 24$$

$$y = -27 + 24$$

$$y = -3$$

$$(0, -3)$$

x-int (y=0)

$$0 = -3(x-3)^2 + 24$$

$$-24 = -3(x-3)^2$$

$$8 = (x-3)^2$$

$$\pm\sqrt{8} = x-3$$

$$\pm 2\sqrt{2} = x-3$$

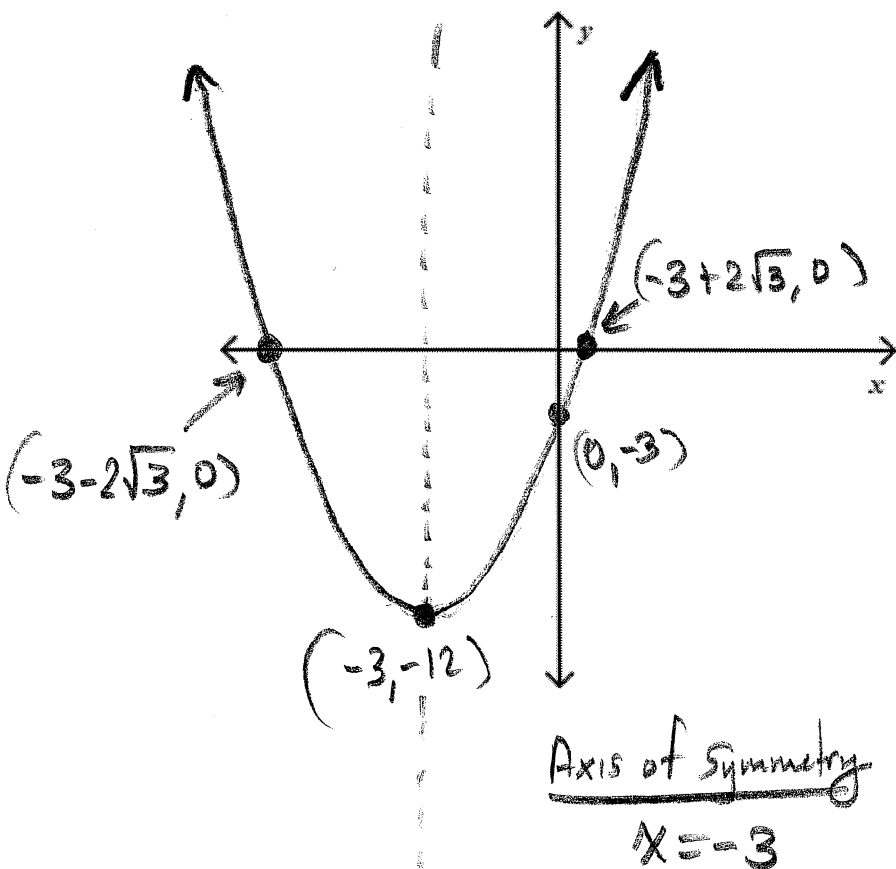


$$x = 3 + 2\sqrt{2}$$

$$x = 3 - 2\sqrt{2}$$

5. Graph:  $f(x) = x^2 + 6x - 3$

Identify all key points : Intercepts, Vertex, Axis of Symmetry.



Rewrite

$$x^2 + 6x = 3$$

$$x^2 + 6x + 9 = 3 + 9$$

$$(x+3)^2 = 12$$

$$f(x) = (x+3)^2 - 12$$

Vertex

$$(-3, -12)$$

Concave up

x-int (y=0)

y-int

$$(0, -3)$$

$$0 = (x+3)^2 - 12$$

$$12 = (x+3)^2$$

$$\pm\sqrt{12} = x+3$$

$$\pm 2\sqrt{3} = x+3$$

$$x = -3 + 2\sqrt{3}$$

$$x = -3 - 2\sqrt{3}$$

Axis of Symmetry

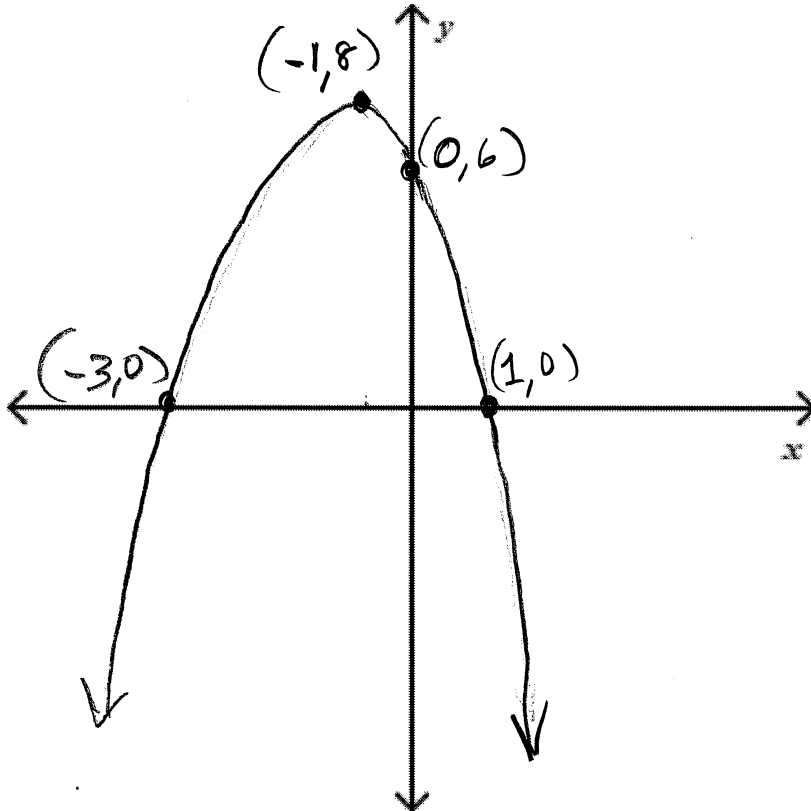
$$x = -3$$

We can approach in two ways:

- ① Rewrite into Vertex form
- ② Use the Standard form and find zeros

6. Graph:  $f(x) = -2x^2 - 4x + 6$

Identify all key points: Intercepts, Vertex, Axis of Symmetry.



