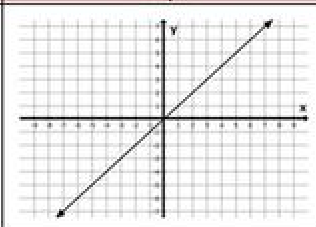
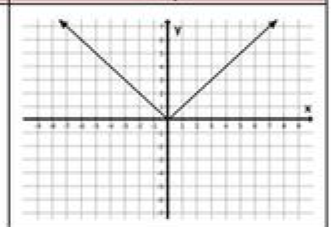
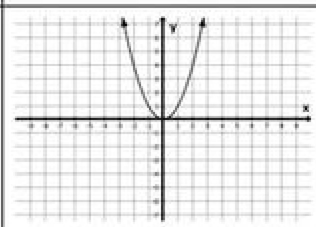
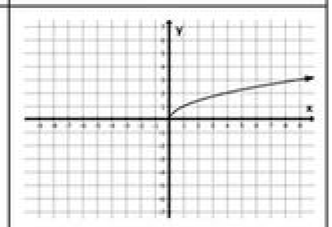
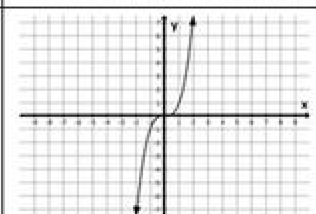
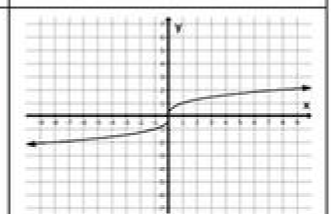
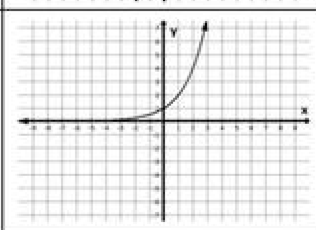
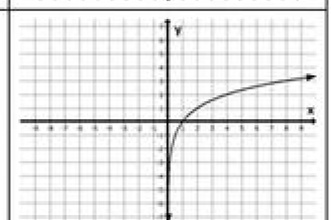
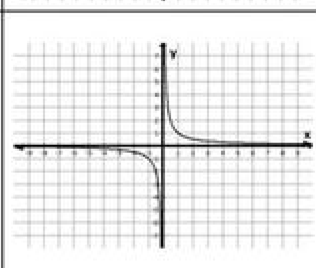
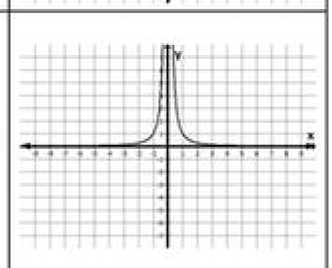
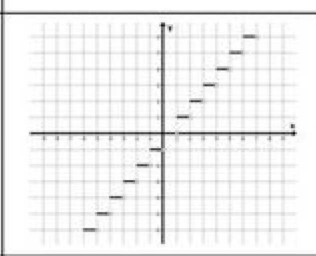
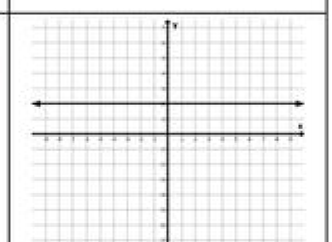
AP Calculus AB Summer Assignment

This packet includes Pre-Calculus topics that are essential to your success in AP Calculus. This packet will not be graded, HOWEVER, you will be tested on this material within the first few days of school. So, you are responsible for knowing this material on the first day of class.

Parent Functions

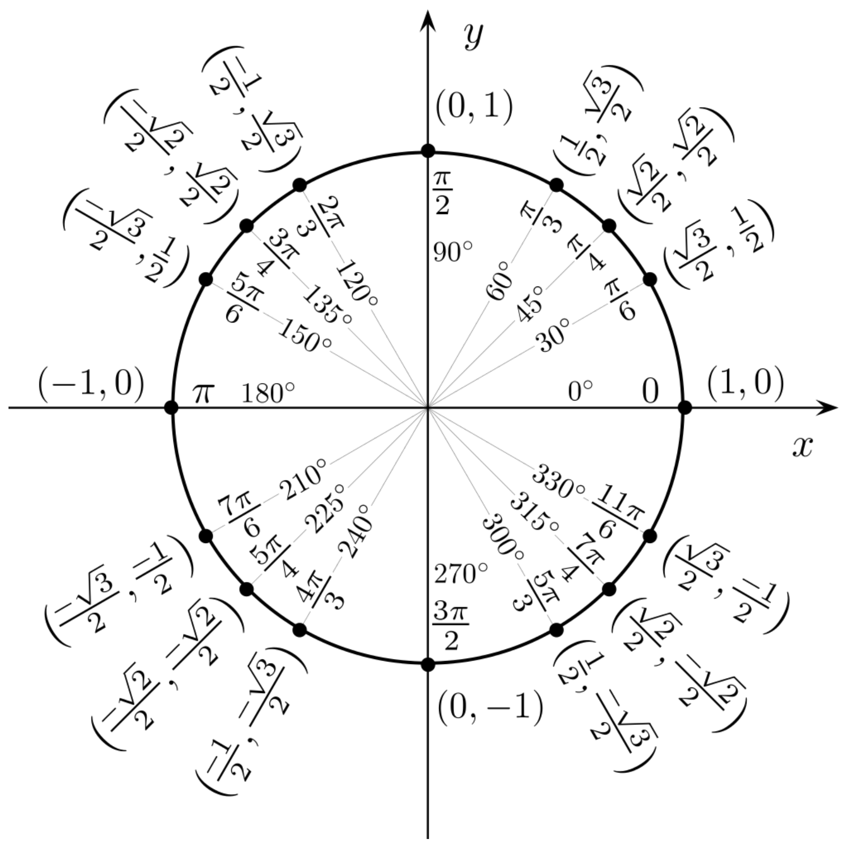
You must know the parent functions below along with key information like domain, range, end behavior, and whether the function is even, odd, or neither.

