FAIRFIELD PUBLIC SCHOOLS

Algebra I

Course Information

Grade(s):	9-12
Discipline/Course:	Math / Algebra I
Course Title:	Algebra I
Prerequisite(s):	Successful completion of one of the following: Pre-Algebra 8 Foundations of Algebra
Course Description: *Program of Studies*	The fundamental purpose of this course is to formalize and extend the mathematics that students learned in the middle grades. The critical areas deepen and extend understanding of linear relationships to quadratics by contrasting them with each other and by applying linear models to data that exhibit a linear trend. Students also engage in methods for analyzing, solving, and using quadratic functions. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations. The course has additional content standards beyond the Algebra I course as well as an increased focus on rigor and depth of study. Strong pre-algebra skills are required.
Course Essential Questions:	• What is a function and how can understanding function families help us to solve problems? • Why are linear, exponential, and quadratic functions useful in real-world settings? • How can we use mathematical modeling to solve problems?
Course Enduring Understandings:	• Real world situations can be represented symbolically and graphically. • Algebraic expressions and equations generalize relationships from specific cases.
Duration/ Credit:	Full Year /1.0 Credit(s)
Course Materials/Resources:	*Pre-AP Algebra 1 Course framework* *Illustrative Mathematics Algebra I* - McGraw Hill (2020)
FPS Course	Exploring and Understanding

Academic Expectation(s):	Conveying Ideas
Year at a Glance (Units):	Unit 1: Linear Functions and Linear Equations (~ 9-11 weeks) Unit 2: Quadratic Functions (~ 9-11 weeks) Unit 3: Exponent Properties and Exponential Functions (~ 5-7 weeks) Unit 4: Systems of Linear Equations and Inequalities (~ 5-7 weeks)

Unit Number and Title:	Unit 1: Linear Functions and Linear Equations
Duration:	~ 9-11 weeks
Unit Overview:	Linear relationships are among the most prevalent and useful relationships in mathematics and the real world. Any equality in two variables that exhibits a constant rate of change for these variables is linear. Real-world contexts that have a constant rate of change and data sets with a nearly constant rate of change can be effectively modeled by a linear function. Students explore all aspects of linear relationships in this unit: contextual problems that involve constant rate of change, lines in the coordinate plane, arithmetic sequences, and algebraic means of expressing a linear relationship between two quantities. Through this unit, students develop deep skills with linear functions and equations and an appreciation for the simplicity and power of linear functions as building blocks of all higher mathematics.
Learning Goals	
Standard(s):	**FUNCTIONS** **Interpreting Functions (IF)** **Understand the concept of a function and use function notation.** F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then *f(x)* denotes the output of f corresponding to the input *x*. The graph of f is the graph of the equation $y = f(x)$. F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. **Interpret functions that arise in applications in terms of the context.** F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

	Analyze functions using different representations. F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear functions ~~and quadratic functions~~ and show intercept~~s, maxima, and minima.~~ b. Graph ~~square root, cube root, and~~ piecewise-defined functions, ~~including step functions and absolute value functions.~~ (linear piecewise only) F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum. **Linear, Quadratic, and Exponential Models (LE)** **Construct and compare linear, quadratic, and exponential models and solve problems.** F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. F.LE.2 Construct linear ~~and exponential functions,~~ including arithmetic ~~and geometric~~ sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). **Interpret expressions for functions in terms of the situation they model.** F.LE.5.

	Interpret the parameters in a linear ~~or exponential function~~ in terms of a context. **Building Functions (BF)** **Build a function that models a relationship between two quantities.** F.BF.1 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). (Only linear functions.) a. Determine an explicit expression, a recursive process, or steps for calculation from a context. F.BF.2 Write arithmetic ~~and geometric~~ sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. **Linear, Quadratic, and Exponential Models (LE)** **Construct and compare linear, quadratic, and exponential models and solve problems.** F.LE 1 Distinguish between situations that can be modeled with linear functions. a. Prove that linear functions grow by equal differences over equal intervals... over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. (Only linear functions.) **ALGEBRA** **Reasoning with Equations and Inequalities (REI)** **Understand solving equations as a process of reasoning and explain the reasoning.** A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. **Solve equations and inequalities in one variable.**

	A.REI.3 Solve linear equations and inequalities and inequalities in one variable, including equations with coefficients represented by letters. **Represent and solve equations and inequalities graphically.** A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line) **Creating Equations (CED)** **Create equations that describe numbers or relationships.** A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear functions A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods. A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.* **Statistics and Probability** **Interpreting Categorical and Quantitative Data (ID)** **Summarize, represent, and interpret data on two categorical and quantitative variables.**