Concave down  
y-intercept  $(0, 6)$

Rewrite into Vertex form

$$\begin{aligned} -2x^2 - 4x &= -6 \\ -2(x^2 + 2x) &= -6 \\ -2(x^2 + 2x + 1) &= -6 - 2 \\ -2(x+1)^2 &= -8 \end{aligned}$$

$$f(x) = -2(x+1)^2 + 8$$

Vertex  
 $(-1, 8)$

X-intercepts ( $y=0$ )

Method 1 (using Vertex form)

$$-2(x+1)^2 + 8 = 0$$

$$-2(x+1)^2 = -8$$

$$(x+1)^2 = 4$$

$$x+1 = \pm 2$$

$$x = -1 + 2$$

$$x = 1$$

$$(1, 0)$$

$$x = -1 - 2$$

$$x = -3$$

$$(-3, 0)$$

Method 2 (Factoring)

$$-2x^2 - 4x + 6 = 0$$

$$-2(x^2 + 2x - 3) = 0$$

$$-2(x+3)(x-1) = 0$$

$$\downarrow$$
  
$$x+3=0$$

$$x = -3$$

$$(-3, 0)$$

$$\downarrow$$
  
$$x-1=0$$

$$x = 1$$

$$(1, 0)$$

Method 3  
(Quadratic Formula)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(-2)(-6)}}{2(-2)}$$

$$x = \frac{4 \pm \sqrt{16 - (-48)}}{-4}$$

$$x = \frac{4 \pm \sqrt{64}}{-4}$$

$$x = \frac{4 \pm 8}{-4}$$

$$x = -3 \quad x = 1$$

## Graphing Absolute Value Functions

Objectives: The Student will be able to –

- Graph absolute value functions.
- Identify the intercepts of an absolute value function.
- Identify the vertex of an absolute value function
- Identify the axis of symmetry of an absolute value function
- Determine the concavity of an absolute value function.

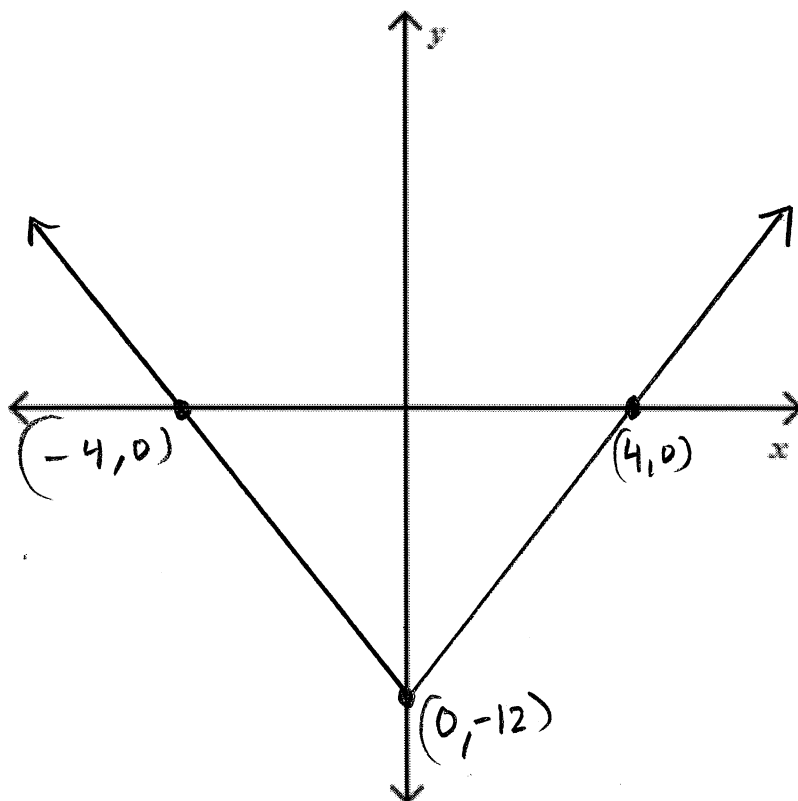
Graphing Absolute Value Functions:

<https://www.khanacademy.org/math/algebra/x2f8bb11595b61c86:absolute-value-piecewise-functions/x2f8bb11595b61c86:graphs-of-absolute-value-functions/v/graphing-absolute-value-functions>

<https://www.youtube.com/watch?v=ld4UD98yHio>

1. Graph:  $f(x) = 3|x| - 12$

Identify all key points: Intercepts, Vertex, Axis of Symmetry



Vertex  $(0, -12)$

X-intercepts  $(y=0)$

$$3|x| - 12 = 0$$

$$3|x| = 12$$

$$|x| = 4$$

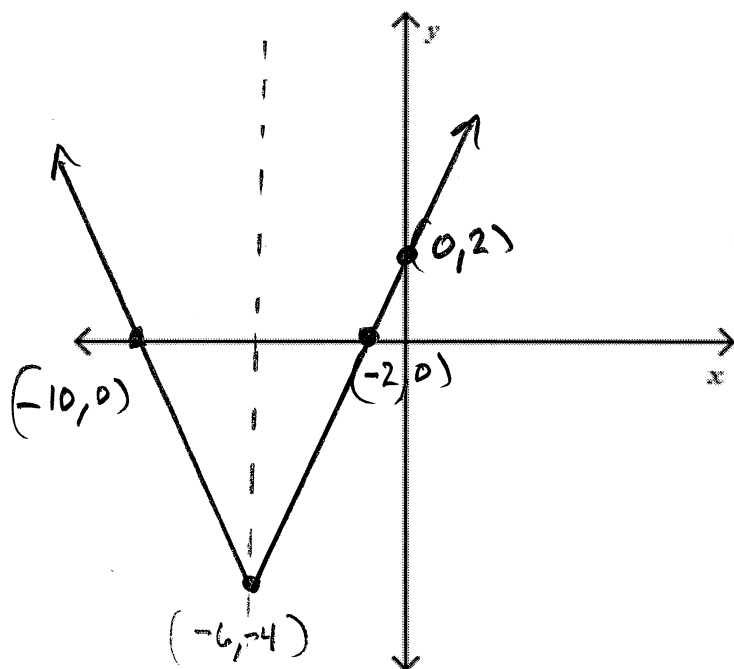
$$x = 4 \quad x = -4$$

Axis of symmetry

$$x = 0$$

2. Graph:  $f(x) = |x+6| - 4$

Identify all key points: Intercepts, Vertex, Axis of Symmetry



Vertex:  $(-6, -4)$

Concave up

y-int ( $x=0$ )

$$y = |x+6| - 4$$

$$y = |6| - 4$$

$$y = 6 - 4$$

$$y = 2$$

$(0, 2)$

x-int ( $y=0$ )

