



Geometry
Fundamentals
Summer Math Packet

Congratulations! You will soon be learning Geometry!

This summer math packet is a review of some of the concepts learned in your Algebra I class which are needed when you begin your Geometry course in August. It will assure that all students begin the school year on the same page and with equal opportunity to learn and build upon mathematical concepts that should have been learned in previous courses.

Instructions for completing the packet:

- Please print the packet or use loose leaf paper to complete the packet by hand showing all work. Work must be neat and legible.
- Please use your Algebra I notes or the websites provided to help you if you need reminders on how to complete some practice problems.
- Take notes as you complete your work. You will be given a quiz on this material the first week of school.
- Work on the packet with your friends. Help each other. Every student is responsible for knowing the material in this packet when you return in August. We will review as a team and everyone will be expected to participate.
- Bring your packet to our first class together. It will be collected for a grade. Only packets done with paper and pencil will be accepted.

Helpful Websites:

<http://www.mathtv.com/>

<http://www.purplemath.com/modules/index.htm>

<https://www.khanacademy.org>

Helpful for graphing functions:

<https://www.education.ti.com/en/resources/family-of-functions>

Name _____

SUMMER PACKET for students entering Geometry

DETERMINING WHETHER A POINT IS ON A LINE

Example 1

Decide whether (3,-2) is a solution of the equation $y = 2x - 8$

$$-2 = 2(3) - 8 \quad \text{Substitute 3 for } x \text{ and } -2 \text{ for } y.$$

$$-2 = -2 \quad \text{Simplify.}$$

The statement is true, so (3,-2) is a solution of the equation $y = 2x - 8$

Exercises: Decide whether the given ordered pair is a solution of the equation.

1. $y = 6x + 4$; (-2, 8) _____

4. $y = \frac{3}{2}x + 10$; (4,12) _____

2. $y = -10x - 2$; (1, -12) _____

5. $y = \frac{5}{9}x + 34$; (-9,27) _____

3. $y = -\frac{1}{4}x - 18$; (-4, -17) _____

6. $y = \frac{2}{3}x - 6$; (9,0) _____

CALCULATING SLOPE

Example 2

Find the slope of a line passing through (3,-9) and (2,-1).

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{Formula for slope}$$

$$m = \frac{-1 - (-9)}{2 - 3} = \frac{-1 + 9}{-1} \quad \text{Substitute values and simplify.}$$

$$m = \frac{8}{-1} = -8 \quad \text{Slope is } -8.$$

Exercises: Find the slope of the line that contains the points

7. (4,1), (3, 6) _____

9. (5, 6), (9,8) _____

11. (-1, 7), (-3, 18) _____

8. (-8, 0), (5, -2) _____

10. (0,-4), (7,3) _____

12. (-6, -4), (1, 10) _____

FINDING THE EQUATION OF A LINE**Example 3**

Find an equation of the line that passes through the point (3, 4) and has a y-intercept of 5.

$$y = mx + b$$

Write the slope-intercept form.

$$4 = 3m + 5$$

Substitute 5 for b, 3 for x, and 4 for y.

$$-1 = 3m$$

Subtract 5 from each side.

$$\frac{-1}{3} = m$$

Divide each side by 3.

The slope is $m = \frac{-1}{3}$. The equation of the line is $y = \frac{-1}{3}x + 5$

Exercises: Write the equation of the line that passes through the given point and has the given y-intercept.

13. (2, 1); $b=5$ _____

16. (7, 0); $b=13$ _____

14. (-5, 3); $b=-12$ _____

17. (-3, -3); $b=-2$ _____

15. (-3, 10); $b=8$ _____

18. (-1, 4); $b=-8$ _____

FINDING THE EQUATION OF A LINE**Example 4**

Write an equation of the line that passes through the points (4, 8) and (3, 1). Find the slope of the line.

$$m = \frac{1-8}{3-4}$$

Substitute values.

$$m = \frac{-7}{-1} = 7$$

Simplify.

$$1 = 7(3) + b$$

Substitute values into $y = mx + b$.

$$1 = 21 + b$$

Multiply.

$$-20 = b$$

Solve for b.

Exercises: Write an equation of the line that passes through the given points.