	S-ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. b. Fit a linear function for a scatter plot that suggests a linear association. **Interpret linear models.** S-ID.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. **NUMBER AND QUANTITY** **Quantities (Q)** **Reason quantitatively and use units to solve problems.** N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. N.Q.2 Define appropriate quantities for the purpose of descriptive modeling. N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
Essential Question(s):	• What is the meaning of the slope and y-intercept of that line in terms of the problem situation? • What is an equation? • What does equality mean? • How can we use linear equations and linear inequalities to solve real world problems? • What is a solution set for a linear equation or linear inequality? • How can models and technology aid in the solving of linear equations and linear inequalities?

	● How are linear functions and equations related?
Enduring Understanding(s):	Students will understand that ... ● A linear relationship has a constant rate of change, which can be visualized as the slope of the associated graph. ● There are many ways to algebraically represent a linear function and each form reveals different aspects of the function. ● Linear functions can be used to model contextual scenarios that involve a constant rate of change or data whose general trend is linear. ● A solution to a two-variable linear equation or inequality is an ordered pair that makes the equation or inequality true.
Learning Goal(s): *Students will know and will be able to use their learning to:* (Content/ Skills)	**Content:** (Students will know/understand…) ● The graph of a direct variation whose domain is all real numbers is a non-vertical and non-horizontal line that contains the origin. ● Direct variation is a special case of a linear relationship where one quantity is proportional to another quantity. Two quantities vary directly if the ratio $\frac{y}{x}$ is constant for all (x,y) pairs. ● A direct variation can be expressed in the algebraic form $y = kx$, where k is a non-zero constant. ● Rate of change describes how two quantities change together. The constant rate of change of a linear relationship is the slope of the line of its associated graph. ● The constant rate of change of a linear relationship can be calculated by finding the ratio of the change in the output to the change in the input using any two distinct ordered pairs and the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ ● Given any point, the slope of a line can be used to generate all points on the graph of the line that passes through the point. ● If the relationship represented in a table of values has a constant rate of change, then the points on the associated graph will lie on a line.

	• In a table of linearly related values where successive input values differ by a constant or varying amount (e.g., differ by 1), successive output values will also differ by a constant or varying amount, respectively. • An arithmetic sequence is a linear relationship whose domain consists of consecutive integers. • The differences between successive terms of an arithmetic sequence are equal. • An arithmetic sequence can be determined using the constant difference and any term in the sequence. • The graph of an arithmetic sequence is a set of discrete points that lie on a line. • Successive terms in an arithmetic sequence are obtained by adding the common difference to the previous term. To find the value of the term that occurs n terms after a specified term, add the common difference n times to the term. • An arithmetic sequence can be algebraically expressed with the $a_n = a_k + d(n - k)$ where a_n is the nth term, a_k is the kth term, and d is the constant difference between successive terms. • A verbal representation of an arithmetic sequence describes a discrete domain and a constant rate of change. • A function is a type of relationship between two quantities where each input is related to one (and only one) value of the output. • The domain of a function is the set of all inputs for the function. The range of a function is the set of all outputs for the function resulting from the set of inputs. • The notation "$f(x)$)" is read as "$f\ of\ x$" is the name of a function, "x" stands for any input value in the domain of the function, and "$f(x)$" represents the output value in the range of the function that corresponds to the input value. • Any solution (x, y) to the equation $y = f(x)$ represents a point that lies on the graph of function f. • A graphical representation of a linear function displays ordered pairs satisfying the relationship. • Any two distinct ordered pairs can be used to generate a graph of the relationship and compute the constant rate of change. • Common algebraic forms of linear functions include point–slope form, $y = y_1 + m(x - x_1)$ and slope–intercept form, $y = mx + b$.

