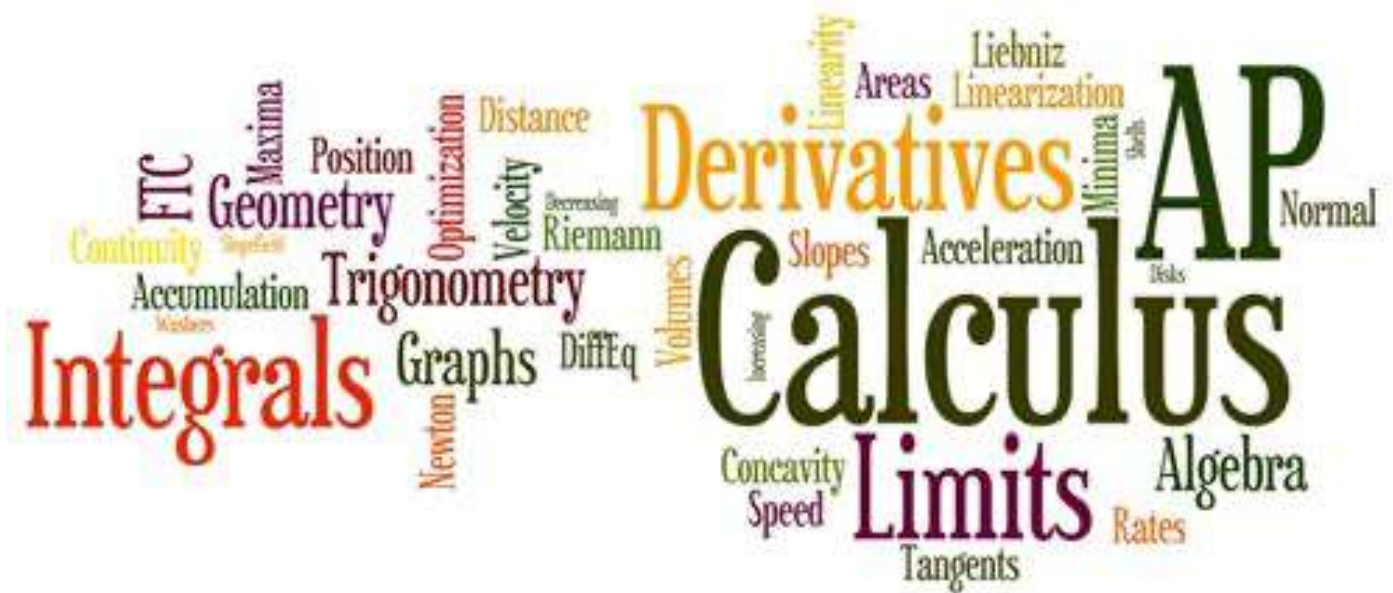# AP Calculus AB
Summer Assignment

June, 2023

Dear Student and Parent(s),

Welcome to **AP Calculus AB**! This review assignment is designed to refresh your Algebra I, Algebra II, Pre-Calculus, and Analytic Geometry skills. It includes information that was taught in these previous courses and will be used throughout the upcoming school year. As you prepare, you may need to seek help by accessing the suggested resources or links provided.

Welcome to your 2023-2024 AP Calculus AB Class! You have electively enrolled in this college level course, where based on your performance on the AP exam in May 2024, you may earn college credits in calculus.

These classes meet promptly four days a week. Attendance will be tracked: 3 tardies *accumulate to one absence. (Please, refer to the BKHS Parent and Student Handbook.)* We need to meet the standardized curriculum by mid-April to be ready for the scheduled national test in May 2024. These advanced placement courses are very demanding and may require additional hours of instruction. Students are strongly encouraged to attend optional office hours in the morning before school. A graphing calculator is required.

We will continue to use the TI-84 plus CE that you have used in your previous classes.