Parent Function	Graph	Parent Function	Graph
$y=x$ **Linear**, Odd Domain: $(-\infty,\infty)$ Range: $(-\infty,\infty)$ End Behavior: $x\to-\infty,\ y\to-\infty$ $x\to\infty,\ y\to\infty$		$y=\lvert x\rvert$ **Absolute Value**, Even Domain: $(-\infty,\infty)$ Range: $[0,\infty)$ End Behavior: $x\to-\infty,\ y\to\infty$ $x\to\infty,\ y\to\infty$	
$y=x^2$ **Quadratic**, Even Domain: $(-\infty,\infty)$ Range: $[0,\infty)$ End Behavior: $x\to-\infty,\ y\to\infty$ $x\to\infty,\ y\to\infty$		$y=\sqrt{x}$ **Radical**, Neither Domain: $[0,\infty)$ Range: $[0,\infty)$ End Behavior: $x\to\infty,\ y\to\infty$	
$y=x^3$ **Cubic**, Odd Domain: $(-\infty,\infty)$ Range: $(-\infty,\infty)$ End Behavior: $x\to-\infty,\ y\to-\infty$ $x\to\infty,\ y\to\infty$		$y=\sqrt[3]{x}$ **Cube Root**, Odd Domain: $(-\infty,\infty)$ Range: $(-\infty,\infty)$ End Behavior: $x\to-\infty,\ y\to-\infty$ $x\to\infty,\ y\to\infty$	
$y=b^x,\ b>1$ **Exponential**, Neither Domain: $(-\infty,\infty)$ Range: $(0,\infty)$ End Behavior: $x\to-\infty,\ y\to0$ $x\to\infty,\ y\to\infty$		$y=\log_b(x),\ b>1$ **Log**, Neither Domain: $(0,\infty)$ Range: $(-\infty,\infty)$ End Behavior: $x\to0^+,\ y\to-\infty$ $x\to\infty,\ y\to\infty$	
$y=\frac{1}{x}$ **Rational (Inverse)**, Odd Domain: $(-\infty,0)\cup(0,\infty)$ Range: $(-\infty,0)\cup(0,\infty)$ End Behavior: $x\to-\infty,\ y\to0$ $x\to\infty,\ y\to0$		$y=\frac{1}{x^2}$ **Rational (Inverse Squared)**, Even Domain: $(-\infty,0)\cup(0,\infty)$ Range: $(0,\infty)$ End Behavior: $x\to-\infty,\ y\to0$ $x\to\infty,\ y\to0$	
$y=\text{int}(x)=[x]$ **Greatest Integer**, Neither Domain: $(-\infty,\infty)$ Range: $\{y: y\in\mathbb{Z}\}$ (integers) End Behavior: $x\to-\infty,\ y\to-\infty$ $x\to\infty,\ y\to\infty$		$y=C$ ($y=2$ in the graph) **Constant**, Even Domain: $(-\infty,\infty)$ Range: $\{y: y=C\}$ End Behavior: $x\to-\infty,\ y\to C$ $x\to\infty,\ y\to C$	

** You must have the point-slope equation of a line **memorized**! It is $y-y_1=m(x-x_1)$

Trigonometry

You must know the unit circle by memory. Along with this, you will need to be able to evaluate trig functions at special angle measures, solve trig equations, and apply trig identities.