$$0 = |x+6| - 4$$

$$4 = |x+6|$$

$$\begin{array}{l} x+6=4 \\ x+6=-4 \end{array}$$

$$\begin{array}{l} x=-2 \\ x=-10 \end{array}$$

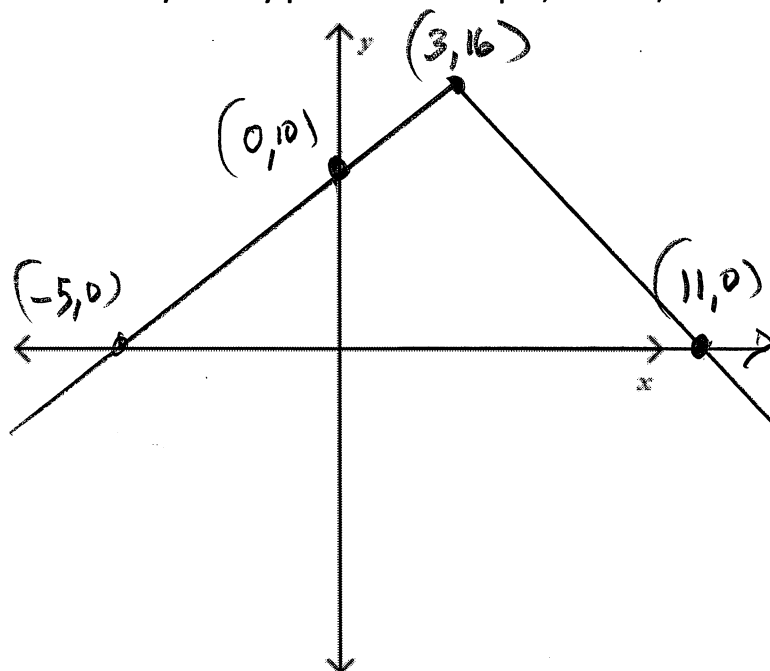
$(-2, 0) \quad (-10, 0)$

Axis of symmetry

$$x = -6$$

3. Graph:  $f(x) = -2|x-3| + 16$

Identify all key points: Intercepts, Vertex, Axis of Symmetry



Vertex:  $(3, 16)$

concave down

y-int ( $x=0$ )

$$y = -2|x-3| + 16$$

$$y = -2|-3| + 16$$

$$y = -2(3) + 16$$

$$y = -6 + 16$$

$$y = 10$$

$(0, 10)$

x-int ( $y=0$ )

$$0 = -2|x-3| + 16$$

$$-16 = -2|x-3|$$

$$8 = |x-3|$$

$$\begin{array}{l} x-3=8 \\ x-3=-8 \end{array}$$

$$\begin{array}{l} x=11 \\ x=-5 \end{array}$$

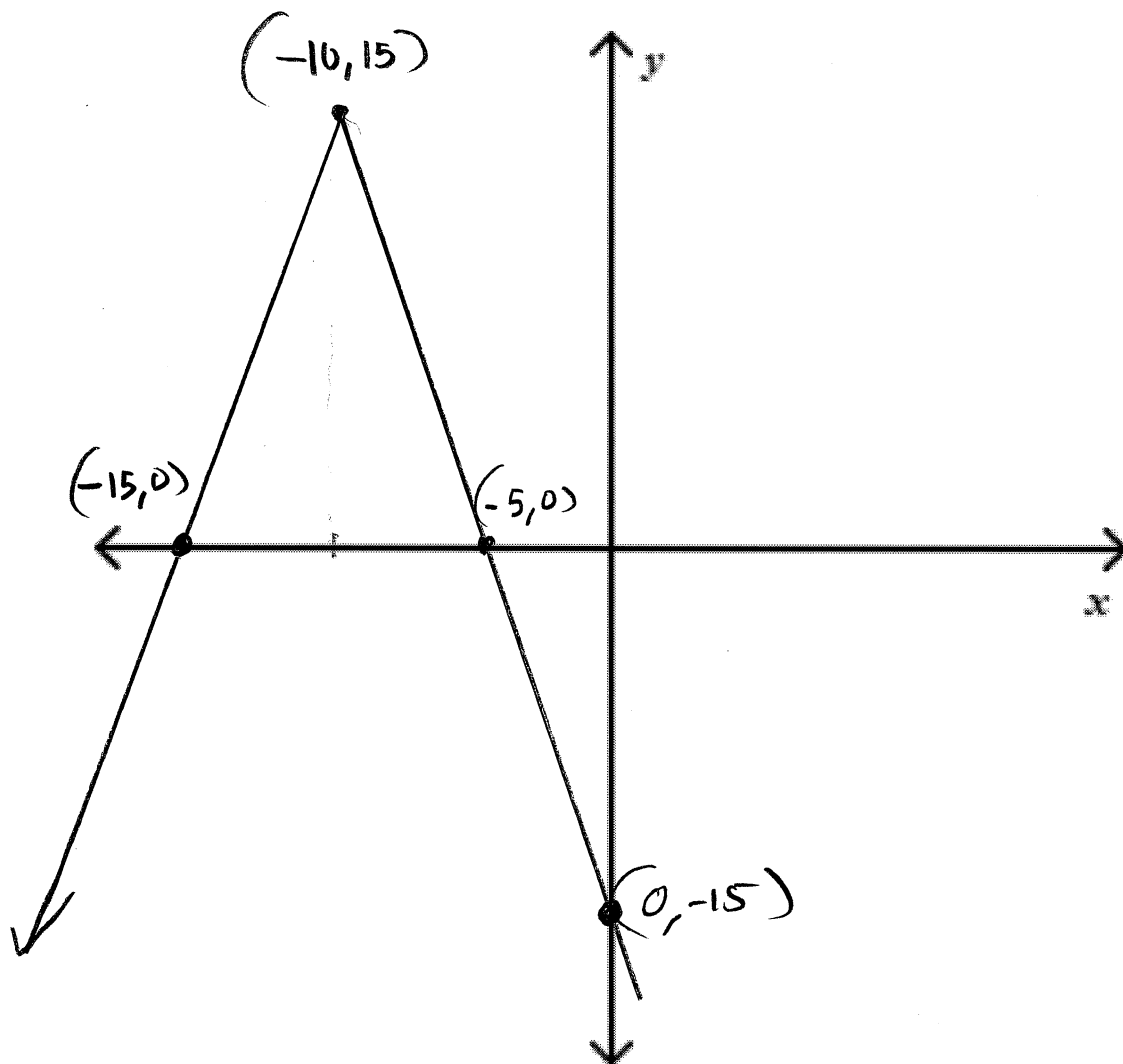
$(11, 0) \quad (-5, 0)$

Axis of symmetry

$$x = 3$$

4. Graph:  $f(x) = -3|x+10|+15$

Identify all key points: Intercepts, Vertex, Axis of Symmetry



Vertex:  $(-10, 15)$

Concave down

y-int ( $x=0$ )