Calculus generalizes ideas and formulas from the constant to varying quantities. To move from the constant to the variable, it involves four stages: pre-calculus, the limit process, derivative calculus, and integral calculus. Congratulations! You have successfully completed the first stage, and you have been introduced to the second phase. I am excited that you are willing to continue to study through the next phases. To be on task for the next stage, I have attached a review assignment **due on the first day of class**, Thursday, August 10th, 2023. This material reviews the major concepts from your previous math classes and highlights the foundational material necessary to be successful in a college level calculus course. To ensure you are prepared, we will go over questions on this material, **and then there will be an assessment of your understanding**.

Have a restful and productive summer. Manage your time wisely. I am looking forward to working with you next year!

Stephanie Jaeger

AP Calculus AB Teacher

Calculus AB - Review Assignment

Flash Cards

     As mentioned above, Calculus truly is a cumulative mathematics class where key problem-solving skills and knowledge of properties and algebraic operations from previous classes is required to be successful. In addition, your verbal language skills are equally important in Calculus as your traditional problem-solving skills. Below is a summary of the most important and commonly occurring vocabulary, formulae, graphs, properties, and concepts from your previous classes. Create Flash Cards for each topic, and I strongly encourage you to begin flipping through them once a week over the summer. The better you know these concepts the easier the year will be! Flash Cards will be <u>checked on the first day of class</u> along with the rest of the summer assignment.

1) Definition: Domain
2) Definition: Range
3) Definition: Average Rate of Change
4) Formula: Point Slope Equation of a Line
5) Definition: Increasing Function
6) Definition: Decreasing Function
7) Definition: Concave Up
8) Definition: Concave Down
9) Formula: Exponential Growth and Decay Equation
10) Even Functions
    a. Examples of Typical Functions
    b. Property of Even Functions
    c. Symmetry of Graph
11) Odd Functions
    a. Examples of Typical Functions
    b. Property of Odd Functions
    c. Symmetry of Graph
12) Function Notation
    a. Definition
    b. Interpreting Units

13) Inverse Functions
    a. Definition
    b. Notation
    c. Interpreting Units
14) Property: Exponent Product Rule
15) Property: Exponent Quotient Rule
16) Property: Exponent Power Rule
17) Property: Negative Exponent Rule
18) Property: Fractional Exponent (Roots) Rule
19) Property: Product Property of Natural Logarithms
20) Property: Quotient Property of Natural Logarithms
21) Property: Power Property of Natural Logarithms
22) Non-Calculator Evaluation: $\ln 1$
23) Non-Calculator Evaluation: $\ln e$
24) Parent Graph: Exponential Functions
25) Parent Graph: Logarithmic Functions
26) Parent Graph: Quadratic Functions
27) Parent Graph: Cubic Functions
28) Parent Graph: Absolute Value Functions
29) Parent Graph: Square Root Functions
30) Parent Graph: Sine Function
31) Parent Graph: Cosine Function
32) Parent Graph: Tangent Function
33) Formula: Pythagorean Theorem
34) Non-Calculator Evaluation: Soh-Cah-Tao
35) Non-Calculator Evaluation: Special Right Triangles (in radians)
36) Property: Converting Trigonometric Angles (Degrees and Radians)
37) Property: Trigonometric Reciprocal Identities
38) Property: Trigonometric Quotient Identities
39) Property: Trigonometric Pythagorean Identities
40) Formula: Trigonometric Double Angle Formulas
41) Formula: Area of Rectangle
42) Formula: Area of a Triangle
43) Formula: Area of a Circle
44) Formula: Circumference of a Circle
45) Formula: Volume of a Cylinder
46) Formula: Volume of a Rectangular Solid
47) Formula: Distance Between Two Points
48) Formula: Factoring $a^2 - b^2$

Review Problems

<u>NON-CALCULATOR</u>

1. Find the Average Rate of Change for the functions $f(x) = 4x^2 - 3x + 2$ over the interval $[1, 4]$

2. Solve the Inequality $x^2 + 5x - 66 \le 0$. Write your answer in interval notation.

3. Find the value of the infinite sum $\displaystyle\sum_{n=0}^{\infty} 18\left(\frac{1}{3}\right)^{n+2}$

4. Evaluate without a calculator $\sin\dfrac{\pi}{3}$, $\cos\dfrac{\pi}{2}$, $\tan\dfrac{\pi}{4}$, $\sin\pi$, $\cos\dfrac{5\pi}{6}$, $\sin\dfrac{3\pi}{2}$

<u>CALCULATOR</u>

5. Find the zeroes for the function $f(x) = \sqrt[3]{e^x} - \sin x - 3$

6. Find the intersection between the functions $f(x) = 2x - 3$ and $g(x) = 4e^{-x+1}$

Part 1: Solving Equations

Solve for x.