	• Slope–intercept form is a special case of point–slope form where $(x_1, y_1) = (0, b)$ • Linear functions and a linear equation can be used to model contextual scenarios that involve a constant rate of change of a dependent variable (the output) with respect to an independent variable (the input). • A linear function derived from a contextual scenario can be solved free of context, but the solution must be interpreted in context to be correctly understood. The solution derived from a contextual scenario should use the same units as the variables in the contextual scenario. • Two distinct input–output pairs from a contextual scenario that involves a constant rate of change can be used to determine a linear function that models the scenario. • A constant rate of change and a corresponding input–output pair from a contextual scenario can be used to determine a linear function that models the scenario.7 • A graphical representation of a linear equation is a set of ordered pairs that satisfy the relationship. • A numerical representation of a linear equation consists of only a subset of the ordered pairs that satisfy the relationship. It can be used to generate a graph of the relationship. • An algebraic representation of a linear equation often takes the form $Ax + By = C$, where the parameters A, B, and C are non-zero constants. This form is called the standard form of a linear equation. • A linear equation in two variables can be used to represent contextual scenarios where there exists a constraint or condition on the variables and neither variable is necessarily considered an input or output. • A solution and solution set to a linear equation, $Ax + By = C$, is an ordered pair (x, y) that makes the equation true. • The slopes of parallel lines are equal, and two distinct lines with equal slopes are parallel. • The slopes of non-vertical and non-horizontal perpendicular lines are multiplicative inverses of each other with opposite signs. • A vertical line is perpendicular to a horizontal line, and vice versa. • An equation for a line parallel or perpendicular to a given line can be determined using the slope of the given line and a point not on the given line. • A piecewise linear function consists of two or more linear functions, each restricted to non-overlapping intervals of input values.

	- A contextual scenario that involves different constant rates of change over different intervals of the domain can be modeled using a piecewise linear function. - Sets of data that show a graphically upward trend (as the input value increases) are said to have a positive association. - Sets of data that show a graphically downward trend (as the input value increases) are said to have a negative association. - The trend line, and its equation, describes an observed relationship between the variables in a scatterplot but may or may not contain any of the data points, so values predicted using the model can be expected to differ from actual values. - Relationships derived from data usually have limited domains beyond which the trend line might become an increasingly poor model. - A graphical representation of a linear inequality is a set of ordered pairs that satisfy the relationship. - An algebraic representation of a linear inequality can relate the expressions $Ax + By$ and C, where the parameters A, B, and C are non-zero constants, with an inequality symbol ,<, ≤, >, or ≥. - A linear inequality is useful for modeling contextual scenarios that include resource limitations, goals, constraints,comparisons, and tolerances. - A solution to an inequality in two variables is an ordered pair that makes the inequality true. - A solution to a two-variable linear inequality that represents a contextual scenario is a pair of numbers that satisfies the constraints of the contextual scenario. - Adding the same real number to or subtracting the same real number from both sides of an inequality does not change the inequality relationship. - Multiplying both sides of an inequality by the same positive real number or dividing both sides of an inequality by the same positive or real number does not change the inequality relationship. - Multiplying both sides of an inequality by the same negative real number or dividing both sides of an inequality by the same negative real number reverses the direction of the inequality relationship. **Skills:** (Students will be able to…)

	• Determine whether two quantities vary directly given a relationship represented graphically, numerically, algebraically, or verbally. • Calculate the constant rate of change of a linear relationship. • Create a graphical or numerical representation of a linear relationship given its constant rate of change. • Determine whether a relationship presented graphically or numerically is linear by examining the rate of change. • Determine whether a relationship is linear or nonlinear based on a numerical sequence whose indices increase by a constant amount. • Convert a given representation of an arithmetic sequence to another representation of the arithmetic sequence. • Use function notation to describe the relationship between an input–output pair of a function. • Convert a given representation of a linear function to another representation of the linear function. • Translate between algebraic forms of a linear function, using purposeful algebraic manipulation. • Model a contextual scenario with a linear function. • Convert a given representation of a linear equation to another representation of that linear equation. • Interpret the solutions to a two-variable linear equation. • Rewrite a two-variable linear equation in terms of one of the variables to preserve the solution set, using purposeful algebraic manipulation. • Construct representations of parallel or perpendicular lines. • Interpret a graphical representation of a piecewise linear function in context. • Construct a graphical representation of a piecewise linear function to model a contextual scenario. • Determine whether the scatterplot of the relationship between two quantities can be reasonably modeled by a linear model. • Determine an equation for a trend line that describes trends in a scatterplot. • Use an equation for a trend line to predict values in context.

	● Convert a given representation of a linear inequality to another representation of the linear inequality. ● Determine solutions to a two-variable inequality. ● Rewrite a linear inequality in terms of one of the variables to preserve the solution set, using purposeful algebraic manipulation.