$$y = -3|x+10| + 15$$

$$y = -3|10| + 15$$

$$y = -3(10) + 15$$

$$y = -30 + 15$$

$$y = -15 \quad (0, -15)$$

x-int ( $y=0$ )

$$y = -3|x+10| + 15$$

$$0 = -3|x+10| + 15$$

$$-15 = -3|x+10|$$

$$5 = |x+10|$$

$$x+10=5$$

$$x=-5$$

$$x+10=-5$$

$$x=-15$$

Axis of symmetry

$$x = -10$$

## Multiplying and Dividing Rational Expressions (Fractions)

**Multiplying and Dividing:** Factor numerator and denominator completely. Cancel any common factors in the top and bottom. If dividing, change divide to multiply and flip the second fraction.

$$\begin{aligned}
 \text{EX: } & \frac{x^2 + 10x + 21}{5 - 4x - x^2} \cdot \frac{x^2 + 2x - 15}{x^3 + 4x^2 - 21x} && \text{Factor everything completely.} \\
 & = \frac{(x+7)(x+3)}{(5+x)(1-x)} \cdot \frac{(x+5)(x-3)}{x(x-3)(x+7)} && \text{Cancel out common factors in the top and bottom.} \\
 & = \frac{(x+3)}{x(1-x)} && \text{Simplify.}
 \end{aligned}$$

Multiplying Rational Expressions:

<https://www.youtube.com/watch?v=RR0Sgr4oXjU>

<https://www.khanacademy.org/math/math3/x5549cc1686316ba5:rational/x5549cc1686316ba5:rational-mul-div/v/multiplying-and-dividing-rational-expressions-2>

[https://www.youtube.com/watch?v=x\\_5hDLe8UL0](https://www.youtube.com/watch?v=x_5hDLe8UL0)

Dividing Rational Expressions:

<https://www.khanacademy.org/math/math3/x5549cc1686316ba5:rational/x5549cc1686316ba5:rational-mul-div/v/multiplying-and-dividing-rational-expressions-3>

<https://www.youtube.com/watch?v=rMVOdIDNjpA>

1.  $\frac{4}{5} \cdot \frac{2}{3}$

2.  $\frac{2}{7} \div \frac{4}{9}$

3.  $\frac{\frac{12}{7}}{-\frac{7}{18}}$

4.  $\frac{x+3}{12x} \cdot \frac{8x}{5x+15}$

5.  $\frac{x^2-25}{4x-20} \cdot \frac{12}{x^2+9x+20}$

6.  $\frac{x^2-x-6}{x^2-4} \div \frac{x^2-8x+15}{x^2-6x+8}$

7.  $\frac{\frac{x^2-x-12}{x^2-9}}{x^2+2x-15}$

See next page!

$$\textcircled{1} \quad \frac{4}{5} \cdot \frac{2}{3} = \textcircled{\frac{8}{15}}$$

$$\begin{aligned} \textcircled{2} \quad \frac{2}{7} \div \frac{4}{9} \\ = \frac{2}{7} \times \frac{9}{4} \\ = \frac{18}{28} = \textcircled{\frac{9}{14}} \end{aligned}$$

$$\begin{aligned} \textcircled{3} \quad \frac{12}{7} \div -\frac{7}{18} \\ = \frac{12}{7} \times \frac{18}{-7} \\ = \textcircled{-\frac{216}{49}} \end{aligned}$$

$$\begin{aligned} \textcircled{4} \quad \frac{x+3}{12x} \cdot \frac{8x}{5x+15} \\ = \frac{\cancel{x+3}}{12\cancel{x}} \cdot \frac{8\cancel{x}}{5(\cancel{x+3})} \\ = \frac{8}{12(5)} \\ = \frac{8}{60} = \textcircled{\frac{2}{15}} \end{aligned}$$

$$\begin{aligned} \textcircled{5} \quad \frac{x^2-25}{4x-20} \cdot \frac{12}{x^2+9x+20} \\ = \frac{\cancel{(x+5)}(\cancel{x-5})}{4(\cancel{x-5})} \cdot \frac{12}{(\cancel{x+5})(x+4)} \\ = \frac{12}{4(x+4)} = \textcircled{\frac{3}{x+4}} \end{aligned}$$

$$\begin{aligned} \textcircled{6} \quad \frac{x^2-x-6}{x^2-4} \div \frac{x^2-8x+15}{x^2-6x+8} \\ = \frac{\cancel{(x-3)}(\cancel{x+2})}{(\cancel{x-2})(\cancel{x+2})} \cdot \frac{(x-4)(\cancel{x-2})}{(x-5)(\cancel{x-3})} \\ = \textcircled{\frac{x-4}{x-5}} \end{aligned}$$

$$\begin{aligned} \textcircled{7} \quad \frac{x^2-x-12}{x^2-9} \cdot \frac{x^2+2x-15}{x^2-16} \\ = \frac{\cancel{(x-4)}(\cancel{x+3})}{(\cancel{x-3})(\cancel{x+3})} \cdot \frac{(x+5)(\cancel{x-3})}{(x+4)(\cancel{x-4})} \\ = \textcircled{\frac{x+5}{x+4}} \end{aligned}$$

## Adding and Subtracting Rational Expressions (Fractions)

### Addition and Subtraction of rational Expressions.

First find the least common denominator. Write each fraction with that LCD. Add/subtract numerators as indicated and leave the denominators as they are.

