



Grade Seven

By the end of grade seven, students are adept at manipulating numbers and equations and understand the general principles at work. Students understand and use factoring of numerators and denominators and properties of exponents. They know the Pythagorean theorem and solve problems in which they compute the length of an unknown side. Students know how to compute the surface area and volume of basic three-dimensional objects and understand how area and volume change with a change in scale. Students make conversions between different units of measurement. They know and use different representations of fractional numbers (fractions, decimals, and percents) and are proficient at changing from one to another. They increase their facility with ratio and proportion, compute percents of increase and decrease, and compute simple and compound interest. They graph linear functions and understand the idea of slope and its relation to ratio.

Number Sense

- 1.0 Students know the properties of, and compute with, rational numbers expressed in a variety of forms:**
- 1.1 Read, write, and compare rational numbers in scientific notation (positive and negative powers of 10) with approximate numbers using scientific notation.
 - 1.2 Add, subtract, multiply, and divide rational numbers (integers, fractions, and terminating decimals) and take positive rational numbers to whole-number powers.
 - 1.3 Convert fractions to decimals and percents and use these representations in estimations, computations, and applications.
 - 1.4 Differentiate between rational and irrational numbers.
 - 1.5 Know that every rational number is either a terminating or repeating decimal and be able to convert terminating decimals into reduced fractions.

- 1.6 Calculate the percentage of increases and decreases of a quantity.
- 1.7 Solve problems that involve discounts, markups, commissions, and profit and compute simple and compound interest.

2.0 Students use exponents, powers, and roots and use exponents in working with fractions:

- 2.1 Understand negative whole-number exponents. Multiply and divide expressions involving exponents with a common base.
- 2.2 Add and subtract fractions by using factoring to find common denominators.
- 2.3 Multiply, divide, and simplify rational numbers by using exponent rules.
- 2.4 Use the inverse relationship between raising to a power and extracting the root of a perfect square integer; for an integer that is not square, determine without a calculator the two integers between which its square root lies and explain why.
- 2.5 Understand the meaning of the absolute value of a number; interpret the absolute value as the distance of the number from zero on a number line; and determine the absolute value of real numbers.

Algebra and Functions

1.0 Students express quantitative relationships by using algebraic terminology, expressions, equations, inequalities, and graphs:

- 1.1 Use variables and appropriate operations to write an expression, an equation, an inequality, or a system of equations or inequalities that represents a verbal description (e.g., three less than a number, half as large as area A).
- 1.2 Use the correct order of operations to evaluate algebraic expressions such as $3(2x + 25)$
- 1.3 Simplify numerical expressions by applying properties of rational numbers (e.g., identity, inverse, distributive, associative, commutative) and justify the process used.
- 1.4 Use algebraic terminology (e.g., variable, equation, term, coefficient, inequality, expression, constant) correctly.
- 1.5 Represent quantitative relationships graphically and interpret the meaning of a specific part of a graph in the situation represented by the graph.

2.0 Students interpret and evaluate expressions involving integer powers and simple roots:

- 2.1 Interpret positive whole-number powers as repeated multiplication and negative whole-number powers as repeated division or multiplication by the multiplicative inverse. Simplify and evaluate expressions that include exponents.
- 2.2 Multiply and divide monomials; extend the process of taking powers and extracting roots to monomials when the latter results in a monomial with an integer exponent.

3.0 Students graph and interpret linear and some nonlinear functions:

- 3.1 Graph functions of the form $y = nx^2$ and $y = nx^3$ and use in solving problems.
- 3.2 Plot the values from the volumes of three-dimensional shapes for various values of the edge lengths (e.g., cubes with varying edge lengths or a triangle prism with a fixed height and an equilateral triangle base of varying lengths).
- 3.3 Graph linear functions, noting that the vertical change (change in y -value) per unit of horizontal change (change in x -value) is always the same and know that the ratio (“rise over run”) is called the slope of a graph.
- 3.4 Plot the values of quantities whose ratios are always the same (e.g., cost to the number of an item, feet to inches, circumference to diameter of a circle). Fit a line to the plot and understand that the slope of the line equals the quantities.