Unit Number and Title:	Unit 2: Quadratic Functions
Duration:	~ 9-11 weeks
Unit Overview:	In this unit, students develop a strong foundation in the important concept of quadratic functions. Students should understand that quadratic functions have a linear rate of change and are often formed by multiplying two linear expressions, and therefore are not linear. Quadratic functions are useful for modeling phenomena that have a linear rate of change and symmetry around a unique minimum or maximum. This foundational understanding of quadratics helps students build their conceptual knowledge of nonlinear functions and prepares them for further study of polynomial and rational functions in Algebra 2.
Learning Goals	
Standard(s):	**ALGEBRA** **Seeing Structure in Expressions (SSE)** **Interpret the structure of expressions.** A.SSE.1 Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1+r)^n$ *as the product of P and a factor not depending on P.* **Write expressions in equivalent forms to solve problems.** A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

	Reasoning with Equations and Inequalities (REI) **Understand solving equations as a process of reasoning and explain the reasoning.** A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. **Solve equations and inequalities in one variable.** A.REI.4 Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. ~~Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b~~. (only real solutions in Algebra-1) **Represent and solve equations and inequalities graphically.** A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line) **Arithmetic with Polynomials and Rational Expressions (APR)** **Perform arithmetic operations on polynomials.** A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. (e.g, multiply two binomials, or a binomial with a trinomial)

	Creating Equations (CED) **Create equations that describe numbers or relationships.** A.CED.1 Create equations and inequalities in one variable and use them to solve problems. A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. **FUNCTIONS** **Interpreting Functions (IF)** **Understand the concept of a function and use function notation.** F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then *f(x)* denotes the output of f corresponding to the input *x*. The graph of f is the graph of the equation $y = f(x)$. F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. **Interpret functions that arise in applications in terms of the context.** F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function *h(n)* gives the number of

	person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Analyze functions using different representations F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph ~~linear and~~ quadratic functions and show intercepts, maxima, and minima. F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). **Building Functions (BF)** **Build a function that models a relationship between two quantities.** F.BF.1 Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. **Build new functions from existing functions.**

	F.BF.3 Identify the effect on the graph of replacing *f(x)* by *f(x) + k*, *kf(x)*, *f(kx)*, and *f(x + k)* for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. **NUMBER AND QUANTITY** **Quantities (Q)** **Reason quantitatively and use units to solve problems.** N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. N.Q.2 Define appropriate quantities for the purpose of descriptive modeling. N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
Essential Question(s):	● What are the similarities and differences between quadratic and linear functions? ● How do changes in the values of the parameters in a quadratic function change the behavior of the graph? ● What is the relationship between the number of real roots and the graph of a quadratic equation? Why does this relationship exist? ● How can you translate among the vertex, standard, and factored forms of quadratic equations? What is useful about each form? ● What are the characteristics of quadratic change as shown in a graph, function rule, table, or real-world situation? ● What is the relationship between the number of real roots and the graph of a quadratic equation? Why does this relationship exist?

	● How can completing the square be used to prove the quadratic formula? ● What are some clues that will allow you to choose an appropriate method to solve a quadratic equation; factoring, completing the square, using the quadratic formula, graphing, using a table? ● What are the possible numbers of solutions for a system of equations where one equation is linear and the other is quadratic?
Enduring Understanding(s):	Students will understand that… ● Quadratic functions have a linear rate of change. ● Quadratic functions can be expressed as a product of linear factors. ● Quadratic functions can be used to model scenarios that involve a linear rate of change and symmetry around a unique minimum or maximum. ● Every quadratic equation, $ax^2 + bx + c = 0$, where a is not zero, has at most two real solutions. These solutions can be determined using the quadratic formula.
Learning Goal(s): *Students will know and will be able to use their learning to:* (Content/ Skills)	**Content:** (Students will know/understand…) ● In a table of values that represents a quadratic relationship and that has constant step sizes, the differences in the values of the relationship, called the first differences, exhibit a linear pattern. The second differences of a quadratic sequence are constant. ● A graphical representation of a quadratic function displays ordered pairs that satisfy the relationship. ● An algebraic form of a quadratic function contains the complete information about the function because any output value can be determined from any given input value.. ● The graph of a quadratic function is a parabola. The parabola is symmetric about a vertical line that passes through the vertex of the parabola. ● The vertex of a parabola is the point on the curve where the outputs of the function change from increasing to decreasing or vice versa. The y-coordinate of the vertex of a parabola is the maximum or minimum value of the function. ● A parabola can have two x-intercepts, one x-intercept, or no x-intercepts. ● If the vertex of a parabola is a minimum value, then the parabola is said to be concave up. If the vertex is a maximum value, then the parabola is said to be concave down. ● Common algebraic forms of a quadratic function include standard form, $f(x)= ax^2 + bx + c$;