EX:  $\frac{3x+1}{x^2+2x} + \frac{5x-4}{2x+4}$  *Factor denominator completely.*

$\frac{3x+1}{x(x+2)} + \frac{5x-4}{2(x+2)}$  *Find LCD, which is  $(2x)(x+2)$*

$\frac{2(3x+1)}{2x(x+2)} + \frac{x(5x-4)}{2x(x+2)}$  *Rewrite each fraction with the LCD in the denominator.*

$\frac{6x+2+5x^2-4x}{2x(x+2)}$  *Write as one fraction.*

$\frac{5x^2+2x+2}{2x(x+2)}$  *Combine like terms.*

Add or Subtract the Following:

8.  $\frac{2}{3} + \frac{5}{7}$

9.  $\frac{5x}{x-5} + \frac{x+5}{x+2}$

10.  $\frac{x}{x^2-9} + \frac{5}{4x-12}$

11.  $\frac{8}{x^2-4x+4} + \frac{2}{x-2}$

12.  $\frac{3}{x^2} - \frac{4}{x}$

Solutions on next page.

$$\textcircled{8} \quad \frac{2}{3} + \frac{5}{7}$$

$$= \frac{14}{21} + \frac{15}{21}$$

$$= \frac{29}{21}$$

$$\textcircled{9} \quad \frac{5x}{x-5} + \frac{x+5}{x+2}$$

$$= \frac{5x(x+2) + (x+5)(x-5)}{(x-5)(x+2)}$$

$$= \frac{5x^2 + 10x + x^2 - 25}{(x-5)(x+2)}$$

$$= \frac{6x^2 + 10x - 25}{(x-5)(x+2)}$$

$$\textcircled{10} \quad \frac{x}{x^2-9} + \frac{5}{4x-12}$$

$$= \frac{x}{(x+3)(x-3)} + \frac{5}{4(x-3)}$$

$$= \frac{4x + 5(x+3)}{4(x+3)(x-3)}$$

$$= \frac{4x + 5x + 15}{4(x+3)(x-3)}$$

$$= \frac{9x + 15}{4(x+3)(x-3)}$$

$$\textcircled{11} \quad \frac{8}{x^2-4x+4} + \frac{2}{x-2}$$

$$= \frac{8}{(x-2)(x-2)} + \frac{2}{x-2}$$

$$= \frac{8 + 2(x-2)}{(x-2)(x-2)}$$

$$= \frac{8 + 2x - 4}{(x-2)(x-2)}$$

$$= \frac{2x + 4}{(x-2)(x-2)}$$

$$\textcircled{12} \quad \frac{3}{x^2} - \frac{4}{x}$$

$$= \frac{3 - 4x}{x^2}$$

## Solving Rational Equations

### Solving Rational Equations:

Multiply each term by the LCD of all the fractions. This should eliminate all of our fractions. Then solve the equation as usual.

$$\frac{5}{x+2} + \frac{1}{x} = \frac{5}{x}$$

*Find LCD first  $x(x+2)$*

$$x(x+2)\frac{5}{x+2} + x(x+2)\frac{1}{x} = \frac{5}{x}x(x+2)$$

*Multiply each term by the LCD.*

$$5x + 1(x+2) = 5(x+2)$$

*Simplify and solve.*

$$5x + x + 2 = 5x + 10$$

$$6x + 2 = 5x + 10$$

$$x = 8 \quad \leftarrow \text{Check your answer! Sometimes they do not check!}$$

Check:  $\frac{5}{8+2} + \frac{1}{8} = \frac{5}{8}$

$$\frac{5}{10} + \frac{1}{8} = \frac{5}{8}$$

$$\frac{5}{8} = \frac{5}{8}$$

Solving Rational Equations:

<https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:equations-of-lines/a2c2f6f830c9fb89:rational-equations/v/rational-equation-intro>

<https://www.youtube.com/watch?v=iZ2-yXzLqCg>

<https://www.youtube.com/watch?v=MFjx52NW5Bw>

<https://www.youtube.com/watch?v=Yaeze9u6Cv8>

Solve the following Rational Equations:

1.  $\frac{12}{x} + \frac{3}{4} = \frac{3}{2}$

2.  $\frac{x}{x-3} = \frac{2}{5}$

3.  $\frac{2}{x} + \frac{3x-1}{x+3} = 4$

4.  $\frac{2}{x+5} + \frac{6}{x^2-25} = \frac{3}{x-5}$

5.  $\frac{13x+20}{x^2+13x+42} - \frac{4}{x+6} = \frac{6}{x+7}$

$$\textcircled{1} \left[ \frac{12}{x} + \frac{3}{4} = \frac{3}{2} \right] 4x$$

$$48 + 3x = 6x$$

$$48 = 3x$$

$$\textcircled{16 = x}$$

$$\textcircled{2} \frac{x}{x-3} = \frac{2}{5}$$

$$5x = 2(x-3)$$

$$5x = 2x - 6$$

$$3x = -6$$

$$\textcircled{x = -2}$$

$$\textcircled{3} \left[ \frac{2}{x} + \frac{3x-1}{x+3} = 4 \right] x(x+3)$$

$$2(x+3) + (3x-1)(x) = 4(x)(x+3)$$

$$2x+6 + 3x^2 - x = 4x^2 + 12x$$

$$x^2 + 11x - 6 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-11 \pm \sqrt{(11)^2 - 4(1)(-6)}}{2(1)}$$

$$x = \frac{-11 \pm \sqrt{121 - (-24)}}{2}$$

$$x = \frac{-11 \pm \sqrt{145}}{2}$$

$$\textcircled{4} \frac{2}{x+5} + \frac{6}{x^2-25} = \frac{3}{x-5}$$

$$(x+5)(x-5) \left[ \frac{2}{x+5} + \frac{6}{(x+5)(x-5)} = \frac{3}{x-5} \right]$$

$$2(x-5) + 6 = 3(x+5)$$

$$2x-10 + 6 = 3x + 15$$

$$2x-4 = 3x + 15$$

$$\textcircled{-19 = x}$$

$$\textcircled{5} \frac{13x+20}{x^2+13x+42} - \frac{4}{x+6} = \frac{6}{x+7}$$

$$(x+7)(x+6) \left[ \frac{13x+20}{(x+7)(x+6)} - \frac{4}{x+6} = \frac{6}{x+7} \right]$$

$$13x+20 - 4(x+7) = 6(x+6)$$

$$13x+20 - 4x-28 = 6x+36$$

$$9x-8 = 6x+36$$

$$3x-8 = 36$$

$$3x = 44$$

$$\textcircled{x = \frac{44}{3}}$$

# Graphing Rational Functions

<https://www.youtube.com/watch?v=XE-Z2-F3oWw>

<https://www.khanacademy.org/math/math3/x5549cc1686316ba5:rational/x5549cc1686316ba5:rational-graphs/v/horizontal-vertical-asymptotes>

