AP CALCULUS AB SUMMER PACKET 2023-2024

This summer math packet is a review of some of the concepts you learned in Pre-calculus that are needed when you begin your Calculus class in August. It will assure that all students will be on the same page as to what they are expected to know.

Instructions for completing the packet:

- ✓ Please print the packet or use loose leaf paper to complete the packet by hand showing all work when necessary. Work must be neat and legible.
- ✓ Please use your Pre-Calculus notes of the websites provided to help you if you need reminders on how to complete some practice problems.
- ✓ Take notes as you complete your work. You will be given a quiz on this material the first week of school.
- ✓ Work on the packet with your friends. Help each other. Every student is responsible for knowing the material in this packet.
- ✓ Bring your packet the first day of school. It will be collected for a grade. Only packets done with paper and pencil will be accepted.

Helpful Websites:

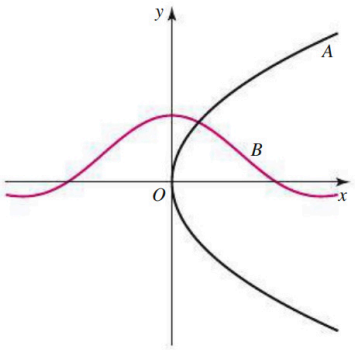
http://www.mathtv.com/

http://www.purplemath.com/modules/index.htm

http://www.khanacademy.org

11–12. Vertical line test *Decide whether graphs A, B, or both represent functions.*

11.



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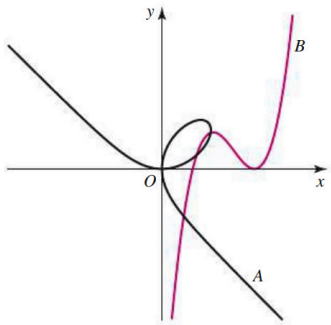


13–20. Domain and range *Graph each function with a graphing utility using the given window. Then state the domain and range of the function.*

13. $f(x) = 3x^4 - 10; \quad [-2, 2] \times [-10, 15]$

14. $g(y) = \dfrac{y + 1}{(y + 2)(y - 3)}; \quad [-4, 6] \times [-3, 3]$

15. $f(x) = \sqrt{4 - x^2}; \quad [-4, 4] \times [-4, 4]$

18. $g(x) = (x^2 - 4)\sqrt{x + 5}; \quad [-5, 5] \times [-10, 50]$

19. $f(x) = (9 - x^2)^{3/2}; \quad [-4, 4] \times [0, 30]$

20. $g(t) = \dfrac{1}{1 + t^2}; \quad [-7, 7] \times [0, 1.5]$

21–24. Domain in context *Determine an appropriate domain of each function. Identify the independent and dependent variables.*

21. A stone is thrown vertically upward from the ground at a speed of 40 m/s at time $t = 0$. Its distance d (in meters) above the ground (neglecting air resistance) is approximated by the function $f(t) = 40t - 5t^2$.

23. A cylindrical water tower with a radius of 10 m and a height of 50 m is filled to a height of h. The volume V of water (in cubic meters) is given by the function $g(h) = 100\pi h$.

37–40. Working with composite functions *Find possible choices for outer and inner functions f and g such that the given function h equals $f \circ g$. Give the domain of h.*

37. $h(x) = (x^3 - 5)^{10}$

38. $h(x) = \dfrac{2}{(x^6 + x^2 + 1)^2}$

39. $h(x) = \sqrt{x^4 + 2}$

40. $h(x) = \dfrac{1}{\sqrt{x^3 - 1}}$

41–48. More composite functions *Let $f(x) = |x|$, $g(x) = x^2 - 4$, $F(x) = \sqrt{x}$, and $G(x) = 1/(x - 2)$. Determine the following composite functions and give their domains.*

41. $f \circ g$ **42.** $g \circ f$ **43.** $f \circ G$

44. $f \circ g \circ G$ **45.** $G \circ g \circ f$ **46.** $F \circ g \circ g$

49–54. Missing piece *Let* $g(x) = x^2 + 3$. *Find a function* f *that produces the given composition.*