4.0 Students solve simple linear equations and inequalities over the rational numbers:

- 4.1 Solve two-step linear equations and inequalities in one variable over the rational numbers, interpret the solution or solutions in the context from which they arose, and verify the reasonableness of the results.
- 4.2 Solve multistep problems involving rate, average speed, distance, and time or a direct variation.

Measurement and Geometry

1.0 Students choose appropriate units of measure and use ratios to convert within and between measurement systems to solve problems:

- 1.1 Compare weights, capacities, geometric measures, times, and temperatures within and between measurement systems (e.g., miles per hour and feet per second, cubic inches to cubic centimeters).
 - 1.2 Construct and read drawings and models made to scale.
 - 1.3 Use measures expressed as rates (e.g., speed, density) and measures expressed as products (e.g., person-days) to solve problems; check the units of the solutions; and use dimensional analysis to check the reasonableness of the answer.
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2.0 Students compute the perimeter, area, and volume of common geometric objects and use the results to find measures of less common objects. They know how perimeter, area, and volume are affected by changes of scale:

- 2.1 Use formulas routinely for finding the perimeter and area of basic two-dimensional figures and the surface area and volume of basic three-dimensional figures, including rectangles, parallelograms, trapezoids, squares, triangles, circles, prisms, and cylinders.
 - 2.2 Estimate and compute the area of more complex or irregular two- and three-dimensional figures by breaking the figures down into more basic geometric objects.
 - 2.3 Compute the length of the perimeter, the surface area of the faces, and the volume of a three-dimensional object built from rectangular solids. Understand that when the lengths of all dimensions are multiplied by a scale factor, the surface area is multiplied by the square of the scale factor and the volume is multiplied by the cube of the scale factor.
 - 2.4 Relate the changes in measurement with a change of scale to the units used (e.g., square inches, cubic feet) and to conversions between units (1 square foot = 144 square inches or $[1 \text{ ft}^2] = [144 \text{ in}^2]$, 1 cubic inch is approximately 16.38 cubic centimeters or $[1 \text{ in}^3] = [16.38 \text{ cm}^3]$).
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3.0 Students know the Pythagorean theorem and deepen their understanding of plane and solid geometric shapes by constructing figures that meet given conditions and by identifying attributes of figures:

- 3.1 Identify and construct basic elements of geometric figures (e.g., altitudes, midpoints, diagonals, angle bisectors, and perpendicular bisectors; central angles, radii, diameters, and chords of circles) by using a compass and straightedge.

- 3.2 Understand and use coordinate graphs to plot simple figures, determine lengths and areas related to them, and determine their image under translations and reflections.
- 3.3 Know and understand the Pythagorean theorem and its converse and use it to find the length of the missing side of a right triangle and the lengths of other line segments and, in some situations, empirically verify the Pythagorean theorem by direct measurement.
- 3.4 Demonstrate an understanding of conditions that indicate two geometrical figures are congruent and what congruence means about the relationships between the sides and angles of the two figures.
- 3.5 Construct two-dimensional patterns for three-dimensional models, such as cylinders, prisms, and cones.
- 3.6 Identify elements of three-dimensional geometric objects (e.g., diagonals of rectangular solids) and describe how two or more objects are related in space (e.g., skew lines, the possible ways three planes might intersect).

Statistics, Data Analysis, and Probability

- 1.0 Students collect, organize, and represent data sets that have one or more variables and identify relationships among variables within a data set by hand and through the use of an electronic spreadsheet software program:**
 - 1.1 Know various forms of display for data sets, including a stem-and-leaf plot or box-and-whisker plot; use the forms to display a single set of data or to compare two sets of data.
 - 1.2 Represent two numerical variables on a scatterplot and informally describe how the data points are distributed and any apparent relationship that exists between the two variables (e.g., between time spent on homework and grade level).
 - 1.3 Understand the meaning of, and be able to compute, the minimum, the lower quartile, the median, the upper quartile, and the maximum of a data set.