	factored form, $f(x) = (x - r)(x - s)$; and vertex form, $f(x) = a(x - h)^2 + k$, where a is not zero. ● Every quadratic function has a standard form and a vertex form, but not all quadratic functions have a factored form over the real numbers. ● The standard form can be purposefully manipulated into the vertex form of the same quadratic function by completing the square. ● The standard form and the factored form of a quadratic function can be translated into each other with purposeful use of the distributive property. ● The graph of a quadratic function whose standard form is $f(x) = ax^2 + bx + c$ has a vertex with the x-coordinate at $x = \frac{-b}{2a}$. The y-coordinate of the vertex can be calculated by evaluating the function rule using the x-coordinate of the vertex. The graph is symmetric about the vertical line $x = \frac{-b}{2a}$. ● The graph of a quadratic function whose factored form is $f(x) = (x - r)(x - s)$, where and, has two x-intercepts, at $(r, 0)$ and $(r, 0)$. ● The graph of a quadratic function whose vertex form is $f(x) = a(x - h)^2 + k$ has a vertex at the coordinate (h, k). ● If two distinct inputs of a quadratic function are associated with equal outputs, then the x-coordinate of the vertex of the parabola is located halfway between these inputs. ● An algebraic rule for a quadratic function can be determined from the vertex and one other point on the graph of the parabola. ● If a quadratic equation $ax^2 + bx + c = 0$ has real solutions $x = r$ and s, then the parabola defined by $y = ax^2 + bx + c = 0$ has x-intercepts at $(r, 0)$and $(s, 0)$. ● For any positive real number a, there are two real numbers that satisfy the equation $x^2 = a$, one positive and one negative. ● The notation “ $\sqrt{a}$” represents the square root of a and refers only to the principal square root, or non-negative, number whose square equals a. ● There is no real number that will satisfy the equation $x^2 = a$,when a is a negative real number.

- Factoring a quadratic expression yields an equivalent form of the expression that can be used to determine the roots of the associated quadratic equation.
- The solutions of a quadratic equation in the form $a(x - r)(x - s) = 0$ are $x = r$ and $x = s$ when $a \neq 0$
- Completing the square is an algebraic process of transforming a quadratic expression into a form that can be solved by adding, multiplying, and taking the square root.
- Every quadratic equation can be solved by completing the square, but the solutions may not be real numbers.
- The quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, can be used to solve any quadratic equation of the form $ax^2 + bx + c = 0$, but the solutions may not be real numbers.
- If a quadratic equation has two rational solutions, then it can be factored into linear factors with integer coefficients.
- Given a quadratic equation of the form $ax^2 + bx + c = 0$, the value of the discriminant of the quadratic equation ($D = b^2 - 4ac$) can be used to determine whether the quadratic equation has two distinct real solutions ($D > 0$), one real solution ($D = 0$), or no real solutions ($D < 0$).

Skills: (Students will be able to…)

- Determine whether a relationship is quadratic or non-quadratic based on a numerical sequence whose indices increase by a constant amount.
- Convert a given representation of a quadratic function to another representation of the quadratic function.
- Identify key characteristics of the graph of a quadratic function.
- Translate between algebraic forms of a quadratic function using purposeful algebraic manipulation.
- Describe key features of the graph of a quadratic function in reference to an algebraic form of the quadratic function.

	● Determine an algebraic rule for a quadratic function given a sufficient number of points from the graph. ● Describe the relationship between the algebraic and graphical representations of a quadratic equation. ● Solve quadratic equations by taking a square root. ● Solve quadratic equations by factoring. ● Solve quadratic equations by completing the square. ● Solve quadratic equations using the quadratic formula. ● Determine the number of real solutions to a quadratic equation. ● Model a contextual scenario with a quadratic function. ● Interpret solutions to quadratic equations derived from contextual scenarios. ● Interpret the vertex and roots of a quadratic model in context.

Unit Number and Title:	Unit 3: Exponent Properties and Exponential Functions
Duration:	~ 5-7 weeks
Unit Overview:	Students explore exponent rules as an extension of geometric sequences and the properties of multiplication and division for real numbers. Students should make sense of exponent rules and not simply memorize them without understanding how they arise. The unit culminates in students investigating how exponential functions can model physical phenomena that exhibit a constant multiplicative growth. Exponential functions are framed as multiplicative analogues of linear functions. Thus, a tight connection should be drawn between these two classes of functions and their shared properties.
Learning Goals	
Standard(s):	**NUMBER AND QUANTITY** **The Real Number System (RN)** **Extend the properties of exponents to rational exponents.** N.RN.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5. N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents N.RN.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. **Quantities (Q)**

Reason quantitatively and use units to solve problems.