49. $(f \circ g)(x) = x^2$

51. $(f \circ g)(x) = x^4 + 6x^2 + 9$

53. $(g \circ f)(x) = x^4 + 3$

56. Composite functions from tables Use the table to evaluate the given compositions.

x	-1	0	1	2	3	4
$f(x)$	3	1	0	-1	-3	-1
$g(x)$	-1	0	2	3	4	5
$h(x)$	0	-1	0	3	0	4

a. $h(g(0))$

d. $g(h(f(4)))$

g. $f(h(g(2)))$

j. $f(f(h(3)))$

57–66. Working with difference quotients *Simplify the difference quotients* $\dfrac{f(x+h)-f(x)}{h}$ *and* $\dfrac{f(x)-f(a)}{x-a}$ *for the following functions.*

57. $f(x) = x^2$

59. $f(x) = 2/x$

61. $f(x) = \dfrac{x}{x+1}$

63. $f(x) = x^3 - 2x$

71–78. Symmetry *Determine whether the graphs of the following equations and functions have symmetry about the x-axis, the y-axis, or the origin. Check your work by graphing.*

71. $f(x) = x^4 + 5x^2 - 12$

72. $f(x) = 3x^5 + 2x^3 - x$

73. $f(x) = x^5 - x^3 - 2$

74. $f(x) = 2|x|$

75. $x^{2/3} + y^{2/3} = 1$

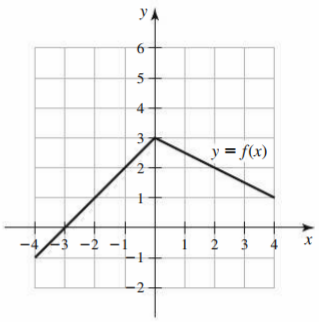
76. $x^3 - y^5 = 0$

77. $f(x) = x|x|$

78. $|x| + |y| = 1$

19–20. Graphs of piecewise functions *Write a definition of the functions whose graphs are given.*

19.



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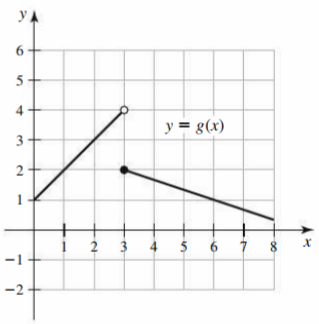


22. Taxicab fees A taxicab ride costs \$3.50 plus \$2.50 per mile for the first 5 miles, with the rate dropping to \$1.50 per mile after the fifth mile. Let m be the distance (in miles) from the airport to a hotel. Find and graph the piecewise linear function $c(m)$ that represents the cost of taking a taxi from the airport to a hotel m miles away.

23–28. Piecewise linear functions *Graph the following functions.*

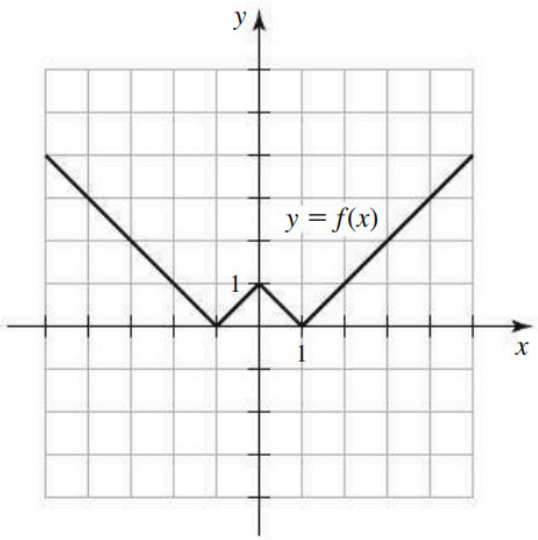
24. $f(x) = \begin{cases} \dfrac{x^2 - x - 2}{x - 2} & \text{if } x \neq 2 \\ 4 & \text{if } x = 2 \end{cases}$

25. $f(x) = \begin{cases} 3x - 1 & \text{if } x \leq 0 \\ -2x + 1 & \text{if } x > 0 \end{cases}$