1. $2(3^{4x-5}) + 4 = 11$

6. $4lne = 4$

2. $\frac{6x-7}{4} + \frac{3x-5}{7} = \frac{5x+78}{28}$

7. $x^4 - 6x^2 + 8 = 0$

3. $x^3 - 6x^2 - 27x = 0$

8. $2\sin 3x - 1 = 0$

4. $\sqrt{x+1} - 3x = 1$

9. $ln(x-3) + ln(x-2) = \ln(2x+24)$

5. $-\frac{2}{x^2} + \frac{1}{2(x-3)^2} = 0$

10. $2cos^2 x - \sin x = 1$ and $0 \le x \le 2\pi$

Part 2: Equation of a Line

Find equation of the lines passing through (2, 4) and having the following characteristics.

11. Slope of $-\frac{2}{3}$.

12. Perpendicular to the line $x + y = 0$.

13. Passing through the point (6,1).

14. Parallel to the x-axis.

Part 3: Transformations

Sketch the following graphs.

15. The function of *f(x)* is defined: $H(x) = \begin{cases} 1, & x > 0 \\ -1, & x < 0 \end{cases}$
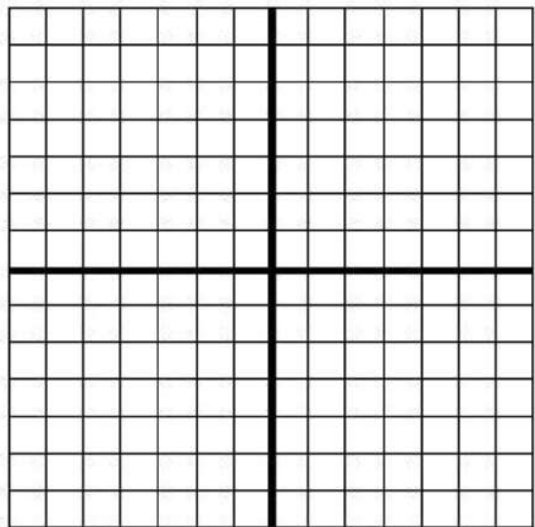
a.) $H(x) + 3$

b.) $H(x - 1)$

c.) $-2H(x)$

d.) $-H(x - 1) + 3$

16. The function of *g(x)* is defined: $g(x) = \begin{cases} \sqrt{x}, & x \geq 0 \\ x, & x < 0 \end{cases}$
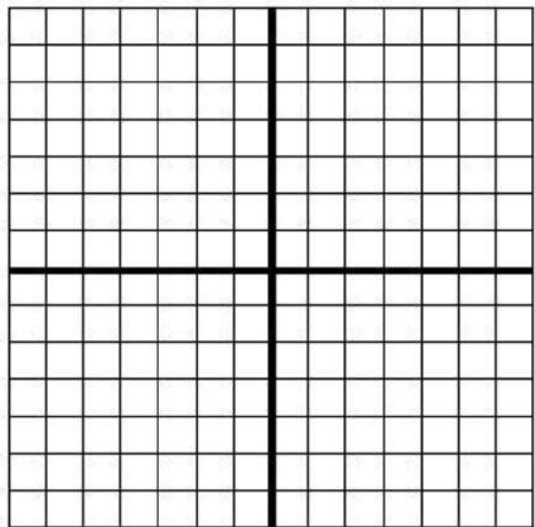
a.) $g(x) - 2$

b.) $g(x - 2)$

c.) $-g(x)$

d.) $-g(x - 2) + 2$

Part 4: Operations with Functions

Find each of the following functions or values given *f(x)* and *g(x)*.