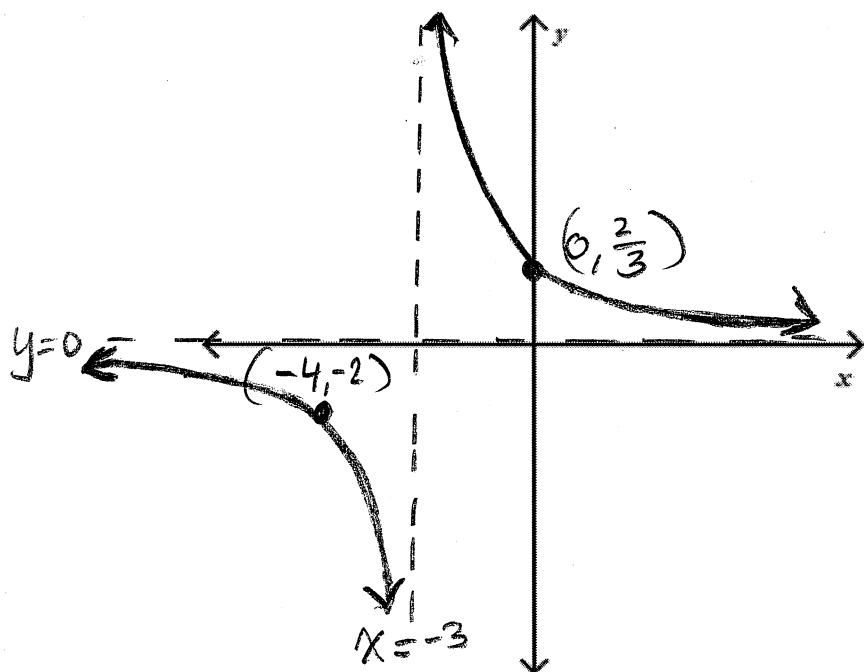
<https://www.khanacademy.org/math/math3/x5549cc1686316ba5:rational/x5549cc1686316ba5:rational-graphs/v/another-rational-function-graph-example>

<https://www.youtube.com/watch?v=GxynsElvjRA>

<https://www.khanacademy.org/math/precalculus/x9e81a4f98389efdf:rational-functions/x9e81a4f98389efdf:graphs-of-rational-functions/v/finding-asymptotes-example>

Graph the following Rational Functions:

1.  $f(x) = \frac{2}{x+3}$



$$\frac{y\text{-int } (x=0)}{(0, \frac{2}{3})}$$

Horizontal  
Asymptote  
 $y=0$

$$\frac{x\text{-int } (y=0)}{0 = \frac{2}{x+3}}$$

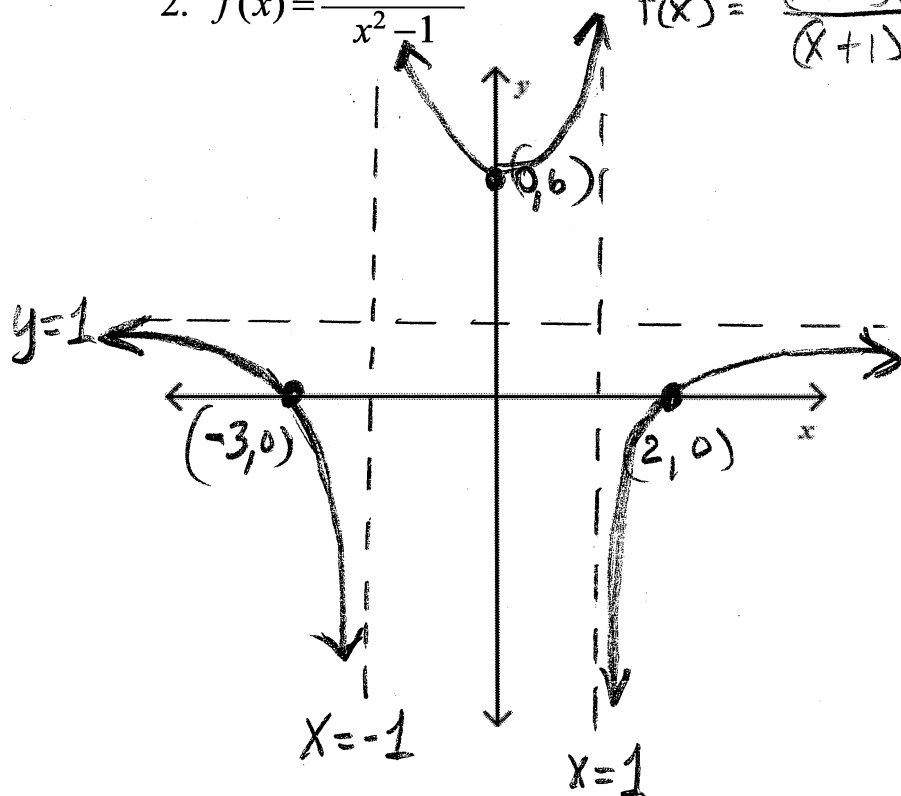
$$0 \neq \frac{2}{x+3}$$

no x-int

Vertical Asymptote  
 $x=-3$

2.  $f(x) = \frac{x^2+x-6}{x^2-1}$

$$f(x) = \frac{(x+3)(x-2)}{(x+1)(x-1)}$$



$$\frac{y\text{-int } (x=0)}{(0, 6)}$$

$$\frac{x\text{-int } (y=0)}{(-3, 0) \text{ and } (2, 0)}$$

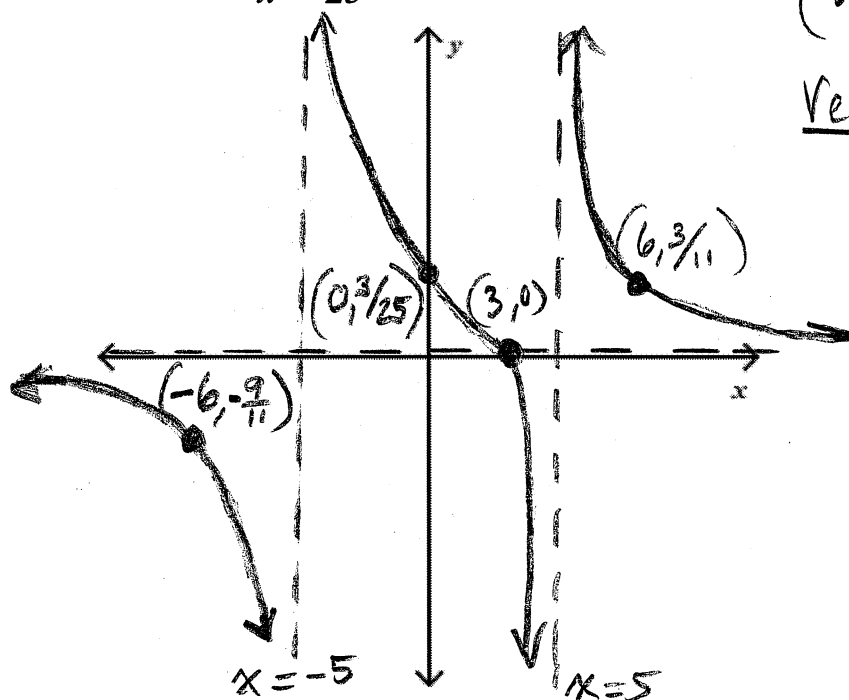
Vertical Asymptotes

$$x=-1$$

$$x=1$$

Horizontal Asymptote  
 $y=1$

3.  $f(x) = \frac{x-3}{x^2-25}$



$$f(x) = \frac{x-3}{(x+5)(x-5)}$$

y-int (x=0)

$(0, -3/25)$

x-int (y=0)

$(3, 0)$

Vertical Asymptotes

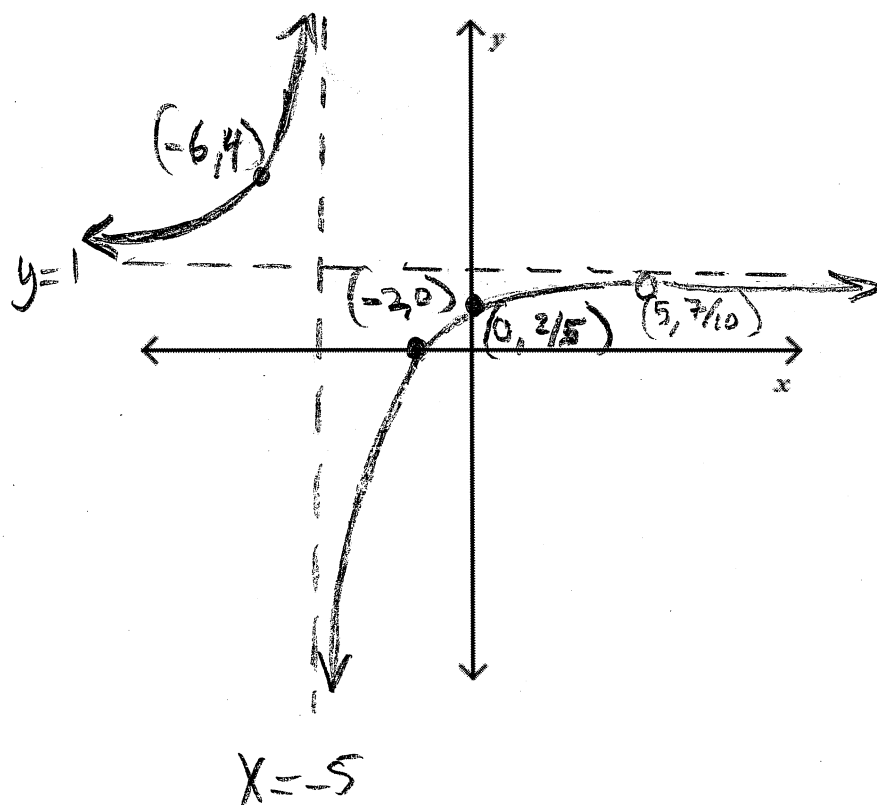
$x = -5$

$x = 5$

Horizontal Asymptote

$y = 0$

4.  $f(x) = \frac{x^2-3x-10}{x^2-25}$



$$f(x) = \frac{(x-5)(x+2)}{(x-5)(x+5)}$$

Hole @  $x=5 \rightarrow (5, 7/10)$

y-int  $(0, 2/5)$

x-int  $(-2, 0)$

Vertical Asymptote:  $x = -5$

Horizontal Asymptote:  $y = 1$

## Radical Equations

Objectives: The Student will be able to –

- Solve Radical Equations
- Identify Extraneous Solutions

Solving Radical Equations

<https://www.youtube.com/watch?v=0gicD4STzpg>

<https://www.youtube.com/watch?v=g3rzugglglw>

<https://www.youtube.com/watch?v=y4C81qAa3pY>

Solving Radical Equations – Extraneous Solution

<https://www.youtube.com/watch?v=m4eiYHL3PP8&v=en-US>

Solve the following Radical Equations:

1.  $\sqrt{x} = 5$

$x = 25$

2.  $\sqrt{x-7} = 8$

$x-7 = 64$

$x = 71$

3.  $\sqrt{2x+3} = 15$

$2x+3 = 225$

$2x = 222$

$x = 111$

4.  $5\sqrt{x+2} = 25$

$\sqrt{x+2} = 5$

$x+2 = 25$

$x = 23$

5.  $\sqrt{x+2} = 1 + \sqrt{x}$

$x+2 = (1+\sqrt{x})^2$

$x+2 = 1 + 2\sqrt{x} + x$

$2 = 1 + 2\sqrt{x}$

$1 = 2\sqrt{x}$

$\frac{1}{2} = \sqrt{x}$

$\frac{1}{4} = x$

$(1+\sqrt{x})(1+\sqrt{x})$

$1 + \sqrt{x} + \sqrt{x} + x$

$1 + 2\sqrt{x} + x$

## Graphing Radical Functions:

Objectives: The Student will be able to –

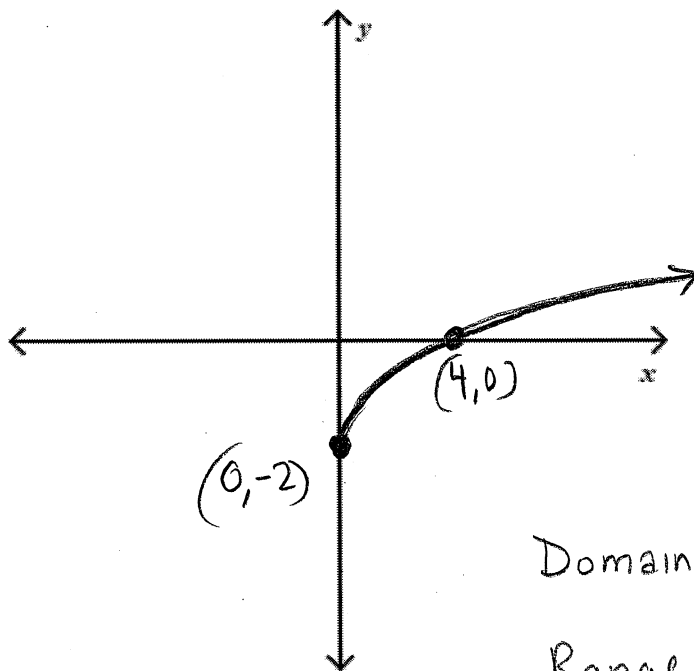
- Identify the intercepts of a radical function.
- Graph a radical function.