28. $f(x) = \begin{cases} 2x + 2 & \text{if } x < 0 \\ x + 2 & \text{if } 0 \leq x \leq 2 \\ 3 - x/2 & \text{if } x > 2 \end{cases}$

44. Transformations Use the graph of f in the figure to plot the following functions.

a. $y = -f(x)$
b. $y = f(x + 2)$
c. $y = f(x - 2)$
d. $y = f(2x)$
e. $y = f(x - 1) + 2$
f. $y = 2f(x)$



T 47–54. Shifting and scaling *Use shifts and scalings to graph the given functions. Then check your work with a graphing utility. Be sure to identify an original function on which the shifts and scalings are performed.*

47. $f(x) = (x - 2)^2 + 1$

48. $f(x) = x^2 - 2x + 3$ (*Hint:* Complete the square first.)

49. $g(x) = -3x^2$

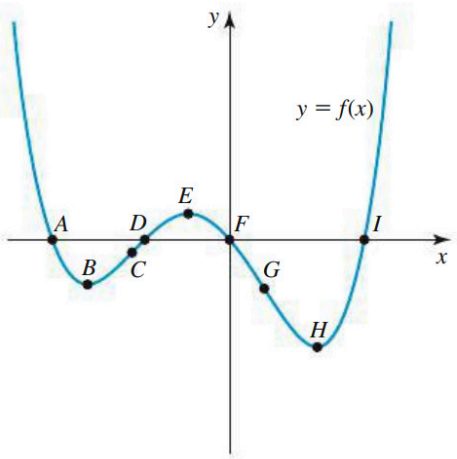
50. $g(x) = 2x^3 - 1$

51. $g(x) = 2(x + 3)^2$

53. $h(x) = -4x^2 - 4x + 12$

54. $h(x) = |3x - 6| + 1$

71. Features of a graph Consider the graph of the function f shown in the figure. Answer the following questions by referring to the points A–I.



a. Which points correspond to the roots (zeros) of f?

b. Which points on the graph correspond to high points of peaks (soon to be called *local maximum* values of f)?

c. Which points on the graph correspond to low points of valleys (soon to be called *local minimum* values of f)?

d. As you move along the curve in the positive x-direction, at which point is the graph rising most rapidly?

e. As you move along the curve in the positive x-direction, at which point is the graph falling most rapidly?

21–28. Finding inverse functions

a. *Find the inverse of each function (on the given interval, if specified) and write it in the form* $y = f^{-1}(x)$.

b. *Verify the relationships* $f(f^{-1}(x)) = x$ *and* $f^{-1}(f(x)) = x$.

21. $f(x) = 2x$

22. $f(x) = x/4 + 1$

24. $f(x) = 3x^3$

26. $f(x) = x^2 + 4$, for $x \geq 0$

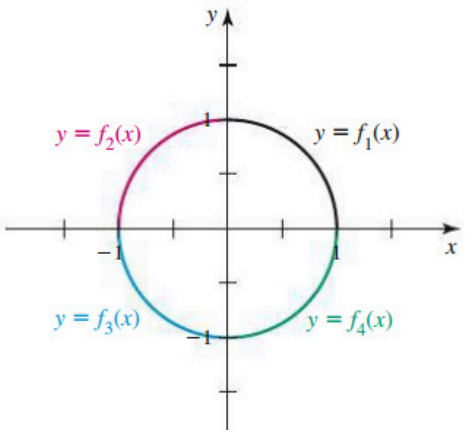
27. $f(x) = \sqrt{x + 2}$, for $x \geq -2$

28. $f(x) = 2/(x^2 + 1)$, for $x \geq 0$

29. Splitting up curves The unit circle $x^2 + y^2 = 1$ consists of four one-to-one functions, $f_1(x)$, $f_2(x)$, $f_3(x)$, and $f_4(x)$ (see figure).

a. Find the domain and a formula for each function.

b. Find the inverse of each function and write it as $y = f^{-1}(x)$.