19. (6, -3), (1, 2) _____

21. (5, -1), (4, -5) _____

23. (-3, -7), (0, 8) _____

20. (-7, 9), (-5, 3) _____

22. (-2, 4), (3, -6) _____

24. (1, 2), (-1, -4) _____

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DISTANCE FORMULA

Example 5

Find the distance between
the points $(-4, 3)$ and $(-7, 8)$

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-7 - (-4))^2 + (8 - 3)^2} \\ &= \sqrt{(-3)^2 + (5)^2} \\ &= \sqrt{34} \end{aligned}$$

Exercises: Find the distance between the points

25. $(3, 6), (0, -2)$ _____

27. $(-3, 4), (1, 4)$ _____

29. $(8, -2), (-3, -6)$ _____

26. $(5, -2), (-6, 5)$ _____

28. $(-6, -6), (-3, -2)$ _____

30. $(-8, 5), (-1, 1)$ _____

COMBINING LIKE TERMS

Example 6

Simplify

$$8x^2 + 16xy - 3x^2 + 3xy - 3x$$

$$8x^2 - 3x^2 + 16xy + 3xy - 3x$$

$$5x^2 - 3x + 19xy$$

Group like terms

Simplify

Exercises: Simplify.

31. $6x + 11y - 4x + y$

33. $-3p - 4t - 5t - 2p$

35. $3x^2y - 5xy^2 + 6x^2y$

32. $-5m + 3q + 4m - q$

34. $9x - 22y + 18x - 3y$

36. $5x^2 + 2xy - 7x^2 + xy$

SOLVING EQUATIONS WITH VARIABLES ON BOTH SIDES**Example 7**

Solve.

$$6a - 12 = 5a + 9$$

$$a - 12 = 9$$

Subtract 5a from both sides

$$a = 21$$

*Add 12 to each side***Exercises: Solve the equation.**

37. $3x + 5 = 2x + 11$

38. $8m + 1 = 7m - 9$

39. $11q - 6 = 3q + 8q$

40. $-14 + 3a = 10 - a$

41. $-2t + 10 = -t$

42. $-7x + 7 = 2x - 11$

SOLVING INEQUALITIES**Example 8**

Solve.

a. $5x - 4 \geq 4x + 6$

b. $10 - 7x < 24$

When you multiply or divide each side of an inequality by a *negative* number, you must *reverse* the inequality symbol to maintain a true statement.

a. $5x - 4 \geq 4x + 6$

$$x - 4 \geq 6$$

$$x \geq 10$$

b. $10 - 7x < 24$

$$-7x < 14$$

$$x > -2$$

Exercises: Solve the inequality.

43. $-x + 2 > 7$

44. $-5 + m < 21$

45. $z + 6 > -2$

46. $c - 18 < 10$

47. $x - 5 < 4$

48. $-3x + 4 \leq -5$

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WRITING AND SIMPLIFYING RATIOS

Example 9

- Train A takes 35 minutes to travel its route. Train B, traveling the same route but making more stops, takes 47 minutes. What is the ratio of the time of Train A to Train B?
- Jennie's height is 4 feet, 7 inches. Her younger sister's height is 25 inches. Find the ratio of Jennie's height to her sister's.

Solutions

- 35 minutes to 47 minutes = $\frac{35 \text{ minutes}}{47 \text{ minutes}} = \frac{35}{47}$
- Convert 4 feet, 7 inches to inches: $4(12) + 7 = 55$ inches

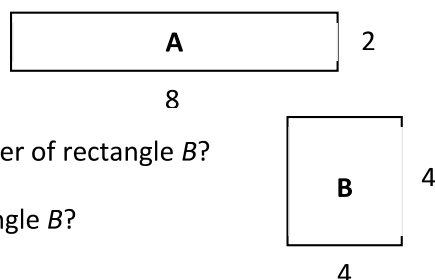
$$55 \text{ inches to } 25 \text{ inches} = \frac{55 \text{ inches}}{25 \text{ inches}} = \frac{55}{25} = \frac{11}{5}$$

Exercises: Write the following ratios.

- Basmati rice needs to cook for 20 minutes, while quinoa (another grain) cooks for 25 minutes. What is the ratio of cooking times for rice to quinoa?
- Jonathan caught 7 fish and Geogeanne caught 4. What is the ratio of fish caught of Jonathan to Geogeanne?
- Two sunflowers' growth was measured daily. At the end of the experiment, Sunflower A had grown from 2 inches to 2 feet, 3 inches. Sunflower B had grown from 3 inches to 2 feet, 6 inches. Find the ratio of the growth in height of Sunflower A to Sunflower B.