$$f(x) = 1 - x^2 \qquad g(x) = 2x + 1$$

17. $f(x) - g(x)$

18. $f(x)g(x)$

19. $f(g(1))$

20. $g(f(5))$

21. $f(g(x))$

22. $g(f(x))$

23. $g(g(x))$

Part 5: Tangent Lines

24. Consider the circle $x^2 + y^2 - 6x - 8y = 0$

    a.) Find the center and the radius of the circle (change into $(x - a)^2 + (y - b)^2 = r^2$ where (a, b) are the coordinates of the center and r is the radius.

    b.) Find an equation of the tangent line to the circle at the point (0,0).

    c.) Find an equation of the tangent line to the circle at the point (6,0).

    d.) Find the coordinates of the point where the two tangent lines intersect.

Part 6: **Odd, Even, and Inverse Functions**

Determine whether each function is odd, even, or neither.

25. Is this function odd, even, or neither?

$$f(x) = x^6$$

27. Is this function odd, even, or neither?

$$f(x) = \sin(x)$$

26. Is this function odd, even, or neither?

$$f(x) = \cos(\cos(x))$$

28. Is this function odd, even, or neither?

$$f(x) = \tan(\sec(x))$$

29. If $f(x) = \sqrt{5x - 7}$, find the inverse.

30. If $f(x) = \frac{6-5x}{7}$, find the inverse.

31. If $f(x) = 6^{3+2x}$, find the inverse.

Part 7: **Finding Zeroes of a Polynomial Function using Synthetic Division**

32. Given the fact that one root of the given polynomial is $x = 2$, find the other zeroes of $f(x) = 2x^3 - \frac{11}{3}x^2 - x + \frac{2}{3}$.

33. If $(x + 5)$ is a factor of $g(x) = x^3 + 9x^2 + 23x + 15$ find the other two factors.

34. Find all zeroes of $f(x) = x^3 - x^2 - 14x + 24$.

35. Find all zeroes of $h(x) = 2x^4 - x^3 - 18x^2 + 9x$.

Part 8: **System of Equations**

Find where the graphs intersect.

36. $f(x) = 2x + 3$

  $g(x) = -0.5x + 7$

38. $f(x) = 6x^4 - 1$

  $g(x) = 3$

37. $f(x) = x^2$

  $g(x) = x + 2$

39. $y = -\frac{1}{5}x + \frac{1}{5}$

  $x = y^2 - 6y + 1$

Part 9: Piecewise Functions

Sketch the graph of $f(x)$ and find the indicated values.

40. $f(x) = \begin{cases} x - 2, & x < 0 \\ x^2 + 1, & x \geq 0 \end{cases}$
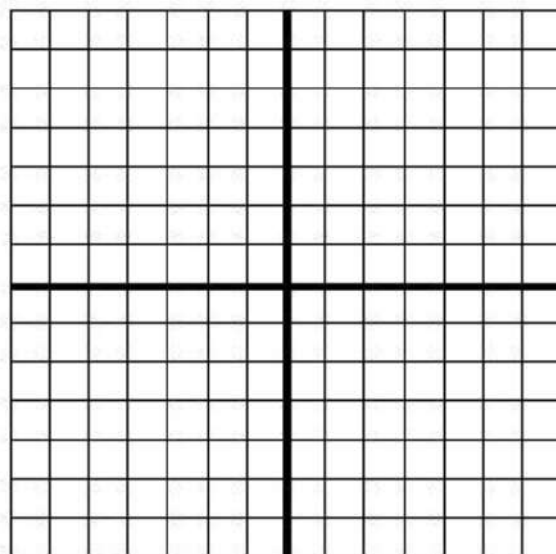
a. $f(-4)$ b. $f(0)$ c. $f(f(-3))$ d. $f(f(0))$