N.Q.1

Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

N.Q.2

Define appropriate quantities for the purpose of descriptive modeling.

N.Q.3

Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

ALGEBRA

Seeing Structure in Expressions (SSE)

Interpret the structure of expressions.

A.SSE.1

Interpret expressions that represent a quantity in terms of its context.

a. Interpret parts of an expression, such as terms, factors, and coefficients.

b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1+r)^n$ *as the product of P and a factor not depending on P.*

Write expressions in equivalent forms to solve problems.

A.SSE.3

Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

c. Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.

Creating Equations (CED)

Create equations that describe numbers or relationships.

A.CED.1
Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear functions
A.CED.2
Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Reasoning with Equations and Inequalities (REI)

Understand solving equations as a process of reasoning and explain the reasoning.

A.REI.1
Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
A.REI.2
~~Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise~~. Represent and solve equations and inequalities graphically.

Represent and solve equations and inequalities graphically.

A.REI.10
Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line)
A.REI.11
Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, ~~rational, absolute value~~, exponential, and ~~logarithmic functions~~.

FUNCTIONS
Interpreting Functions (IF)

Understand the concept of a function and use function notation.

F.IF.1

Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then *f(x)* denotes the output of f corresponding to the input *x*. The graph of f is the graph of the equation *y = f(x)*.

F.IF.2

Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Understand the concept of a function and use function notation.

F.IF.3

Recognize that sequences are functions, ~~sometimes defined recursively~~, whose domain is a subset of the integers. ~~For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1~~

Interpret functions that arise in applications in terms of the context.

F.IF.4

For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; ~~and periodicity.~~

F.IF.5

Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function *h(n)* gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

Analyze functions using different representations.

F.IF.7

Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

e. Graph exponential ~~and logarithmic functions,~~ showing intercepts and end behavior, ~~and trigonometric functions, showing period, midline, and amplitude~~.

F.IF.8

Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.

F.IF.9

Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

Linear, Quadratic, and Exponential Models (LE)

Construct and compare linear, quadratic, and exponential models and solve problems.

F.LE.1

Distinguish between situations that can be modeled with linear functions and with exponential functions.

a. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

F.LE.2

Construct ~~linear and~~ exponential functions~~, including arithmetic and~~ geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

F.LE.3

	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. **Interpret expressions for functions in terms of the situation they model.** F.LE.5. Interpret the parameters in a ~~linear or~~ exponential function in terms of a context. **Building Functions (BF)** **Build a function that models a relationship between two quantities.** F.BF.1 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). b. Determine an explicit expression, a recursive process, or steps for calculation from a context. F.BF.2 Write ~~arithmetic and~~ geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
Essential Question(s):	● How do exponential functions help us create models to describe data and physical phenomenon and solve problems ● How do we apply properties of exponents to whole number and rational exponents? ● How do we apply properties of exponents to whole number and rational exponents? ● How do we use technology to work with graphs and tables to solve problems and make sense of our world? ● How do we extend known rules of functions to exponential functions (transformations and operations)?
Enduring Understanding(s):	Students will understand that ... ● Properties of exponents are derived from the properties of multiplication and division. ● An exponential function has constant multiplicative growth or decay.

	● Exponential functions can be used to model contextual scenarios that involve constant multiplicative growth or decay. ● Graphs and tables can be used to estimate the solution to an equation that involves exponential expressions.
Learning Goal(s): *Students will know and will be able to use their learning to:* (Content/ Skills)	**Content:** (Students will know/understand…) ● Exponential expressions involving multiplication can be rewritten by invoking the rule $n^a \cdot n^b = n^{a+b}$, where n>0. ● Exponential expressions involving division can be rewritten by invoking the rule $\frac{n^a}{n^b} = n^{a-b}$, where n > 0. ● Exponential expressions involving powers of powers can be rewritten by invoking the rule $(n^a)^b = n^{ab}$, where n>0. ● Any non-zero real number raised to the zero power is equal to 1. That is, $n^0 = 1$, where n does not equal zero. ● Zero raised to the zero power is not defined in the real number system. ● A negative exponent of -1 can be used to represent a reciprocal. That is, $n^{-k} = \frac{1}{n^k}$, where n does not equal zero. ● The properties of negative integer exponents, and those of an exponent of zero, are extensions of the properties of positive integer exponents. ● The value of an irrational number cannot be expressed exactly as a ratio of integers or as a nonrepeating, non-terminating decimal, and is often represented exactly by a symbol such as π or $\sqrt{2}$. ● An irrational number can be approximated to any specified degree of precision by a rational number. ● The square root of a squared real number a is equivalent to the absolute value of the number.