Mathematical Reasoning

1.0 Students make decisions about how to approach problems:

- 1.1 Analyze problems by identifying relationships, distinguishing relevant from irrelevant information, identifying missing information, sequencing and prioritizing information, and observing patterns.
 - 1.2 Formulate and justify mathematical conjectures based on a general description of the mathematical question or problem posed.
 - 1.3 Determine when and how to break a problem into simpler parts.
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2.0 Students use strategies, skills, and concepts in finding solutions:

- 2.1 Use estimation to verify the reasonableness of calculated results.
 - 2.2 Apply strategies and results from simpler problems to more complex problems.
 - 2.3 Estimate unknown quantities graphically and solve for them by using logical reasoning and arithmetic and algebraic techniques.
 - 2.4 Make and test conjectures by using both inductive and deductive reasoning.
 - 2.5 Use a variety of methods, such as words, numbers, symbols, charts, graphs, tables, diagrams, and models, to explain mathematical reasoning.
 - 2.6 Express the solution clearly and logically by using the appropriate mathematical notation and terms and clear language; support solutions with evidence in both verbal and symbolic work.
 - 2.7 Indicate the relative advantages of exact and approximate solutions to problems and give answers to a specified degree of accuracy.
 - 2.8 Make precise calculations and check the validity of the results from the context of the problem.
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3.0 Students determine a solution is complete and move beyond a particular problem by generalizing to other situations:

- 3.1 Evaluate the reasonableness of the solution in the context of the original situation.
- 3.2 Note the method of deriving the solution and demonstrate a conceptual understanding of the derivation by solving similar problems.
- 3.3 Develop generalizations of the results obtained and the strategies used and apply them to new problem situations.



Glossary

absolute value. A number's distance from zero on the number line. The absolute value of -4 is 4 ; the absolute value of 4 is 4 .

algorithm. An organized procedure for performing a given type of calculation or solving a given type of problem. An example is long division.

arithmetic sequence. A sequence of elements, a_1, a_2, a_3, \dots , such that the difference of successive terms is a constant $a_{i+1} - a_i = k$; for example, the sequence $\{2, 5, 8, 11, 14, \dots\}$ where the common difference is 3 .

asymptotes. Straight lines that have the property of becoming and staying arbitrarily close to the curve as the distance from the origin increases to infinity. For example, the x -axis is the only asymptote to the graph of $\sin(x)/x$.

axiom. A basic assumption about a mathematical system from which theorems can be deduced. For example, the system could be the points and lines in the plane. Then an axiom would be that given any two distinct points in the plane, there is a unique line through them.

binomial. In algebra, an expression consisting of the sum or difference of two monomials (see the definition of *monomial*), such as $4a - 8b$.

binomial distribution. In probability, a binomial distribution gives the probabilities of k outcomes A (or $n - k$ outcomes B) in n independent trials for a two-outcome experiment in which the possible outcomes are denoted A and B .

binomial theorem. In mathematics, a theorem that specifies the complete expansion of a binomial raised to any positive integer power.

box-and-whisker plot. A graphical method for showing the median, quartiles, and extremes of data. A box plot shows where the data are spread out and where they are concentrated.

complex numbers. Numbers that have the form $a + bi$ where a and b are real numbers and i satisfies the equation $i^2 = -1$. Multiplication is denoted by $(a+bi)(c+di) = (ac-bd) + (ad+bc)i$, and addition is denoted by $(a+bi) + (c+di) = (a+c) + (b+d)i$.

congruent. Two shapes in the plane or in space are congruent if there is a rigid motion that identifies one with the other (see the definition of *rigid motion*).