41. $f(x) = \begin{cases} 6 - x, & x \leq 3 \\ \ln x, & x > 3 \end{cases}$
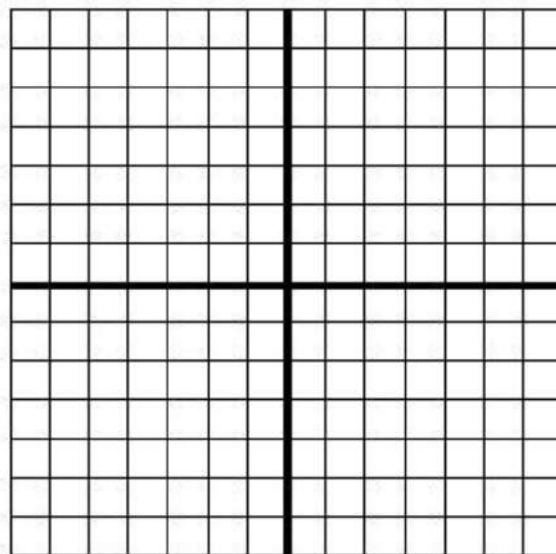
a. $f(3)$ b. $f(5)$ c. $f(f(0))$ d. $f(f(-1))$

42. $f(x) = \begin{cases} \sin x, & x \le 0 \\ x^3, & x > 0 \end{cases}$
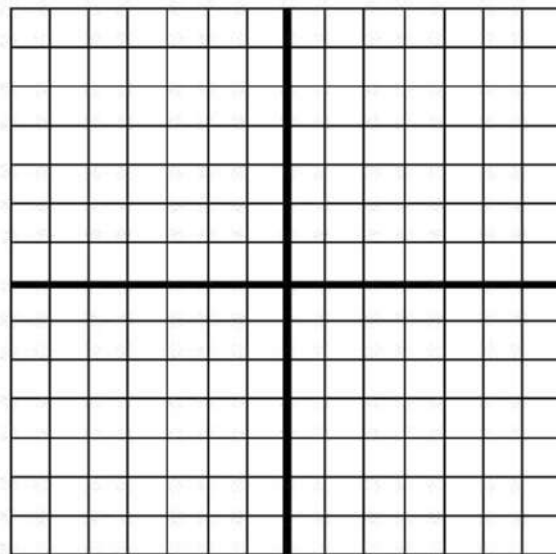
a. $f(-\pi)$ b. $f(2)$ c. $f\left(f\left(-\frac{3\pi}{2}\right)\right)$ d. $f(f(0))$