<https://www.khanacademy.org/math/algebra-home/alg-radical-eq-func/alg-graphs-of-radical-functions/v/graphs-of-square-root-functions>

<https://www.youtube.com/watch?v=SLV600BpHpE>

Graph the Following:

1.  $f(x) = \sqrt{x} - 2$

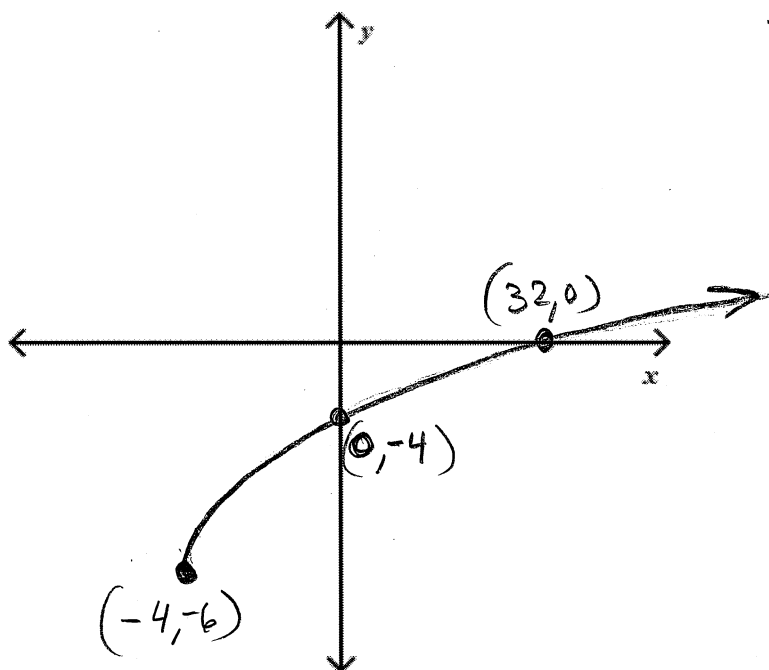


Domain:  $[0, \infty)$

Range:  $[-2, \infty)$

<u>y-int (<math>x=0</math>)</u>	<u>x-int (<math>y=0</math>)</u>
$y = \sqrt{x} - 2$	$0 = \sqrt{x} - 2$
$y = \sqrt{0} - 2$	$2 = \sqrt{x}$
$y = -2$	$4 = x$
$(0, -2)$	$(4, 0)$

2.  $f(x) = \sqrt{x+4} - 6$



y-int (x=0)

$$y = \sqrt{x+4} - 6$$

$$y = \sqrt{4} - 6$$

$$y = 2 - 6$$

$$y = -4$$

$$(0, -4)$$

x-int (y=0)

$$0 = \sqrt{x+4} - 6$$

$$6 = \sqrt{x+4}$$

$$36 = x+4$$

$$32 = x$$

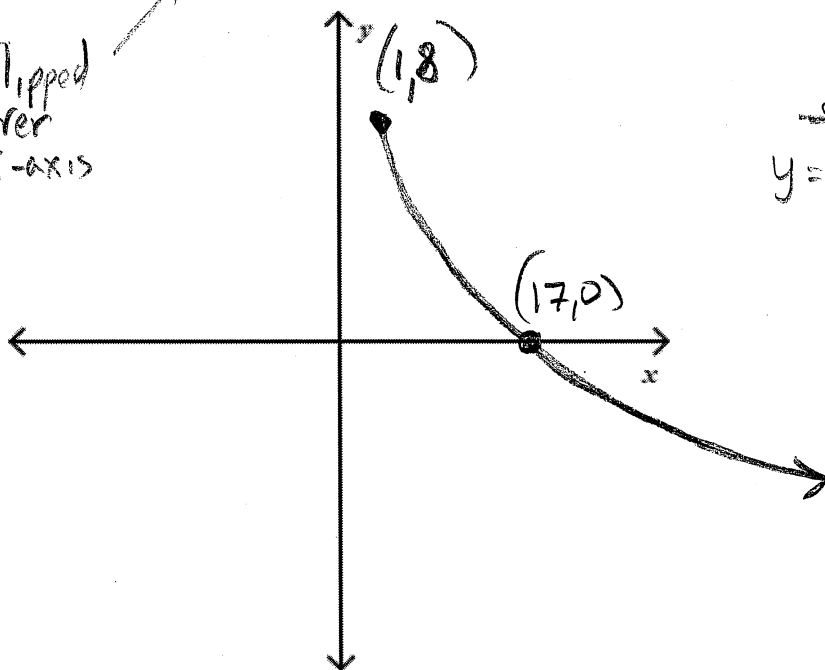
$$(32, 0)$$

Domain:  $[-4, \infty)$

Range:  $[-6, \infty)$

3.  $f(x) = -2\sqrt{x-1} + 8$

Flipped  
over  
x-axis



y-int (x=0)

$$y = -2\sqrt{-1} + 8$$

not real

no y-int

x-int (y=0)

$$0 = -2\sqrt{x-1} + 8$$

$$-8 = -2\sqrt{x-1}$$

$$4 = \sqrt{x-1}$$

$$16 = x-1$$

$$17 = x$$

$$(17, 0)$$

Domain:  $[1, \infty)$

Range:  $(-\infty, 8]$

## Special Triangles:

<https://www.khanacademy.org/math/geometry-home/right-triangles-topic/special-right-triangles/v/45-45-90-triangles>

<https://www.youtube.com/watch?v=p70UBGCHZrQ>