	That is, $\sqrt{a^2} = \|a\|$. ● For any two nonnegative real numbers a and b, the product of their square roots is equal to the square root of their product. That is, the $\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$. ● If a positive real number a can be written as the product of the square of a positive number b and another positive number c—that is, if $a = b^2c$ then the square root of a is equal to the square root of the product of b^2 and c. That is, $\sqrt{a} = \sqrt{b^2c} = \sqrt{b^2} \cdot \sqrt{c} = b\sqrt{c}$ where a, b, and c are all positive. ● The square root of a nonnegative real number can be expressed with the exponent ½. That is, $\sqrt{n} = n^{1/2}$ where $n \geq 0$. ● The cube root of any real number can be expressed with the exponent ⅓. That is, $\sqrt[3]{n} = n^{1/3}$. ● The properties of rational exponents are extensions of the properties of integer exponents. ● A geometric sequence is an exponential relationship whose domain consists of consecutive integers. ● The ratios of successive terms of a geometric sequence are equivalent. ● A geometric sequence can be determined from the common ratio and any term in the sequence. ● The graph of a geometric sequence is a set of discrete points that lie on a curve. ● Successive terms in a geometric sequence are obtained by multiplying the previous term by the common ratio. To find the value of the term that occurs n terms after a specified term, multiply the specified term by the common ratio n times. ● A geometric sequence can be algebraically expressed with the formula $a_n = a_k \cdot b^{(n-k)}$, where a_n is the n^{th} term, a_k is the k^{th} term, and c is the common ratio between successive terms. ● A verbal representation of a geometric sequence describes a discrete domain and a constant multiplicative growth or decay. ● A graph of an exponential function is a curve that exhibits asymptotic behavior to the left or

right.

- The numerical representation of an exponential function will have ordered pairs where, if the inputs differ by a constant amount, then the ratios of corresponding outputs are equivalent.
- An algebraic representation of an exponential function often takes the form $f(x) = a \cdot b^{x}$, $b > 0$, where b is the constant growth or decay factor.
- A verbal representation of an exponential function describes a constant multiplicative growth or decay and known values from the relationship.
- In an exponential relationship, the output values grow by equal factors over equal intervals.
- Given two input–output pairs in an exponential relationship, (a, f(a)) and (b, f(b)) where b is n units more than a, the n-unit growth (or decay) factor is the quotient of the corresponding outputs, $\frac{f(b)}{f(a)}$. If the quotient of any two outputs $\frac{f(b)}{f(a)}$, where $b > a$, is greater than 1, then the value is called a growth factor. If the quotient of any two outputs $\frac{f(b)}{f(a)}$, where $b > a$, is between 0 and 1, then the value is called a decay factor.
- Given any point on the graph of an exponential function, the common growth or decay factor can be used to generate all points on the graph of the curve that contains the point.
- The graphical representation of an exponential function displays ordered pairs that satisfy the relationship.
- A numerical representation of an exponential function consists of only a subset of the ordered pairs that satisfy the relationship, but can be used to compute the common ratio of growth or decay and determine all other aspects of the exponential function.
- An algebraic representation of an exponential function contains the complete information about the function because any output value can be determined from any given input value, or can be used to approximate an input for a specific output.
- A contextual scenario that exhibits constant multiplicative growth or decay in its outputs over equal differences in the corresponding inputs can be modeled by an exponential function.
- Estimated inputs and outputs for exponential functions derived from a contextual scenario can

	be determined free of context, but the values must be interpreted in context to be correctly understood. The estimated inputs and outputs should involve the same units as the variables in the contextual scenario. **Skills:** (Students will be able to…) ● Use exponent rules to express products and quotients of exponential expressions in equivalent forms. ● Use exponent rules to express numerical and variable expressions that involve negative exponents using positive exponents, and vice versa. ● Perform operations with rational and irrational numbers. ● Represent square roots of real numbers in equivalent forms. ● Use the laws of exponents to represent roots of real numbers in terms of rational number powers. ● Determine whether a relationship is exponential or nonexponential based on a numerical sequence whose indices increase by a constant amount. ● Determine whether a relationship is exponential by analyzing a graphical, numerical, algebraic, or verbal representation. ● Calculate a growth or decay factor of an exponential relationship. ● Create graphical or numerical representations of an exponential function using the common growth or decay factor. ● Approximate input and output values of an exponential function using representations of the exponential function. ● Model a contextual scenario with an exponential function.