Part 10: Function Type

Name the function, find its domain and range, zeroes, and graph it.
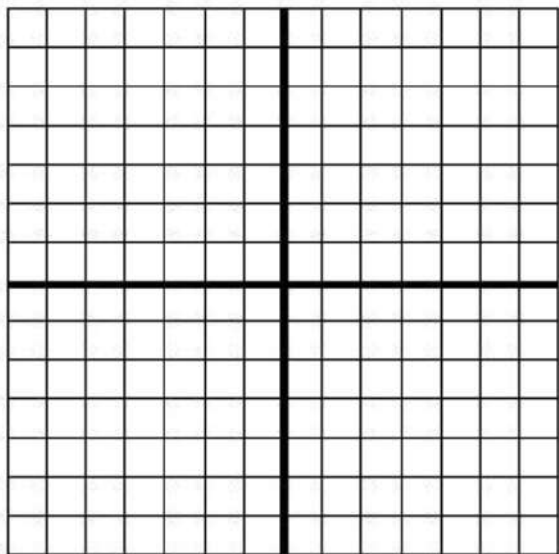
43. $4x - 3y = 6$ 44. $f(x) = x^2 - 2x - 3$
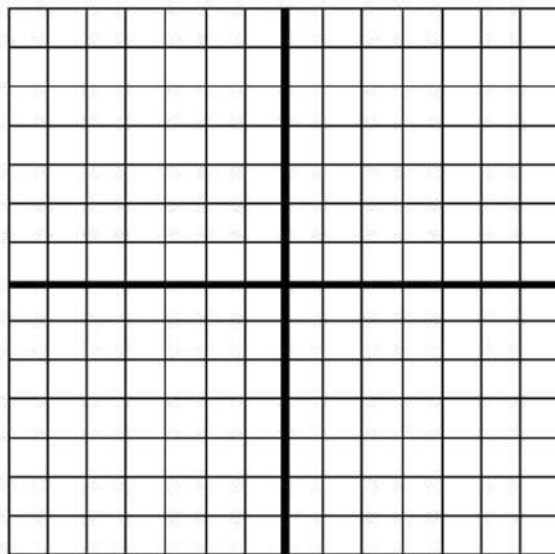
45. $y = x^3 - 8$
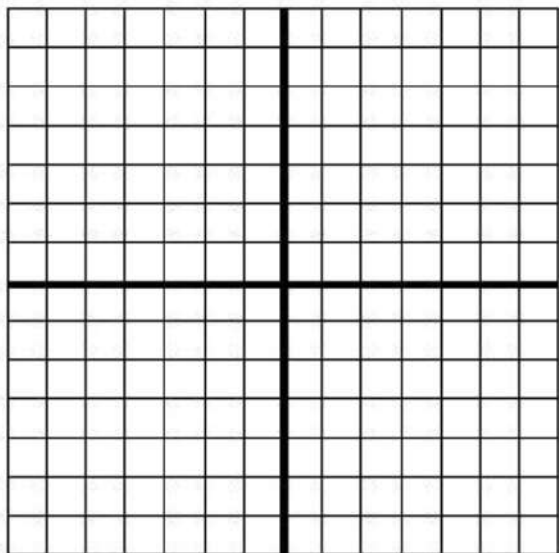
46. $y = 2^x$
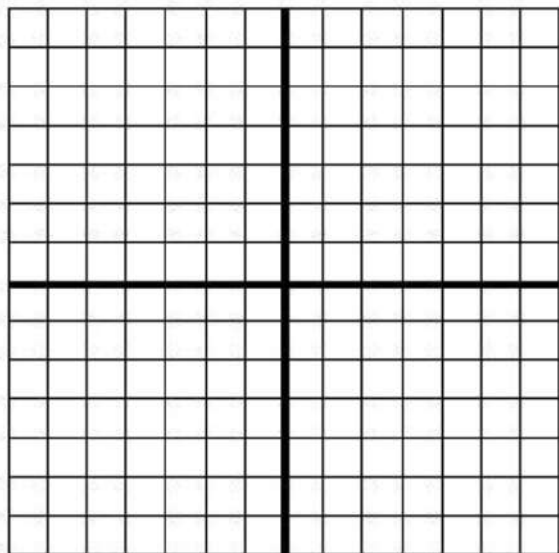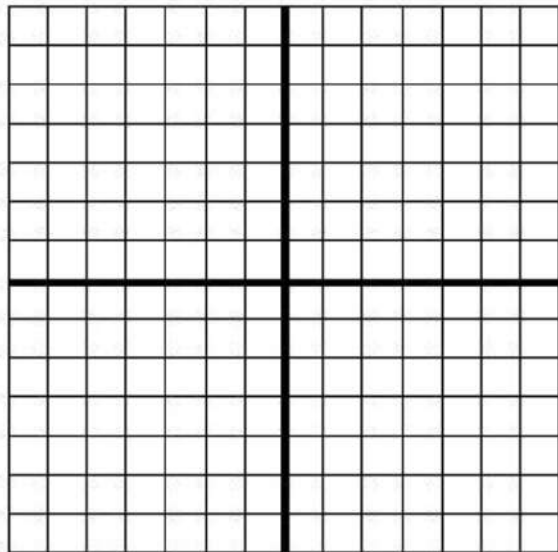
47. $f(x) = \sin x$
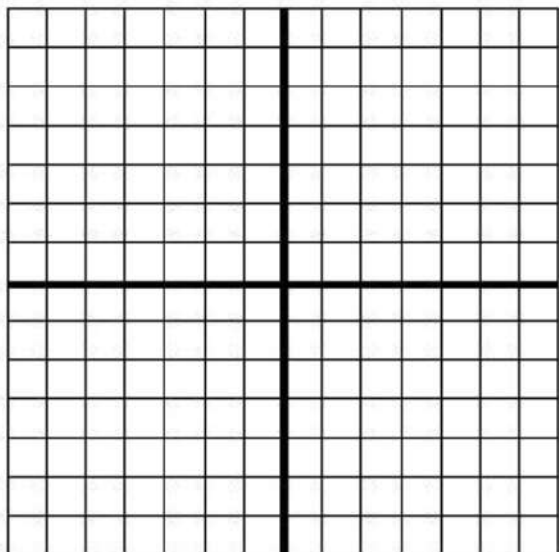