31–38. Graphing inverse functions *Find the inverse function (on the given interval, if specified) and graph both f and f^{-1} on the same set of axes. Check your work by looking for the required symmetry in the graphs.*

31. $f(x) = 8 - 4x$

32. $f(x) = 4x - 12$

33. $f(x) = \sqrt{x}$, for $x \geq 0$

34. $f(x) = \sqrt{3 - x}$, for $x \leq 3$

35. $f(x) = x^4 + 4$, for $x \geq 0$

36. $f(x) = 6/(x^2 - 9)$, for $x > 3$

37. $f(x) = x^2 - 2x + 6$, for $x \geq 1$ (*Hint:* Complete the square.)

38. $f(x) = -x^2 - 4x - 3$, for $x \leq -2$ (*Hint:* Complete the square.)

41–46. Solving logarithmic equations *Solve the following equations.*

41. $\log_{10} x = 3$

42. $\log_5 x = -1$

43. $\log_8 x = \frac{1}{3}$

44. $\log_b 125 = 3$

45. $\ln x = -1$

46. $\ln y = 3$

47–52. Properties of logarithms *Assume* $\log_b x = 0.36$, $\log_b y = 0.56$, *and* $\log_b z = 0.83$. *Evaluate the following expressions.*

47. $\log_b \dfrac{x}{y}$

48. $\log_b x^2$

49. $\log_b xz$

50. $\log_b \dfrac{\sqrt{xy}}{z}$

51. $\log_b \dfrac{\sqrt{x}}{\sqrt[3]{z}}$

52. $\log_b \dfrac{b^2 x^{5/2}}{\sqrt{y}}$

53–56. Solving equations *Solve the following equations.*

53. $7^x = 21$

54. $2^x = 55$

55. $3^{3x-4} = 15$

56. $5^{3x} = 29$

15–22. Evaluating trigonometric functions *Evaluate the following expressions by drawing the unit circle and the appropriate right triangle. Use a calculator only to check your work. All angles are in radians.*

15. $\cos(2\pi/3)$ **16.** $\sin(2\pi/3)$ **17.** $\tan(-3\pi/4)$

18. $\tan(15\pi/4)$ **19.** $\cot(-13\pi/3)$ **20.** $\sec(7\pi/6)$

21. $\cot(-17\pi/3)$ **22.** $\sin(16\pi/3)$

23–28. Evaluating trigonometric functions *Evaluate the following expressions or state that the quantity is undefined. Use a calculator only to check your work.*

23. $\cos 0$ **24.** $\sin(-\pi/2)$ **25.** $\cos(-\pi)$

26. $\tan 3\pi$ **27.** $\sec(5\pi/2)$ **28.** $\cot \pi$

23–28. Evaluating trigonometric functions *Evaluate the following expressions or state that the quantity is undefined. Use a calculator only to check your work.*

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29–36. Trigonometric identities

29. Prove that $\sec\theta = \dfrac{1}{\cos\theta}$.

30. Prove that $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$.

31. Prove that $\tan^2\theta + 1 = \sec^2\theta$.

32. Prove that $\dfrac{\sin\theta}{\csc\theta} + \dfrac{\cos\theta}{\sec\theta} = 1$.

33. Prove that $\sec(\pi/2 - \theta) = \csc\theta$.

34. Prove that $\sec(x + \pi) = -\sec x$.

35. Find the exact value of $\cos(\pi/12)$.

36. Find the exact value of $\tan(3\pi/8)$.

37–46. Solving trigonometric equations *Solve the following equations.*

37. $\tan x = 1$

39. $\sin^2 \theta = \frac{1}{4}, 0 \leq \theta < 2\pi$

41. $\sqrt{2} \sin x - 1 = 0$

42. $\sin 3x = \frac{\sqrt{2}}{2}, 0 \leq x < 2\pi$

45. $\sin \theta \cos \theta = 0, 0 \leq \theta < 2\pi$

67–74. Evaluating inverse trigonometric functions *Without using a calculator, evaluate or simplify the following expressions.*

67. $\tan^{-1} \sqrt{3}$

68. $\cot^{-1} (-1/\sqrt{3})$

69. $\sec^{-1} 2$

70. $\csc^{-1} (-1)$

71. $\tan^{-1} (\tan \pi/4)$

72. $\tan^{-1} (\tan 3\pi/4)$

73. $\csc^{-1} (\sec 2)$

74. $\tan (\tan^{-1} 1)$