θ	$0^{\circ}/\,0$	$30^{\circ}/\,\frac{\pi}{6}$	$45^{\circ}/\,\frac{\pi}{4}$	$60^{\circ}/\,\frac{\pi}{3}$	$90^{\circ}/\,\frac{\pi}{2}$	$180^{\circ}/\,\pi$	$270^{\circ}/\,\frac{3\pi}{2}$	$360^{\circ}/\,2\pi$
$\sin\theta$								
$\cos\theta$								
$\tan\theta$								
$\csc\theta$								
$\sec\theta$								
$\cot\theta$								

*This table includes special angles in Quadrant I and the Quadrantal angles. You must also be able to evaluate trig functions for angles in Quadrants II, III, and IV (e.g. $\sin\frac{5\pi}{3}$, $\cos\frac{5\pi}{6}$, etc.)

Solve the following on the interval $[0, 2\pi]$.

1. $2\cos x + 4 = 5$
2. $5\sin x - \sqrt{3} = 3\sin x$
3. $2\cos^2\theta - \cos\theta = 0$
4. $\sin^2\theta - 1 = 0$
5. $2\cos^2 B - \cos B - 1 = 0$
6. $4\cos^2\alpha - 3 = 0$

Know and be able to apply trig identities.

Quotient Identities

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\cot\theta = \frac{\cos\theta}{\sin\theta}$

Reciprocal Identities

$\cot\theta = \frac{1}{\tan\theta}$

$\csc\theta = \frac{1}{\sin\theta}$

$\sec\theta = \frac{1}{\cos\theta}$

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

Sum Identities
Addition Formulas

$\sin(a+b) = \sin a\cos b + \cos a\sin b$

$\cos(a+b) = \cos a\cos b - \sin a\sin b$

$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a\tan b}$

Difference Identities
Subtraction Formulas

$\sin(a-b) = \sin a\cos b - \cos a\sin b$

$\cos(a-b) = \cos a\cos b + \sin a\sin b$

$\tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a\tan b}$

Double Angle Formulas

$\sin 2a = 2\sin a\cos a$

$\cos 2a = \cos^2 a - \sin^2 a$

$= 2\cos^2 a - 1$

$= 1 - 2\sin^2 a$

$\tan 2a = \frac{2\tan a}{1 - \tan^2 a}$

Co-function Identities

$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$

$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$

$\cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$

$\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$

$\csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta$

$\sec\left(\frac{\pi}{2} - \theta\right) = \csc\theta$

Even-Odd Identities

$\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta$

$\csc(-\theta) = -\csc\theta$

$\sec(-\theta) = \sec\theta$

$\cot(-\theta) = -\cot\theta$

Half-Angle Formulas

$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}$

$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$

$\tan\left(\frac{\theta}{2}\right) = \frac{1-\cos\theta}{\sin\theta}$

$= \frac{\sin\theta}{1+\cos\theta}$

$= \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$

Other Important Topics

Algebra is used heavily in Calculus. Be able to solve these types of problems. It would not hurt to brush up on any other algebra skills not included in this packet.

Domain and Range

Example problem: Find the domain and range of each function. Write your answer in interval notation. $f(x) = \sqrt{4 - x^2}$ (See solution on the next page)

Solution Domain: $f(x)$ is defined only when $4 - x^2 \geq 0$. This avoids negative values under the square root (complex numbers). This is true when $-2 \leq x \leq 2$. So, in interval notation, the domain is $[-2, 2]$. Any x values outside of this interval result in negative values under the square root.

Range: the y values range from $[0, 2]$. The smallest $f(x)$ can be is 0. This happens when $x = -2$ and $x = 2$. The largest $f(x)$ can be is $\sqrt{4} = 2$. This happens when $x = 0$ This is the equation of the top half of a semicircle with radius 2. So, the range of $f(x)$ is $[0, 2]$. You try now!

7. $f(x) = x^2 - 5$
8. $f(x) = \sqrt{x + 3}$
9. $f(x) = 3 \sin x$
10. $f(x) = \frac{2}{x-1}$

Logarithms

$y = b^x$ can be written as $x = \log_b y$. These mean the same thing. A logarithm is an exponent!

Recall that $\ln x = \log_e x$, where the value of $e = 2.718281828...$

Evaluate the following logarithms.

11. $\log_7 7$
12. $\log_3 27$
13. $\log_2 \frac{1}{32}$
14. $\log_{25} 5$
15. $\log_9 1$
16. $\ln \sqrt{e}$
17. $\ln \frac{1}{e}$

Properties of Logarithms

Be able to apply the following log properties.

$\log_b xy = \log_b x + \log_b y$

$\log_b \frac{x}{y} = \log_b x - \log_b y$

$\log_b x^y = y \log_b x$

$b^{\log_b x} = x$

Use the properties of logarithms to evaluate the following.

18. $\log_2 2^5$
19. $\ln e^3$
20. $\log_2 8^3$
21. $2^{\log_2 10}$
22. $e^{\ln 8}$
23. $\log_3 \sqrt[5]{9}$
24. $\log 25 + \log 4$
25. $\log_2 40 - \log_2 5$
26. $\log_2 \left(\sqrt{2}\right)^5$

You must have the following **MEMORIZED.

$\ln 1 = 0$

$\ln e = 1$