48. $f(x) = \tan x$
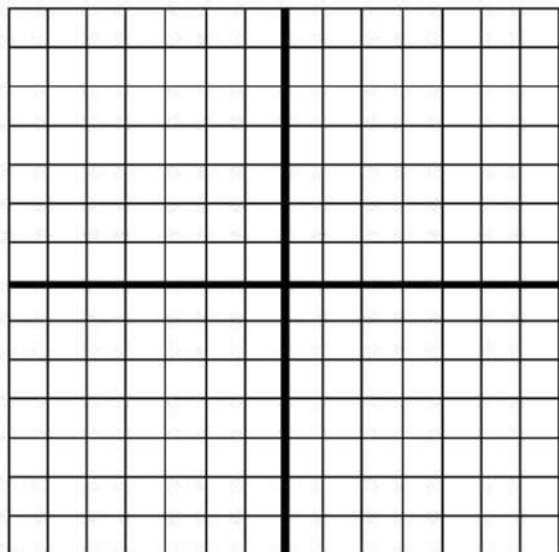
49. $f(x) = \cos x$
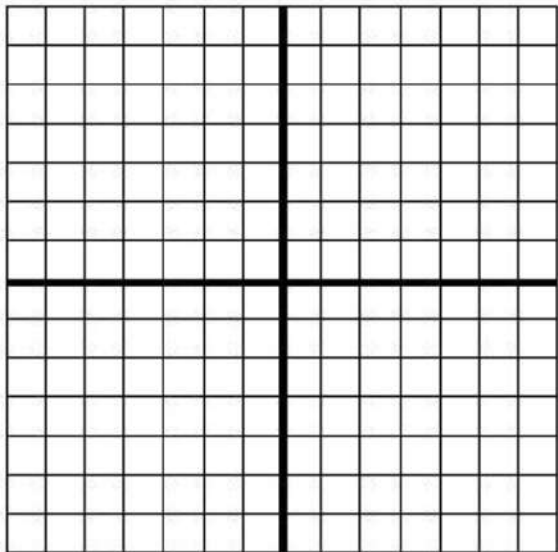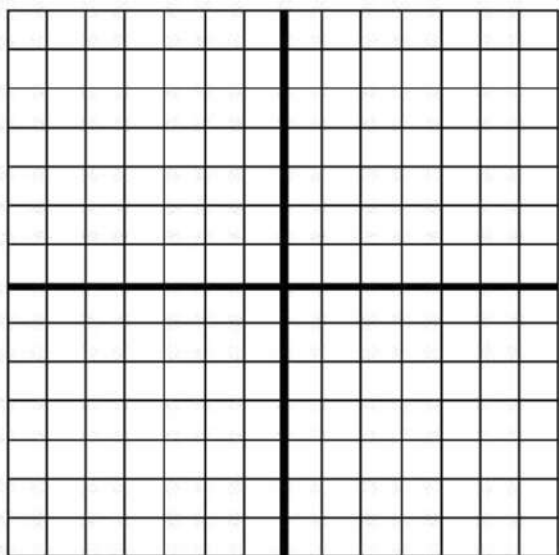
50. $y = \sqrt{x - 3}$
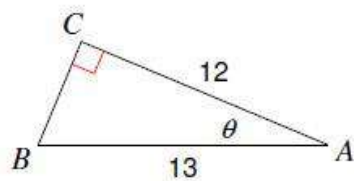
51. $f(x) = \dfrac{2}{x-3}$
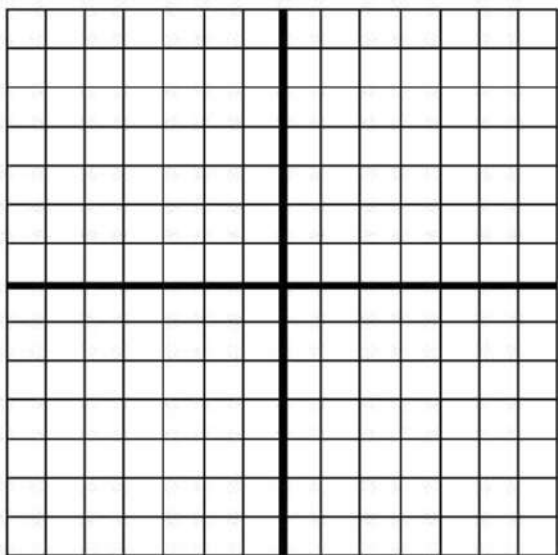
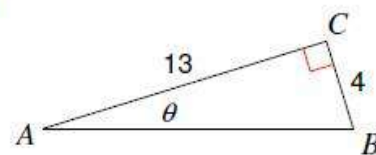52. $f(x) = e^x + 3$
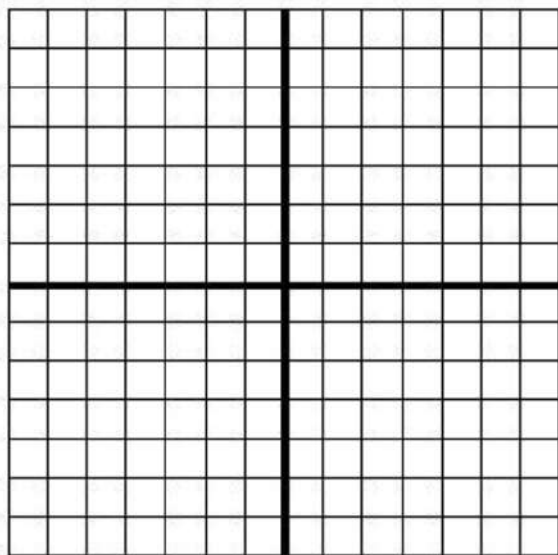
53. $f(x) = \ln x$