Use the diagram at the right.

- What is the ratio of length to width of rectangle A?
- What is the ratio of the perimeter of rectangle A to the perimeter of rectangle B?
- What is the ratio of the area of rectangle A to the area of rectangle B?



DISTRIBUTIVE PROPERTY

Example 10

Solve.

- $$4(x + 3) = 36$$

$$4x + 12 = 36$$

$$4x = 24$$

$$x = 6$$
- $$6(x + 4) + 12 = 5(x + 3) + 7$$

$$6x + 24 + 12 = 5x + 15 + 7$$

$$6x + 36 = 5x + 22$$

$$x = -14$$

Exercises: Solve.

55. $2(x + 7) = 20$

56. $-10(y + 8) - 40$

57. $7(2 - x) = 5x$

58. $-4(x - 6) = 28$

SOLVING PROPORTIONS**Example 11**

Solve.

$$\begin{aligned} \text{a. } \frac{x}{8} &= \frac{3}{4} \\ 4x &= 8 \cdot 3 \\ 4x &= 24 \\ x &= 6 \end{aligned}$$

$$\begin{aligned} \text{b. } \frac{6}{x+4} &= \frac{1}{9} \\ 6 \cdot 9 &= x + 4 \\ 54 &= x + 4 \\ 50 &= x \end{aligned}$$

Exercises: Solve.

59. $\frac{y}{50} = \frac{3}{100}$

60. $\frac{6}{45} = \frac{2z + 10}{15}$

61. $\frac{3}{p - 6} = \frac{1}{p}$

62. $\frac{3}{8} = \frac{3}{2d}$

63. $\frac{1}{18} = \frac{5}{-4(x - 1)}$

64. $\frac{r}{3r + 1} = \frac{2}{3}$

65. $\frac{3w + 6}{28} = \frac{3}{4}$

66. $\frac{3}{m + 4} = \frac{9}{14}$

67. $\frac{w}{4} = \frac{9}{w}$

SIMPLIFYING RADICALS**Example 12**Simplify the expression $\sqrt{20}$

$$\begin{aligned} \sqrt{20} &= \sqrt{4} \cdot \sqrt{5} \\ &= 2\sqrt{5} \end{aligned}$$

Exercises: Simplify the expression.

68. $\sqrt{121}$

69. $\sqrt{40}$

70. $\sqrt{243}$

71. $\sqrt{52}$

72. $\sqrt{27}$

73. $\sqrt{288}$

74. $\sqrt{45}$

75. $\sqrt{80}$

76. $\sqrt{320}$

77. $\sqrt{72}$

78. $\sqrt{50}$

79. $\sqrt{225}$

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SIMPLIFYING RADICAL EXPRESSIONS

Example 13

$$\begin{aligned} \text{a. } 5\sqrt{3} - \sqrt{3} - \sqrt{2} \\ = 4\sqrt{3} - \sqrt{2} \end{aligned}$$

$$\begin{aligned} \text{b. } (2\sqrt{2})(5\sqrt{3}) \\ = 2 \cdot 5 \cdot \sqrt{2} \cdot \sqrt{3} \\ = 10\sqrt{6} \end{aligned}$$

$$\begin{aligned} \text{c. } (5\sqrt{7})^2 \\ = 5^2 \sqrt{7^2} \\ = 25 \cdot 7 \\ = 175 \end{aligned}$$

Exercises: Simplify the radical expression.

80. $\sqrt{75} + \sqrt{3}$

81. $-\sqrt{147} - \sqrt{243}$

82. $(5\sqrt{4})(2\sqrt{4})$

83. $\sqrt{50} - \sqrt{18}$

84. $(3\sqrt{14})(\sqrt{35})$

85. $(6\sqrt{5})^2$

86. $\sqrt{64} - \sqrt{28}$

87. $(\sqrt{363})(\sqrt{300})$

88. $(4\sqrt{2})^2$

89. $\sqrt{44} + 2\sqrt{11}$

90. $(\sqrt{32})(\sqrt{2})$

91. $(8\sqrt{3})^2$

92. $\sqrt{125} - \sqrt{80}$

93. $(\sqrt{98})(\sqrt{128})$

94. $(10\sqrt{11})^2$

95. $\sqrt{242} + \sqrt{200}$

SIMPLIFYING QUOTIENTS WITH RADICALS

Example 14

Simplify the quotient $\frac{6}{\sqrt{5}}$

$$\begin{aligned} \frac{6}{\sqrt{5}} &= \frac{6}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} \\ &= \frac{6\sqrt{5}}{\sqrt{5}\sqrt{5}} \\ &= \frac{6\sqrt{5}}{5} \end{aligned}$$

Exercises: Simplify the quotient.