Unit Number and Title:	Unit 4: Systems of Linear Equations and Inequalities
Duration:	~ 5-7 weeks
Unit Overview:	Across this unit, students are asked to solve systems of equations in support of two goals: to determine the solution to the system of equations and to become strategic and efficient in choosing a method to solve the system. Students use systems of linear equations and systems of linear inequalities to model physical phenomena, especially those with multiple constraints where an optimal solution to an objective function is desired. Through these contexts students build upon their prior knowledge of solving systems of equations and develop more sophisticated understandings about what the solution(s) to a system means in the context of the problem.
Learning Goals	
Standard(s):	**ALGEBRA** **Creating Equations (CED)** **Create equations that describe numbers or relationships.** A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods. A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R. (i.e., rearrange an equation for a particular variable and use this new equation to solve the system of equations by substitution)

	Reasoning with Equations and Inequalities (REI) **Understand solving equations as a process of reasoning and explain the reasoning.** A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. **Solve systems of equations.** A.REI.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. A.REI.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$. **Represent and solve equations and inequalities graphically.** A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line) A.REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half.planes. **NUMBER AND QUANTITY** **Quantities (Q)**

	Reason quantitatively and use units to solve problems. N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. N.Q.2 Define appropriate quantities for the purpose of descriptive modeling. N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
Essential Question(s):	● What does the number of solutions (none, one or infinite) of a system of linear equations represent? ● What are the advantages and disadvantages of solving a system of linear equations graphically versus algebraically? ● Given equations of two lines, how can you determine whether the lines are parallel, perpendicular, intersecting or concurrent? ● What are the possible types of solutions for a system of two linear equations? Give sample equations that demonstrate these types?
Enduring Understanding(s):	Students will understand that ... ● A solution to a system of linear equations or inequalities is an ordered pair of numbers that satisfies all the equations or inequalities simultaneously. ● Solving a system of linear equations or inequalities is a process of determining the value or values that make the equation or inequality true. ● Systems of linear equations or inequalities can be used to model scenarios that include multiple constraints, such as resource limitations, goals, comparisons, and tolerances.
Learning Goal(s): *Students will know and will be able to use their*	**Content:** (Students will know/understand…) ● A solution to a system of linear equations, if one exists, is an intersection point of the lines

learning to: (Content/ Skills)	corresponding to the equations. ● If the graphs of two linear equations in a system are parallel, then the system has no solutions. If the graphs of two linear equations are not parallel and do not coincide, then the system has one solution. If the graphs of two linear equations coincide, then the system has infinitely many solutions. ● Algebraic methods of solving a system of equations include the substitution method and the elimination method. ● A system of linear equations can be used to determine when two linear functions that model a contextual scenario have the same input–output pair. ● A system of linear equations derived from a contextual scenario can be solved free of context, but the solution must be interpreted in context to be correctly understood and should use the same units as the variables. ● If an ordered pair is a solution to a system of inequalities, it will make all inequalities that constitute the system true. ● Every point located in the solution region, or on the boundary if the boundary is included, is a solution to the system of linear inequalities. ● A system of linear inequalities can be used to model a contextual scenario in which the relationship between two quantities is subject to multiple constraints, such as resource limitations, goals, comparisons, and tolerances. ● A solution, if one exists, to a system of inequalities used to model a contextual scenario is a set of values that satisfies the constraints of the scenario. ● A system of linear inequalities that models a contextual scenario can be solved free of context, but any solution must be interpreted in context to be correctly understood and should involve the same units as the variables. **Skills: (Students will be able to…)** ● Use a graph or tables of values to estimate the solution to a system of equations. ● Determine the number of real solutions to a system of two linear equations.

	- Determine the intersection point(s) of the graphs of two functions. - Solve a system of linear equations using algebraic methods. - Model a contextual scenario with a system of linear equations. - Use the rates of change to draw conclusions about a contextual scenario modeled by a system of linear equations. - Use algebra to determine if an ordered pair is a solution to a system of linear inequalities. - Graphically represent the solution to a system of two-variable inequalities. - Model a contextual scenario with a system of linear inequalities.