Part 11: Solving Problems involving Trigonometry

54. Triangles: Solve for the missing side and θ.
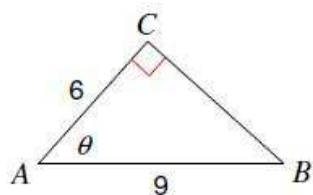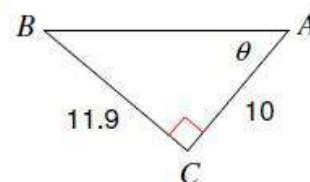
1)

2)

3)

4)

55. Find the equation of a line (in $y = mx + b$ form) through the point $(-2,5)$ with an angle of inclination of $45°$.

56. If $\sin x = \frac{3}{5}$, find $\sin(2x)$.

57. The angle of elevation of the top of a building from point on the ground 40 feet from the foot of the building is $60°$. How tall is the building?

Part 12: Trig Identities

Prove that both sides of each equation are equal to each other.

58. $\sin^5 x = (1 - 2\cos^2 x + \cos^4 x) \sin x$

59. $\frac{\cos^2 x}{\sin x} + \sin x = \csc x$

60. $\frac{1}{1-\sin x} + \frac{1}{1+\sin x} = 2\sec^2 x$

61. $\cot x = \frac{1+\cos^2 x}{\sin^2 x}$

Part 13: Unit Circle

Fill in the unit circle, label degrees, and radians *(cos x, sin x)*.

Unit Circle

Label degrees, radians, (cos x, sin x)   NAME_____