96. $\frac{4}{\sqrt{3}}$

97. $\frac{2\sqrt{3}}{\sqrt{5}}$

98. $\frac{\sqrt{32}}{\sqrt{5}}$

99. $\frac{5}{\sqrt{7}}$

100. $\frac{\sqrt{12}}{\sqrt{24}}$

101. $\frac{\sqrt{27}}{\sqrt{45}}$

102. $\frac{2\sqrt{3}}{\sqrt{6}}$

103. $\frac{\sqrt{18}}{\sqrt{10}}$

104. $\frac{\sqrt{50}}{\sqrt{75}}$

SOLVING LITERAL EQUATIONS**Example 16**

Given the formula for the surface area of a right cylinder, solve for h . $S = 2\pi r^2 + 2\pi rh$

$$S = 2\pi r(r + h)$$

$$\frac{S}{2\pi r} = r + h$$

$$\frac{S}{2\pi r} - r = h$$

or

$$S - 2\pi r^2 = 2\pi rh$$

$$\frac{(S - 2\pi r^2)}{2\pi r} = h$$

Exercises: Solve the literal equation for the indicated variable. Assume variables are positive.

105. $V = \frac{4}{3}\pi r^3; r$

106. $V = s^3; s$

107. $V = \pi r^2 h; h$

108. $A = \frac{1}{2}bh; h$

109. $P = 2l + 2w; l$

110. $S = 6s^2; s$

112. $V = lwh; h$

113. $a^2 + b^2 = c^2; b$

111. $A = \frac{1}{2}h(b_1 + b_2); b_1$

ALGEBRAIC EXPRESSIONS**Example 17**

- a. Write an expression for seven less than a number
 $x - 7$

- b. Write an equation for three times less than six times a number is five times the same number plus 5, then solve.

$$6x - 3 = 5x + 5$$

$$x - 3 = 5$$

$$x = 8$$

Exercises: Write the expression or equation. Solve the equations.

114. Half of a number plus three times the number

115. The product of five and a number decreased by seven equals thirteen.

116. Sixteen less than twice a number is 10.

117. Twice a number increased by the product of the number and fourteen results in forty-eight.

118. Half of a number is three times the sum of the number and five.

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PERCENT PROBLEMS

Example 18

- a. What number is 12% of 75?

$$x = 0.12(75)$$

$$x = 9$$

- b. 6 is what percent of 40?

$$6 = 40p$$

$$0.15 = p$$

$$p = 15\%$$

Exercises:

119. What number is 30% of 120?

120. 11 dogs is what percent of 50 dogs?

121. What distance is 15% of 340 miles?

122. 200 is what percent of 50?

123. 34 is what percent of 136?

124. 8 weeks is what percent of a year?

SIMPLIFYING RATIONAL EXPRESSIONS

Example 19

Simplify.

a. $\frac{8x^2+12x}{4x^2+16x} = \frac{4x(2x+3)}{4x(x+4)} = \frac{2x+3}{x+4}$

b. $\frac{y^2-9}{y^2+6y+9} = \frac{(y+3)(y-3)}{(y+3)(y+3)} = \frac{y-3}{y+3}$

Exercises: Simplify.

125. $\frac{5x}{10x^2}$

126. $\frac{14d^2-2d}{6d^2+8d}$

127. $\frac{-5h+1}{h+1}$

128. $\frac{16a^3}{8a}$

129. $\frac{2y-12}{24-2y}$

130. $\frac{t^2-1}{t^2+2t+1}$

131. $\frac{(5x^2+x)}{(5x+1)}$

132. $\frac{36s^2-4s}{4s^2-12s}$

133. $\frac{m^2-4m+4}{m^2-4}$