conjecture. An educated guess.

coordinate system. A rule of correspondence by which two or more quantities locate points unambiguously and which satisfies the further property that points unambiguously determine the quantities; for example, the usual Cartesian coordinates x, y in the plane.

cosine. $\cos(\theta)$ is the x -coordinate of the point on the unit circle so that the ray connecting the point with the origin makes an angle of θ with the positive x -axis. When θ is an angle of a right triangle, then $\cos(\theta)$ is the ratio of the adjacent side with the hypotenuse.

dilation. In geometry, a transformation D of the plane or space is a dilation at a point P if it takes P to itself, preserves angles, multiplies distances from P by a positive real number r , and takes every ray through P onto itself. In case P is the origin for a Cartesian coordinate system in the plane, then the dilation D maps the point (x, y) to the point (rx, ry) .

dimensional analysis. A method of manipulating unit measures algebraically to determine the proper units for a quantity computed algebraically. For example, velocity has units of the form length over time (e.g., meters per second [m/sec]), and acceleration has units of velocity over time; so it follows that acceleration has units $(m/sec)/sec = m/(sec^2)$.

expanded form. The expanded form of an algebraic expression is the *equivalent expression* without parentheses. For example, the expanded form of $(a + b)^2$ is $a^2 + 2ab + b^2$.

exponent. The power to which a number or variable is raised (the exponent may be any real number).

exponential function. A function commonly used to study growth and decay. It has the form $y = a^x$ with a positive.

factors. Any of two or more quantities that are multiplied together. In the expression 3.712×11.315 , the factors are 3.712 and 11.315.

function. A correspondence in which values of one variable determine the values of another.

geometric sequence. A sequence in which there is a common ratio between successive terms. Each successive term of a geometric sequence is found by multiplying the preceding term by the common ratio. For example, in the sequence $\{1, 3, 9, 27, 81, \dots\}$ the common ratio is 3.

histogram. A vertical block graph with no spaces between the blocks. It is used to represent frequency data in statistics.

inequality. A relationship between two quantities indicating that one is strictly *less than* or *less than or equal* to the other.

integers. The set consisting of the positive and negative whole numbers and zero; for example, $\{\dots, -2, -1, 0, 1, 2, \dots\}$.

irrational number. A number that cannot be represented as an exact ratio of two integers. For example, the square root of 2 or π .

linear expression. An expression of the form $ax + b$ where x is variable and a and b are constants; or in more variables, an expression of the form $ax + by + c$, $ax + by + cz + d$, etc.

linear equation. An equation containing linear expressions.

logarithm. The inverse of exponentiation; for example, $a^{\log_a x} = x$.

mean. In statistics, the average obtained by dividing the sum of two or more quantities by the number of these quantities.

median. In statistics, the quantity designating the middle value in a set of numbers.

mode. In statistics, the value that occurs most frequently in a given series of numbers.

monomial. In the variables x, y, z , a monomial is an expression of the form $ax^m y^n z^k$, in which m, n , and k are nonnegative integers and a is a constant (e.g., $5x^2, 3x^2y$ or $7x^3yz^2$).

nonstandard unit. Unit of measurement expressed in terms of objects (such as paper clips, sticks of gum, shoes, etc.).

parallel. Given distinct lines in the plane that are infinite in both directions, the lines are parallel if they never meet. Two distinct lines in the coordinate plane are parallel if and only if they have the same slope.

permutation. A permutation of the set of numbers $\{1, 2, \dots, n\}$ is a reordering of these numbers.

polar coordinates. The coordinate system for the plane based on $r\theta$, the distance from the origin and θ , and the angle between the positive x -axis and the ray from the origin to the point.

polar equation. Any relation between the polar coordinates (r, θ) of a set of points (e.g., $r = 2\cos\theta$ is the polar equation of a circle).

polynomial. In algebra, a sum of monomials; for example, $x^2 + 2xy + y^2$.

prime. A natural number p greater than 1 is prime if and only if the only positive integer factors of p are 1 and p . The first seven primes are 2, 3, 5, 7, 11, 13, 17.

quadratic function. A function given by a polynomial of degree 2.

random variable. A function on a probability space.

range. In statistics, the difference between the greatest and smallest values in a data set. In mathematics, the image of a function.

ratio. A comparison expressed as a fraction. For example, there is a ratio of three boys to two girls in a class $(3/2, 3:2)$.

rational numbers. Numbers that can be expressed as the quotient of two integers; for example, $7/3, 5/11, -5/13, 7 = 7/1$.

real numbers. All rational and irrational numbers.

reflection. The reflection through a line in the plane or a plane in space is the transformation that takes each point in the plane to its mirror image with respect to the line or its mirror image with respect to the plane in space. It produces a mirror image of a geometric figure.

rigid motion. A transformation of the plane or space, which preserves distance and angles.

root extraction. Finding a number that can be used as a factor a given number of times to produce the original number; for example, the fifth root of $32 = 2$ because $2 \times 2 \times 2 \times 2 \times 2 = 32$.

rotation. A rotation in the plane through an angle θ and about a point P is a rigid motion T fixing P so that if Q is distinct from P , then the angle between the lines PQ and $PT(Q)$ is always θ . A rotation through an angle θ in space is a rigid motion T fixing the points of a line l so that it is a rotation through θ in the plane perpendicular to l through some point on l .

scalar matrix. A matrix whose diagonal elements are all equal while the nondiagonal elements are all 0. The identity matrix is an example.

scatterplot. A graph of the points representing a collection of data.

scientific notation. A shorthand way of writing very large or very small numbers. A number expressed in scientific notation is expressed as a decimal number between 1 and 10 multiplied by a power of 10 (e.g., $7000 = 7 \times 10^3$ or $0.0000019 = 1.9 \times 10^{-6}$).

similarity. In geometry, two shapes R and S are similar if there is a dilation D (see the definition of *dilation*) that takes S to a shape congruent to R . It follows that R and S are similar if they are congruent after one of them is expanded or shrunk.

sine. $\sin(\theta)$ is the y -coordinate of the point on the unit circle so that the ray connecting the point with the origin makes an angle of θ with the positive x -axis. When θ is an angle of a right triangle, then $\sin(\theta)$ is the ratio of the opposite side with the hypotenuse.

square root. The square roots of n are all the numbers m so that $m^2 = n$. The square roots of 16 are 4 and -4. The square roots of -16 are $4i$ and $-4i$.

standard deviation. A statistic that measures the dispersion of a sample.

symmetry. A symmetry of a shape S in the plane or space is a rigid motion T that takes S onto itself ($T(S) = S$). For example, reflection through a diagonal and a rotation through a right angle about the center are both symmetries of the square.

system of linear equations. Set of equations of the first degree (e.g., $x + y = 7$ and $x - y = 1$). A solution of a set of linear equations is a set of numbers a, b, c, \dots so that when the variables

are replaced by the numbers all the equations are satisfied. For example, in the equations above, $x = 4$ and $y = 3$ is a solution.

translation. A rigid motion of the plane or space of the form X goes to $X + V$ for a fixed vector V .

transversal. In geometry, given two or more lines in the plane a transversal is a line distinct from the original lines and intersects each of the given lines in a single point.

unit fraction. A fraction whose numerator is 1 (e.g., $\frac{1}{\pi}, \frac{1}{3}, \frac{1}{x}$). Every nonzero number may be written as a unit fraction since, for n not equal to 0, $n = 1/(1/n)$.

variable. A placeholder in algebraic expressions; for example, in $3x + y = 23$, x and y are variables.

vector. Quantity that has magnitude (length) and direction. It may be represented as a directed line segment.

zeros of a function. The points at which the value of a function is zero.